Evolutionary game theory

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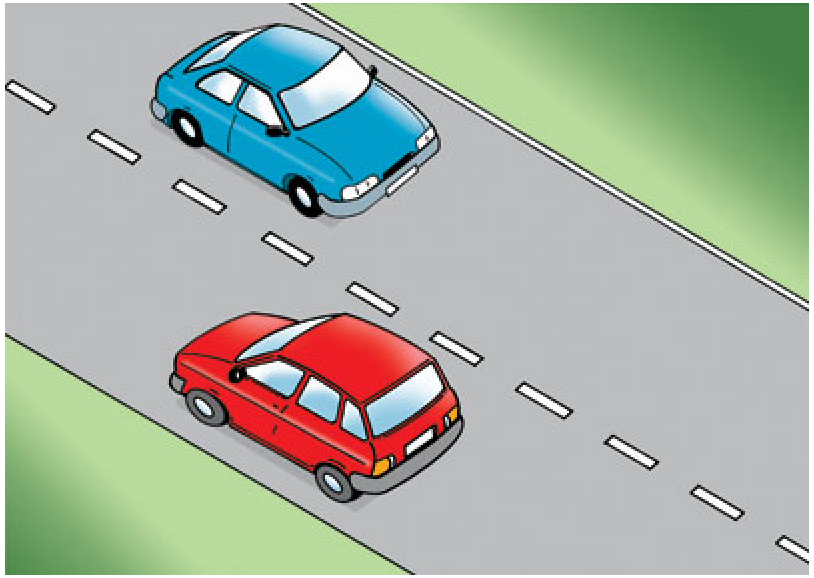
Outline

- What do we mean by "game"?
- Brief introduction to game theory.
- An example game: the Prisoner's Dilemma.
- Representing games: matrices and trees.
- Solving games via Nash equilibria.
- Using game theory in evolutionary biology.
- A biological example: the Hawk-Dove game.
- Evolutionarily stable strategies and mixed-strategy equilibria.
- The replicator dynamics.
- Other aspects of evolutionary game theory: advantages over traditional theory, repeated games, spatial games, etc.

What is a game?

- Strategic interaction between two or more players that has consequences for all.
- Everyday examples: chess, poker, Monopoly, rock-paper-scissors.
- Economics: rival petrol station owners deciding what to charge per litre.
- Biology: a young male chimpanzee deciding whether to challenge the alpha male for access to females.

A coordination game



- Two cars approach each other at high speed on a two-lane road.
- Each driver must decide whether to stay to the left or the right.
- If both choose left, or both choose right, they pass safely.
- If one chooses left and the other right, there is a crash.

- In the UK, this game is handled via the social convention of driving on the left.
- Note that this kind of problem is different to optimizing against the environment (e.g., finding the shortest route, designing a more streamlined aeroplane) because the optimal strategy depends on what your opponent will do.

History of game theory

- Ideas mentioned as early as Plato: soldiers considering whether to fight or flee.
- Important in the political philosophy of Thomas Hobbes (1651): the social contract.
- Formalized by John von Neumann and Oskar Morgenstern (1944).
- Hugely influential during the Cold War: the doctrine of "mutually assured destruction".

Maximizing utility

- To get started, we need to know how each agent ranks the outcomes of a game, i.e., its preferences.
- In the coordination game, for instance, it's assumed that we all prefer passing safely to crashing.
- Even better if we can score game outcomes on a single dimension: *utility*.
- In psychological contexts, this roughly equates to happiness. In economic contexts, it could be money.
- In biological applications utility maps very well to fitness.

The Prisoner's Dilemma

- Two people have been arrested for a robbery.
- The police don't have enough evidence to convict them for the robbery, but can get them for theft of the getaway car (2 year sentence).
- Both are questioned in separate rooms.
- Each is offered the deal "testify against your partner, and you can go free."

