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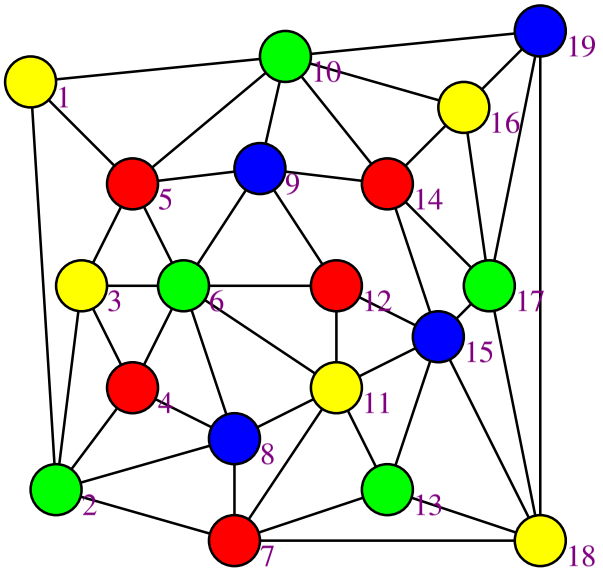
- Many problems have very simple, natural representations as a string
- Graph colouring is an example of this
- We can use standard string based mutation and crossover operators on these natural representation



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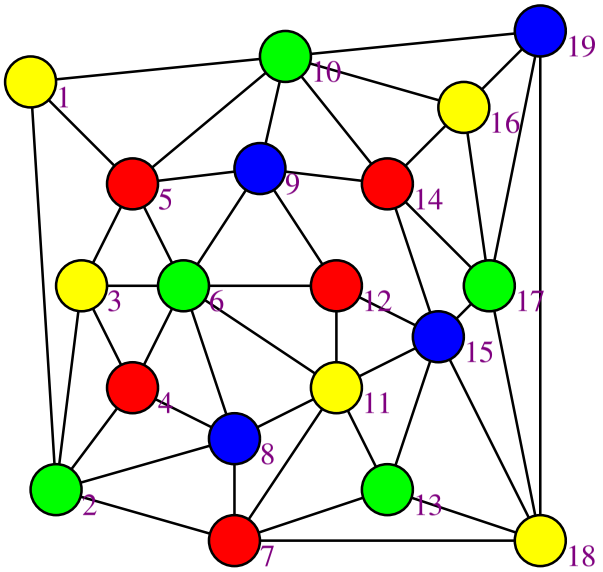

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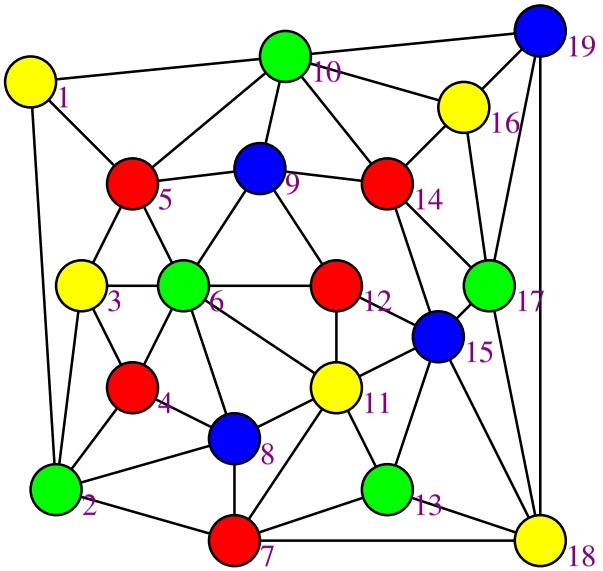

