School of Electronics and Computer Science
University of Southampton

COMP6212 Computational Finance 2015/16, Assignment 2 (of 4) 30%

Issue	02 March 2016
Due	19 April 2016

Data

Some prices of call and put options written on the `FTSE100` Index are available in `http://users.ecs.soton.ac.uk/mn/assignment2data/`. These are daily data during the period February to December 1994. The date of maturity is the day following the last date in the time series. There are ten data files with names `c2925.prn` etc., where the initial letter `c` stands for call option and `p` denotes a put option. The number (`2925`) is the strike price of the option. Each file has three columns: the date, the price of the option and the value of the underlying asset (the FTSE index). The annual interst rate during this period may be assumed to be 6.0%.

Tasks

[**5 Marks**] (a) A trader buys a call option with a strike price of £45 and a put option with a strike price of £40. Both options are on the same underlying asset and have the same maturity. Draw a graph of the trader's profit as a function of the asset price when the options mature.

(b) Study how the Black-Scholes model of pricing options was derived before attempting this task. The notation is : K, the strike price; S, the value of the underlying asset; r the risk-free interest rate; T, the time of maturity; t, the current time and $\mathcal{N}(x)$, the cummulative normal distribution.

1. Write the expression for $\mathcal{N}'(x)$ (the derivative of $\mathcal{N}(x)$).
2. Show that $S\mathcal{N}'(d_1) = K\exp(-r(T-t))\mathcal{N}'(d_2)$ where d_1 and d_2 were defined as:

$$\begin{aligned} d_1 &= \frac{\log(S/K) + (r+\sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}} \\ d_2 &= d_1 - \sigma\sqrt{(T-t)} \end{aligned}$$

3. Calculate the derivaties $\partial d_1/\partial S$ and $\partial d_2/\partial S$
4. With the solution for the call option price given by

$$c = S\mathcal{N}(d_1) - K\exp(-r(T-t))\mathcal{N}(d_2)$$

show that its derivative with respect to time is

$$\frac{\partial c}{\partial t} = -rK\exp(-r(T-t))\mathcal{N}(d_2) - S\mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{(T-t)}}$$

Show that $\partial c/\partial S = \mathcal{N}(d_1)$

5. Differentiating again to get

$$\frac{\partial^2 c}{\partial S^2} = \mathcal{N}'(d_1)\frac{1}{S\sigma\sqrt{(T-t)}},$$

and substituting in the relevant expression, show that the expression for the call option price indeed the solution to the Black-Scholes differential equation.

[**5 Marks**] Evaluate how well the option prices in the given data satisfy the Black-Scholes model. For each option, evaluate the price given by Black-Scholes from $T/4 + 1$ to T (the length of the time series), using volatility estimated using a sliding window of range $t - T/4$ to t. Compare the price obtained by using the formula with the true price of the option. Are there any systematic differences? For estimating volatility from historic data, see for example Hull [**?**] Section 13.4 or a similar source.

[**5 Marks**] On a random set of 30 days in the range $T/4 + 1 : T$, compute the *implied volatilities* and plot them as scatter plot against the corresponding volatilities you estimated from data. For prices on any particular day, is there any systematic variation of implied volatilities computed from options with different strike prices (Hint: Look up the term *volatility smile*)?

[**5 Marks**] Explain *in your own words* why an option with American style exercise is always worth at least as much as an option with European style exercise on the same asset with the same strike price and date of maturity.

[**5 Marks**] Compare pricing a call option with European style exercise using the Black-Scholes and Binomial lattice methods. Using *one* random set of values for the parameters (strike price, time to maturity, interest rate and volatility) taken from the data given to you, evaluate how the binomial lattice method approximates Black-Scholes as the step time δt is decreased. Plot a graph of the absolute difference between the two methods as a function of δt.

[**5 Marks**] Consider pricing an option using a binomial lattice. The code for pricing a put option with American style exercise included the following lines[**?**]:

```
[...]
for tau=1:N
    for i= (tau+1):2:(2*N+1-tau)
        hold = p_u*PVals(i+1) + p_d*PVals(i-1);
        PVals(i) = max(hold, K-SVals(i));
    end
end
[...]
```

Explain *in your own words* the steps involved in this part of the code. How will this change if you were pricing a call option (with American style exercise)?

Report

Write a report of no more than eight pages describing the work you have done, and answering any questions above.

Note this assignment is worth 30% of the assessment for the module. It is recommended you spend 30 hours on this assignment.

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