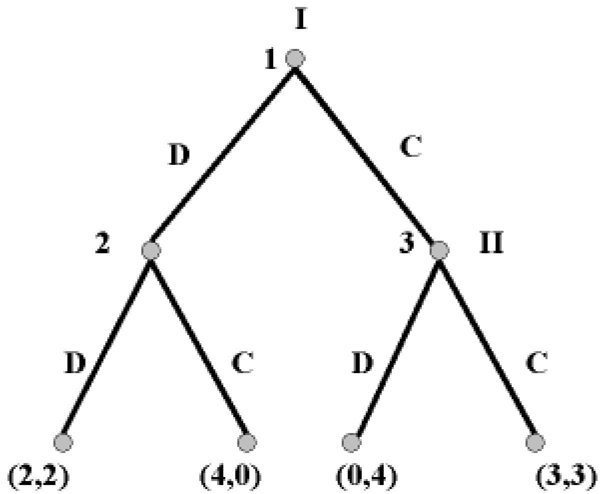
- But if they both betray each other, they'll both go to jail (5 years).
- If one betrays the other, the partner gets 10 years and they go free.
- "Do I trust my partner to keep quiet, then we both get only 2 years, or do I betray him, hoping he won't betray me, and try to go free?"
- Here is their utility function:
    - Go free = 4 units (most preferred option).
    - 2 years in prison = 3 units.
    - 5 years in prison = 2 units.
    - 10 years in prison = 0 units.
- What should they do?

Representing the game as a matrix

	Defect	Cooperate
Defect	(2,2)	(4,0)
Cooperate	(0,4)	(3,3)

- Also called the normal-form or strategic-form representation.
- Player one's strategy determines the row; player two's strategy determines the column.
- Payoffs written as (P1, P2) or sometimes just as P1 if symmetrical.

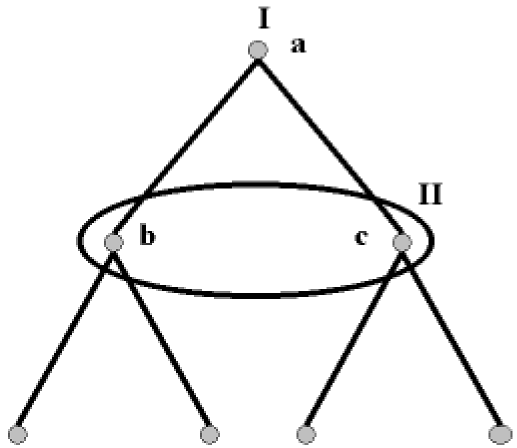
Representing the game as a tree



- Nodes represent decision points, labelled by player.
- Terminal nodes represent game outcomes.

- The normal-form loses some detail and implies simultaneous decision-making.
- The extensive form spells out the sequence of actions.
- In both cases, a "strategy" is a full description of behaviour at every possible choice point.
- Note the difficulty of representing some everyday games (e.g., chess, monopoly) in either normal or extended format: the matrix or tree would have to be very large.

Games and information



- The circled nodes are an "information set".
- Player 2 must make a choice not knowing what player 1 has done.

"Solving" games: Nash equilibria

- John Nash (1950): a solution to a game is a set of strategies where no player is tempted to deviate.
- Nobody can, through a unilateral change in strategy, get any more utility than what they're getting now.
- "Rational" players will reason the same way and get to one of these equilibria.

- Look at the payoff matrix again:

	Defect	Cooperate
Defect	(2,2)	(4,0)
Cooperate	(0,4)	(3,3)

- C-C may look tempting, but each player has an incentive to switch strategies to D, as 4 > 3.
- C-D and D-C are no good as the player getting exploited has an incentive to switch (2 > 0).
- Only D-D is a Nash equilibrium. (This was seen by many as a depressing result!)

Evolution and game theory

- John Maynard Smith (1982): can we apply game theory to problems concerning the evolution of animal behaviour?
- No need to consider rational deliberation over strategies.
- A population of individuals repeatedly play games over evolutionary time: all sorts of different strategies are present initially.
- By standard evolutionary logic, strategies that do well will tend to become more prevalent: biological fitness is our utility function.
- When the population reaches a stable state, which strategies will be present?

Games in the biological world

- Contests over resources.
- Mate choice and sexual display.
- Sex ratio "decisions" by mothers.
- Parent-offspring conflict.
- Herding behaviour.
- Collective anti-predator vigilance and signalling.
- Reciprocal grooming.

The Hawk-Dove game

- Animals in a population must decide whether to be aggressive (Hawk) or passive (Dove) in resolving contests over resources.
- Two animals meet to contest a resource worth 10 units.
- Hawks always fight, Doves always surrender the resource.
- When two Hawks fight, the winner gets the resource but the loser is injured (-50 units). Each is equally likely to win or lose.
- A Hawk takes the resource from a Dove.
- Two Doves split the resource equally.

- The payoff matrix for the Hawk-Dove game:

	Hawk	Dove
Hawk	(-20, -20)	(10, 0)
Dove	(0, 10)	(5, 5)

- Note that -20 is the average of +10 and -50.
- Where will the population end up? All Hawks? All Doves?