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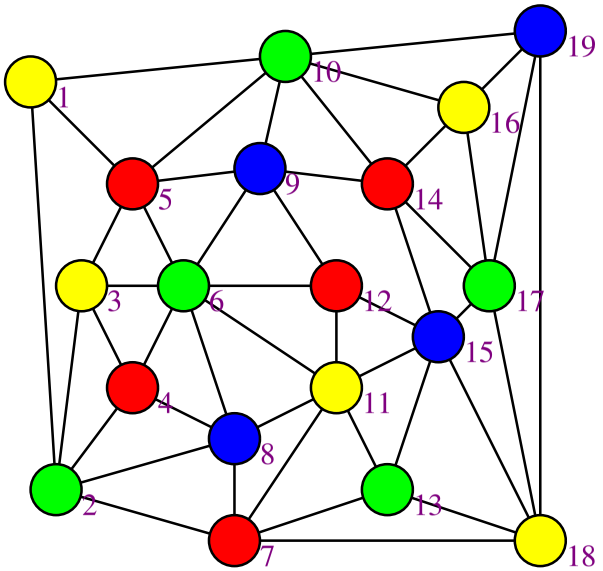
- For graph colouring these operators don't work well (c.f. Galinier and Hao's crossover)



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TSP and GAs

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- The most natural representation is a permutation of $(1, 2, 3, \cdots, n)$

- However normal mutation and crossover doesn't work

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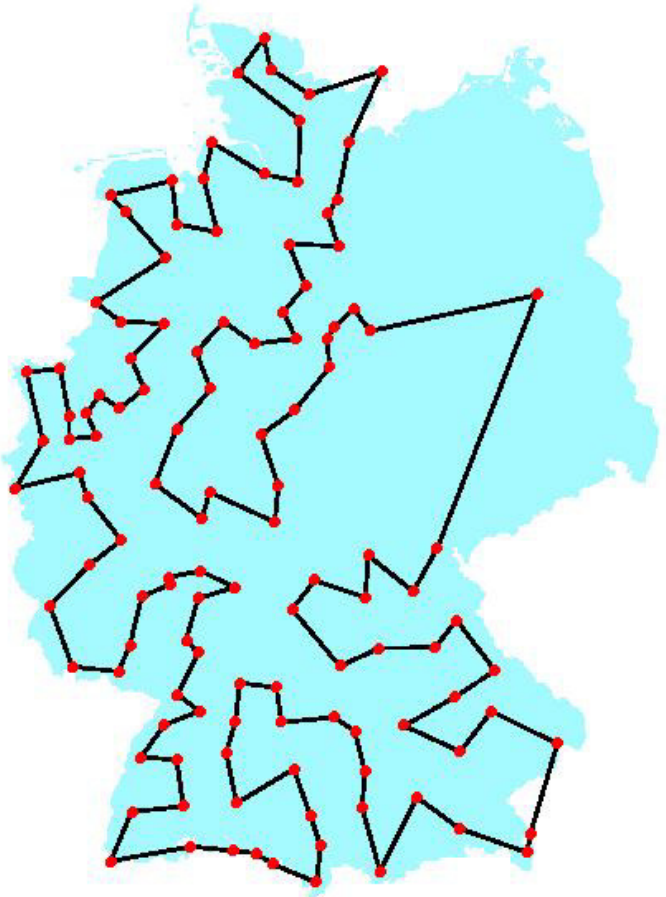

