School of Electronics and Computer Science
University of Southampton

COMP6212 Computational Finance 2015/16, Assignment 3 (of 4) | 15% |

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The relationship between the price of an option contract and the underlying parameters relating to the exercise of the option is a complex one. Hutchinson *et al.* [1] show that this relationship can be reasonably approximated by nonparametric neural network type models. In particular, they train a Radial Basis Functions (RBF) model on data simulated from a Black-Scholes model and show that not only good approximations to the options price can be obtained, but also the sensitivity *Delta* ($\Delta = \partial C / \partial S$) can be relaibly extracted from it. In this assignment we will explore if some of the claims made in their paper are true.

1. Study the paper by Hutchinson *et al.* [1]. You need to focus on Figures 4 and 5, and how these were arrived at. Skip their evaluation part beyond this point.
2. Construct a dataset by generating call option prices from the Black-Scholes formula. You may take input parameters (strike prices, interest rates, volatilities and the underlying asset price) from the data used in Assignment 2.

   With this data, reproduce Fig. 4 and Fig. 5 in [1] by training an RBF model (Eqn. 9 in [1]).

   - With the data vector $\boldsymbol{x} = [\, S/X \quad (T-t)\,]^T$, the RBF model used is
   $$c = \sum_{j=1}^{J} \lambda_j \, \phi_j(\boldsymbol{x}) + \boldsymbol{w}^T \boldsymbol{x} + w_0$$
   where the nonlinear function $\phi_j(\boldsymbol{x}) = \Big[(\boldsymbol{x} - \boldsymbol{m}_j)^T \, \boldsymbol{\Sigma_j} \, (\boldsymbol{x} - \boldsymbol{m}_j) + b_j \Big]^{1/2}$ is a local Mahalanobis distance with a bias term and $J = 4$. In this task, we will ignore the bias terms within the nonlinearity (*i.e.* $b_j = 0$).
   - Use the `MATLAB` function `fitgmdist` for fitting a Gaussian mixture model and extract the parameters $\boldsymbol{m}_j$ and $\boldsymbol{\Sigma}_j$ of the nonlinear terms in the model.
   - Now construct the *design matrix* of inputs – this matrix will have as many rows as you have data points and seven columns (four for mapping the data to each of the nonlinear distances, two for the two data dimensions of the linear part and the last column will have ones which multiply w_0).
   - Estimate the weights of the model ($\lambda_j, j = 1, ..., 4;\ \boldsymbol{w}$ and w_0) by solving the resulting least squares problem.
3. How well does such a non-parametric model learn the data given to you in Assignment 2?

Report

Write a report of no more than four pages describing the work you have done, and answering any questions above.

This assignment is worth 15% of the assessment for the module. It is recommended you spend no more than 15 hours on this assignment. There will be a special help session for students who have not done one of the Machine Learning modules where RBF models were taught.

References

[1] J. Hutchinson, A. Lo, and T. Poggio, "A nonparametric approach to pricing and hedging derivative securities via learning networks," *The Journal of Finance*, vol. 49, no. 3, pp. 851–889, 1994.

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