Two views of strategy evolution

- Approach 1: inspired by the Nash equilibrium concept, focus on the stable states of the population.
- Over evolutionary time, we should end up at a stable state so this is a reasonable simplification.
- Approach 2: treat the competing strategies as part of a dynamical system and study the trajectory through state-space.

Evolutionarily stable strategies

- A strategy is an ESS if no mutant could invade a population playing that strategy.
- Call the population strategy S and the mutant strategy M.
- F(X,Y) is the fitness expected by an animal playing X meeting an animal playing Y.
- S is an ESS if F(S,S) > F(S,M) (i.e., S is a "best reply" to itself).
- Or, if F(S,S) = F(S,M) but F(S,M) > F(M,M).

	Hawk	Dove
Hawk	-20	10
Dove	0	5

- Hawk cannot be an ESS: Hawks meeting Hawks expect a payoff of -20, but a Dove mutant would get 0 against Hawks and could therefore invade.
- Dove cannot be an ESS: Doves meeting Doves expect 5, but a Hawk mutant would get 10 against Doves and could thus invade.
- There is no ESS in the Hawk-Dove game if we are limited to "pure strategies". So a real population would have both Hawks and Doves. Can we say any more than this?

Mixed strategy equilibria

- What if the animals can randomize their strategies, playing Hawk some of the time and Dove the rest of the time?
- This expansion of the strategy space changes things.
- Assume there is an ESS where animals play Hawk with probability p and Dove with probability (1-p).
- At this ESS there must be no temptation to play either strategy a little more often, or that's where the population would go.
- We are therefore looking for a value of p where the expected payoffs for playing Hawk and for playing Dove ***are the same***.

- Expected payoff for playing Hawk = (p)(-20) + (1-p)(10), because I will meet a Hawk (p) times and a Dove (1-p) times.
- Expected payoff for playing Dove = (p)(0) + (1-p)(5)... same logic.
- Set the two expected payoffs equal and solve for p:
    - $-20p + 10 - 10p = 5 - 5p$
    - $p = -5 / -25$
    - $p = 0.2$.
- So the ESS is a mixed strategy equilibrium where the animals play Hawk 20% of the time and Dove 80% of the time.
- This could also be interpreted as a polymorphic population: 20% Hawks and 80% Doves.

The replicator dynamics

$$\frac{dp_c}{dt} = \frac{p_c(W_C - \overline{W})}{\overline{W}}$$

- A system of differential equations describing the change in the frequency of strategies over time.
- The rate of change of a strategy's frequency is proportional to its frequency multiplied by its relative difference from average fitness.
- In the Hawk-Dove game the predictions are the same, but this is not always so as trajectories can be complex.

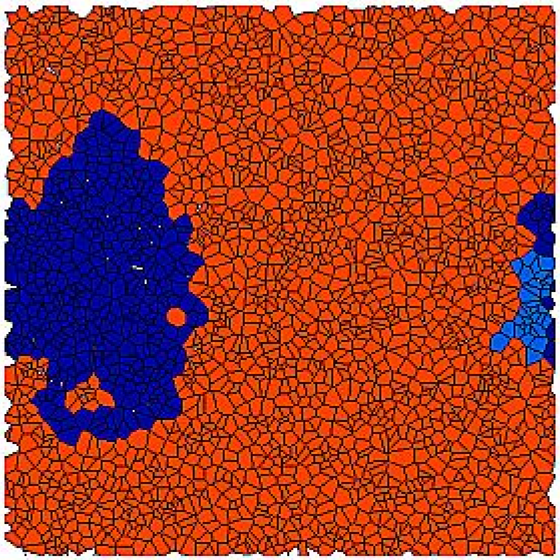
Advantages of evolutionary game theory

- The evolutionary process does not have to be biological: it could be a model of learning or of cultural evolution.
- There is no need for debates on how to define a "rational player". Strategies persist if they do well.
- Ironically these two factors have meant that evolutionary game theory is now applied in areas like economics.
- There is room to move beyond the static equilibrium focus of traditional game theory, through the replicator dynamics.

Repeated games

- Games like the Prisoner's Dilemma can be very different, strategically, if they are played repeatedly by the same players, and those players can remember the results of previous rounds.
- The equilibria of a repeated game may be very different from those of its single-shot version.

Correlated play and spatial games



- A prisoner's dilemma, cooperators in red, defectors in blue.
- Play in a two-dimensional space means that you are more likely to encounter a genetic relative who plays the same strategy as you.
- This makes it possible for cooperation to survive.
- Image from Flache & Hegselmann (2001), JASSS.

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