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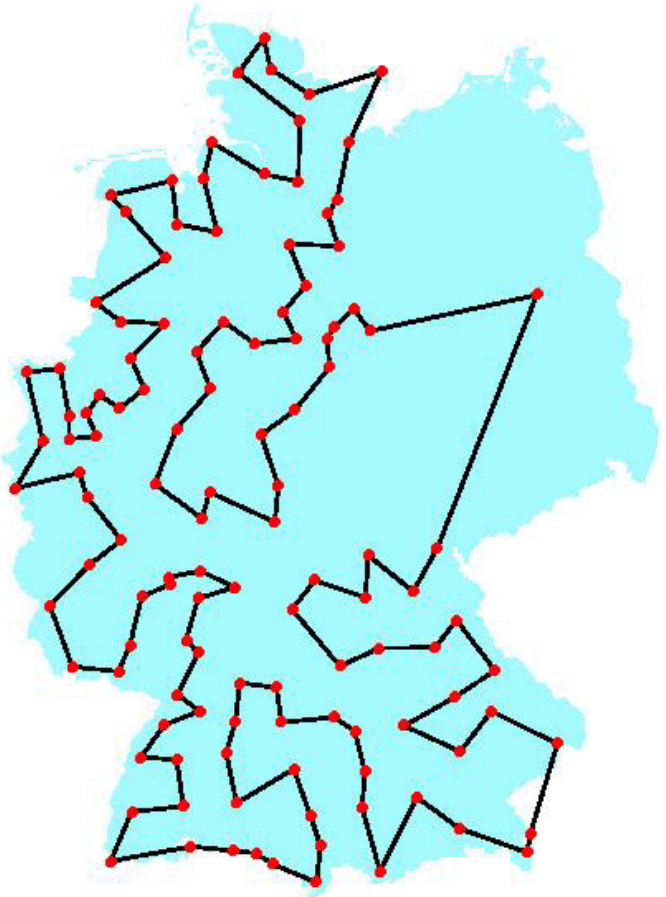

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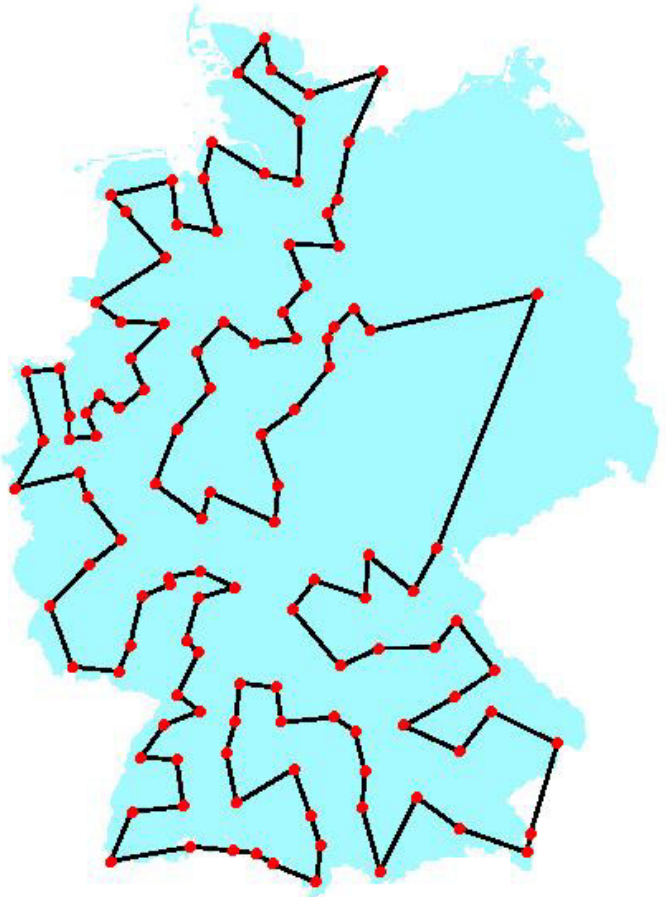

