ECE/CS/ME 539 Introduction to Artificial Neural Networks

Homework 02

The homework is graded on *completion only* scale. If an answer to the problem exists, full credit will be given regardless the correctness of the answer. No answer will receive no credit though.

Coverage: Probability, Statistics, Information Theory

1. (6 pts) Read *Dive into Deep Learning,* 2.6 Probability and Statistics.
   $2.6.5 An example (have been worked out in-class).

   Assume that a doctor administers an HIV test to a patient. This test is fairly accurate, and it fails only with 1% probability if the patient is healthy but reporting him as diseased. Moreover, it never fails to detect HIV if the patient has it. We use $D \in \{0,1\}$ to indicate the diagnosis (0 if negative and 1 if positive) and $H \in \{0,1\}$ to denote the HIV status.

Conditional Probability	$H = 1$	$H = 0$
$P(D = 1 \mid H)$	1	0.01
$P(D = 0 \mid H)$	0	0.99

   Note that the column sums are all 1 (but the row sums don't) since they are conditional probabilities. Let's compute the probability of the patient having HIV if the test comes back positive, i.e., $P(H = 1 \mid D = 1)$. Intuitively this is going to depend on how common the disease is, since it affects the number of false alarms. Assume that the population is fairly healthy, e.g., $P(H = 1) = 0.0015$. To apply Bayes' theorem, we need to apply marginalization to determine

$$
\begin{aligned}
P(D=1) &= P(D=1, H=0) + P(D=1, H=1) \\
&= P(D=1 \mid H=0)P(H=0) + P(D=1 \mid H=1)P(H=1) \\
&= (0.01)\cdot(1-0.0015) + (1)\cdot(0.0015) = 0.009985 + 0.0015 = 0.11485
\end{aligned}
$$

	H=0	H=1
D=0	0.9	0.01
D=1	0.1	0.99

   This leads to

$$
P(H=1 \mid D=1) = \frac{P(D=1 \mid H=1)P(H=1)}{P(D=1)} = \frac{(1)\cdot(0.0015)}{0.11485} = 0.13061
$$

   Problem 7, $2.6.8 Exercise:

   Assume that the outcomes of the two tests are not independent that either test on its own has a false positive rate of 10% and a false negative rate of 1%. That is, assume that $P(D = 1 \mid H = 0) = 0.1$ and that $P(D = 0 \mid H=1) = 0.01$. Moreover, assume that for $H = 1$ (infected) the test outcomes are conditionally independent, i.e., that $P(D_1, D_2 \mid H = 1) = P(D_1 \mid H = 1) \cdot P(D_2 \mid H = 1)$ but that for healthy patients the outcomes are coupled via $P(D_1 = D_2 = 1 \mid H = 0) = 0.02$.

   (a) (3 pts) Work out the joint probability table for D_1 and D_2, given $H = 0$ based on the information you have so far.

D_1, D_2	$P(D_1, D_2 \mid H = 0)$	$P(D_1, D_2 \mid H = 1)$
0, 0	0.82	(0.01)(0.01) = 0.0001
0, 1	0.08	(0.01)(0.99) = 0.0099
1, 0	0.08	(0.99)(0.01) = 0.0099
1, 1	0.02	(0.99)(0.99) = 0.9801

(b) (3 pts) Derive the probability of the patient being positive ($H = 1$) after both tests return positive. *Hint: Refer to D2L, $2.6.5.*

2. (4 points) The table below lists the weather outlook, temperature, humidity, and wind conditions of 14 days and the outcome of whether a junior league will play footfall nor not.

Outlook	Temperature	Humidity	Wind	Played football(yes/no)
Sunny	Hot	High	Weak	No
Sunny	Hot	High	Strong	No
Overcast	Hot	High	Weak	Yes
Rain	Mild	High	Weak	Yes
Rain	Cool	Normal	Weak	Yes
Rain	Cool	Normal	Strong	No
Overcast	Cool	Normal	Strong	Yes
Sunny	Mild	High	Weak	No
Sunny	Cool	Normal	Weak	Yes
Rain	Mild	Normal	Weak	Yes
Sunny	Mild	Normal	Strong	Yes
Overcast	Mild	High	Strong	Yes
Overcast	Hot	Normal	Weak	Yes
Rain	Mild	High	Strong	No

Find the following empirical probabilities:

(a) (0.5 point) Pr.{Humidity = High} = $\boxed{1/2}$

(b) (0.5 point) Pr.{Outlook = Sunny AND Humidity = Normal} $\boxed{(5/\cancel{14}_2)(7/14) = \frac{5}{28}}$

(c) (0.5 points) Pr.{Temperature = Cool OR Wind = Weak} $\boxed{4/14 + 8/14 = 6/7}$

(d) (0.5 points) Pr.{Play = Yes | Humidity = High} = $\boxed{3/7}$

(e) (1 points) Pr.{Humidity = High | Play = Yes} = $\boxed{1/3}$

(f) (1 points) The entropy of the outcome of Play = $\boxed{0.282}$

(1a) $P(D=1 \mid H=0) = 0\ 1$

$P(D=0 \mid H=1) = 0\ 01$

$P(D_1=1, D_2=0 \mid H=0) = P(D_1=1 \mid H=0) - P(D_1=1, D_2=1 \mid H=0)$

$= 0\ 1 - 0\ 02$

$= 0\ 08$

$P(D_1=0, D_2=1 \mid H=0) = 0.08$

$P(D_1=0, D_2=0 \mid H=0) = P(D_1=0 \mid H=0) - P(D_1=0, D_2=1 \mid H=0)$

$= 0\ 9 - 0\ 08$

$= 0\ 82$

(1b) $P(H=1 \mid D_1=D_2=1) = \dfrac{P(D_1=1, D_2=1 \mid H=1)\, P(H=1)}{P(D_1=1, D_2=1)}$

$P(D_1=D_2=1) = P(D_1=D_2=1, H=1) + P(D_1=D_2=1, H=1)$

$= P(D_1=D_2=1 \mid H=1)P(H=1) + P(D_1=D_2=1 \mid H=0)P(H=0)$

$= (0\ 9801)(0\ 0015) + (0\ 02)(0\ 9985)$

$= 0\ 00147015 + 0\ 01997$

$= 0\ 02144015$

$= \dfrac{(0\ 9801)(0\ 0015)}{0\ 02144015}$

$= 0\ 069$

(2f) $E_n[X] = \sum_i -P(x)\log(P(x))$

$= -\left(\frac{9}{14}\right)\left(\log\left(\frac{9}{14}\right)\right) - \left(\frac{5}{14}\right)\left(\log\left(\frac{5}{14}\right)\right)$

$= 0\ 123 + 0\ 159$

$= 0\ 282$