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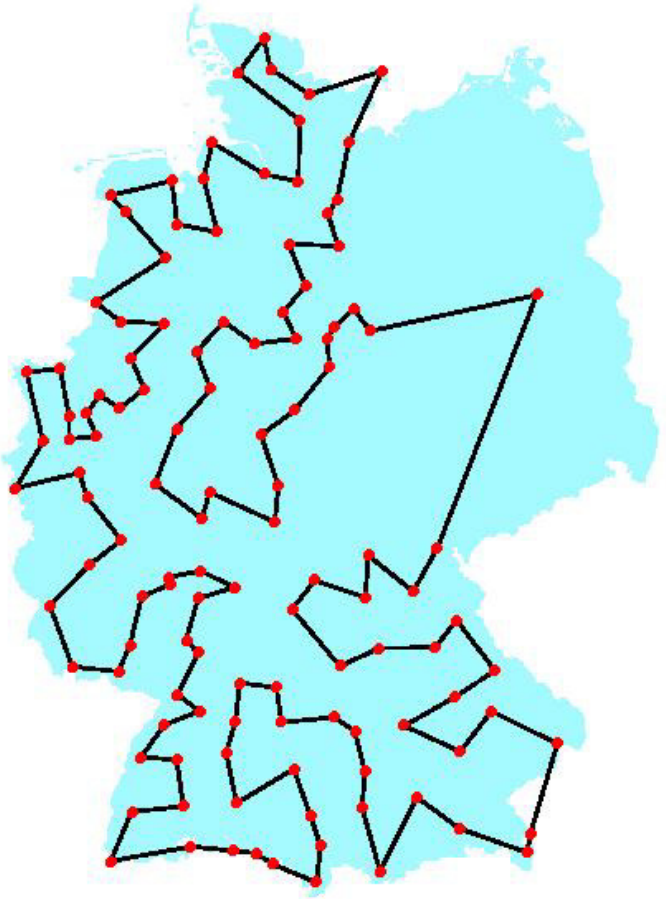

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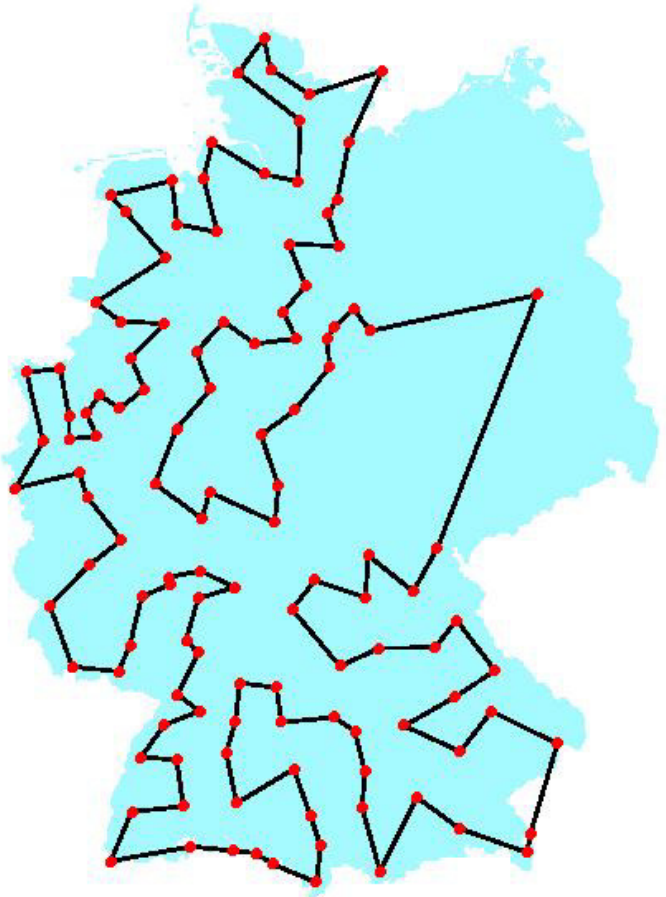

Changing the Representation

- A number of different representations for the TSP have been proposed so that point mutation and one or two-point crossover work
- Unfortunately, such representations don't lead to good GAs
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- These attempts miss the main point of why GAs (sometimes) work
- Instead they try to make traditional string based operators work on a permutation based problem
- GAs are a heuristic search mechanism which work by exploring points in the search space that have similar cost to previous points
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Edge-Based Operators

- Since it is the edges which determines the cost it seems sensible to build operators to preserve as many edges as possible
- An example of an operator which does this is *edge recombination*
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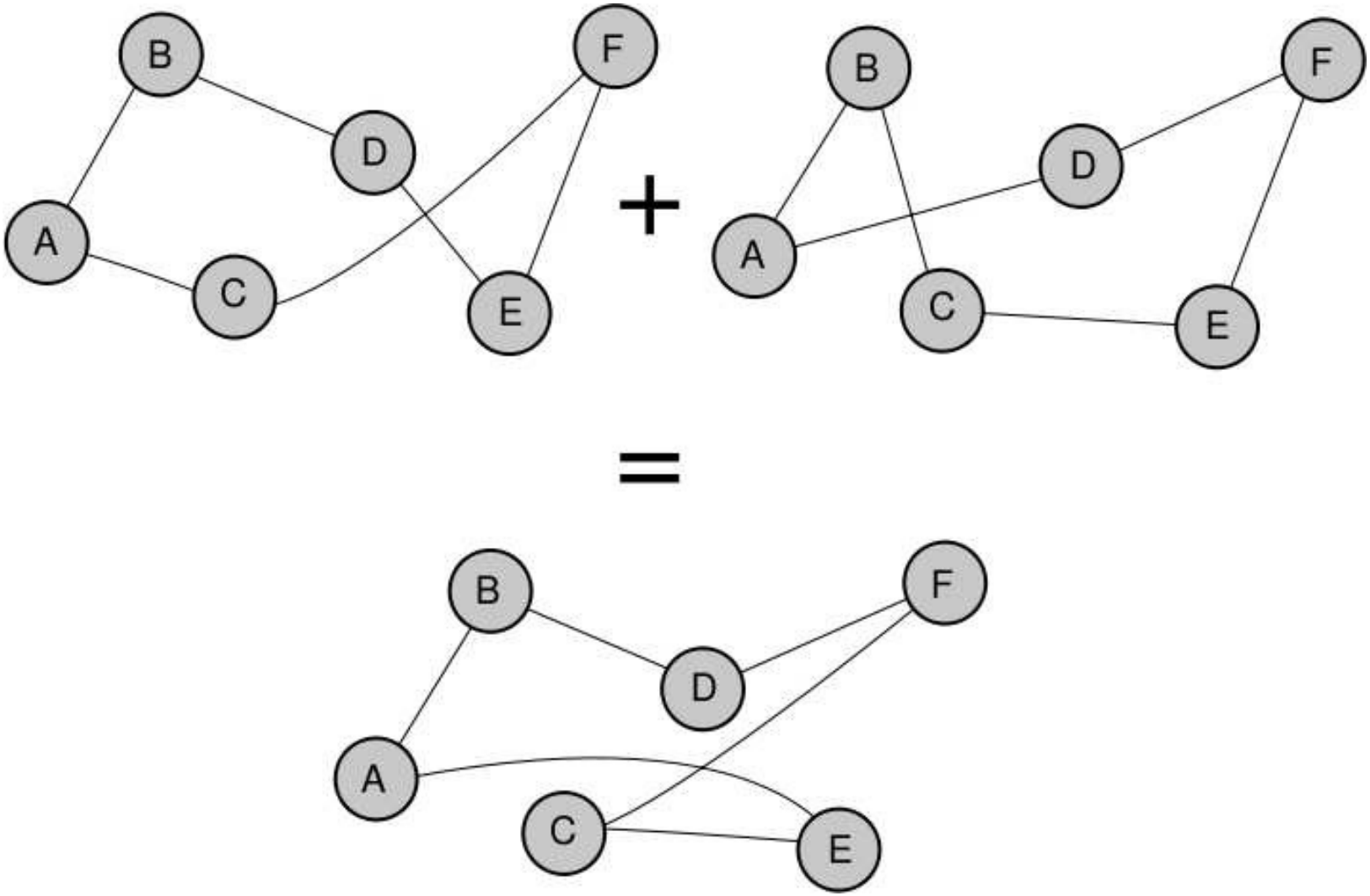

Mutation

- Mutation is also non-trivial in TSP
- Point mutations make no sense
- Swapping two cities

$$(1,2,3,4,5,6) \overset{swap\ mutation}{\longrightarrow} (1,5,3,4,2,6)$$

  disrupts four edges 1–5, 5–3, 4–2 and 2–6

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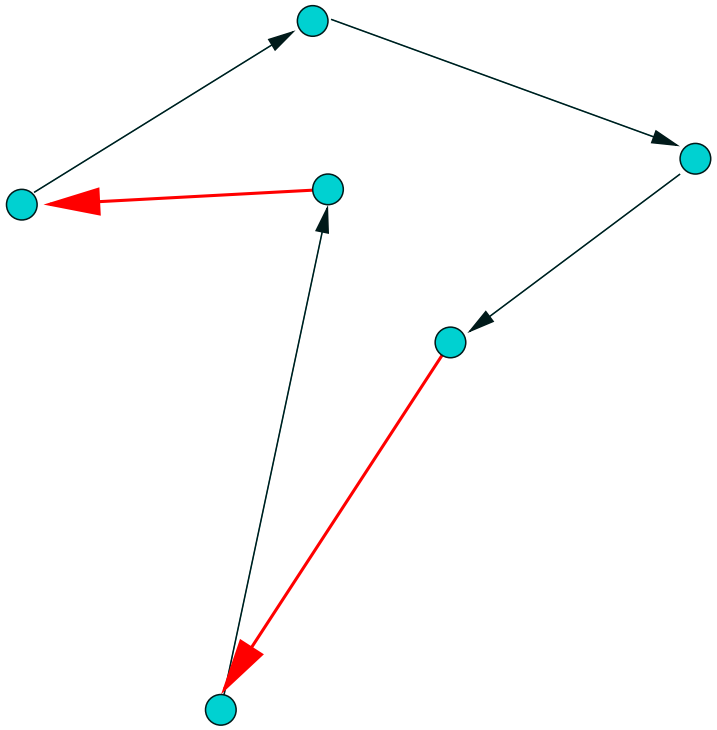

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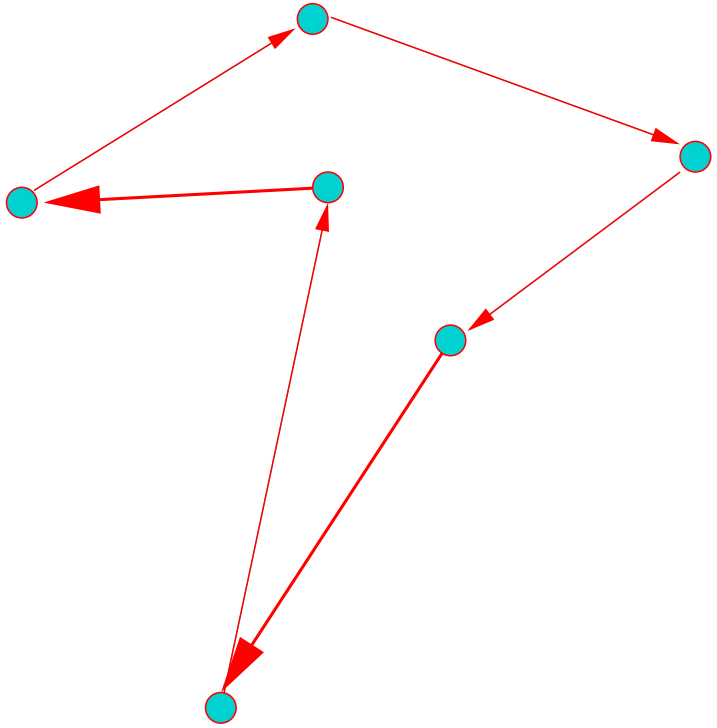

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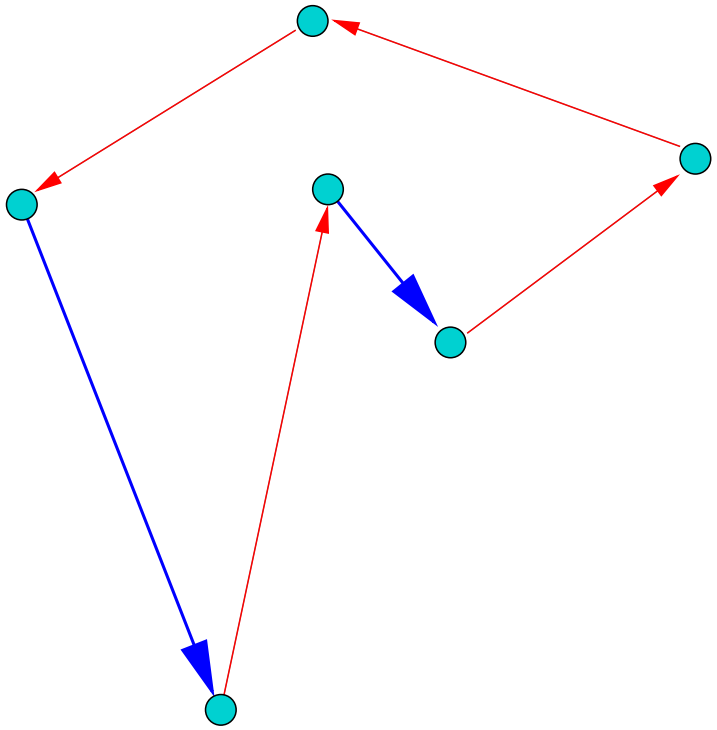

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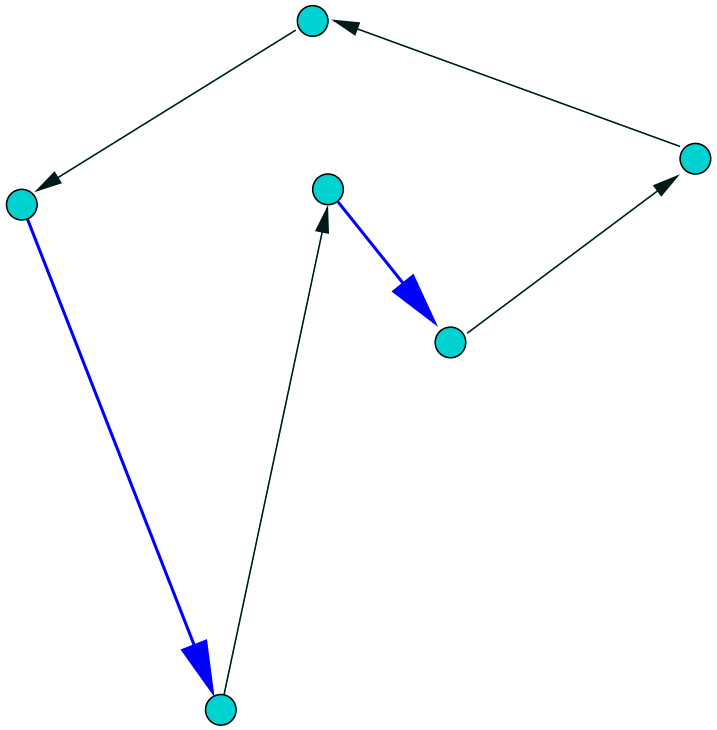

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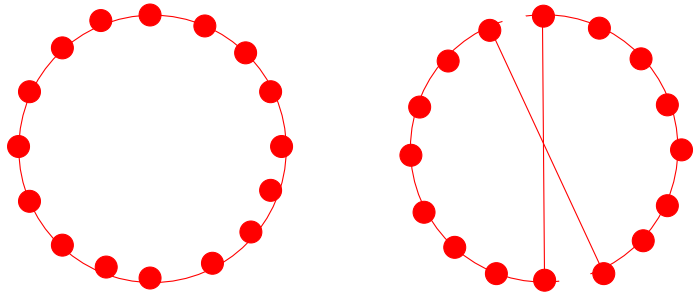

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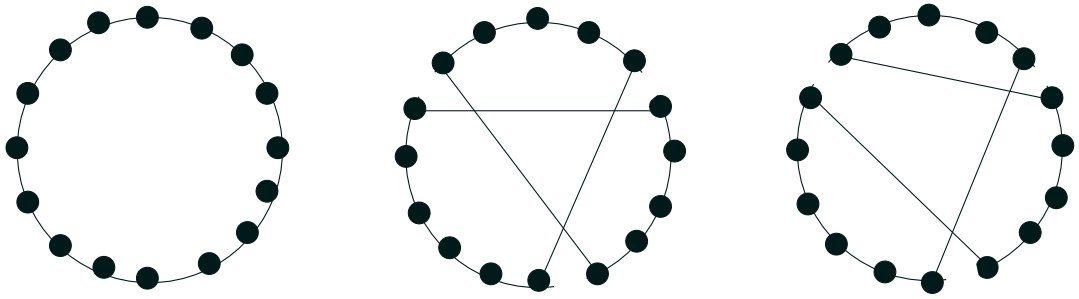

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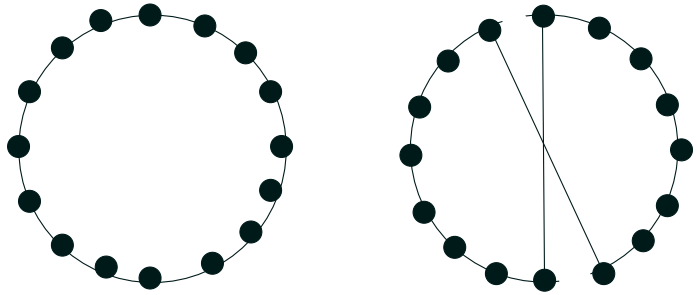

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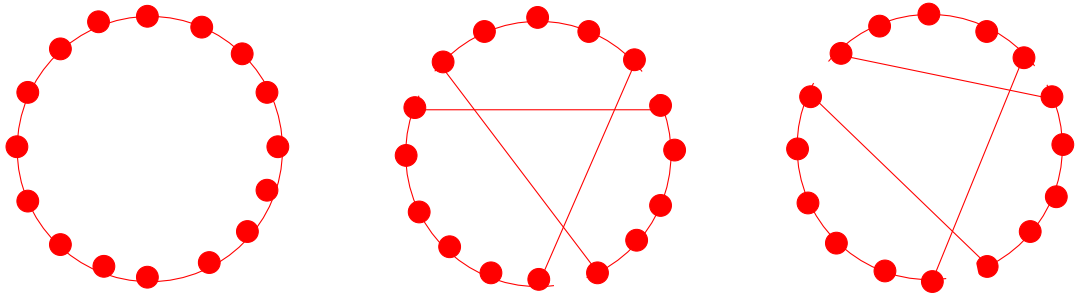

Neighbourhood Search

- Using 2-Opt and 3-Opt provides a fast neighbourhood search
- Neighbourhood search is fast because it is relatively cheap to recompute the cost after performing a 2-Opt or 3-Opt move
- The classic search method for TSP is the $Lin\text{-}Kernighan\ method$ which uses 2-Opts, 3-Opts and a specialised 4-Opt which combines two 2-Opts
- The best heuristic to date for finding good solutions for TSP is $Iterated\ Lin\text{-}Kernighan$
- For any GA to be competitive with the state-of-the-art it has to include a fast neighbourhood search