

(2b) Let  $A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ ,  $D = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}$ ,  $B = \begin{bmatrix} s & t & u & v \\ w & x & y & z \end{bmatrix}$

$$\begin{aligned} ADB &= \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} s & t & u & v \\ w & x & y & z \end{bmatrix} \\ &= \begin{bmatrix} am & bn \\ cm & dn \\ em & fn \end{bmatrix} \begin{bmatrix} s & t & u & v \\ w & x & y & z \end{bmatrix} \\ &= \begin{bmatrix} amst+bnw & amt+bnx & amu+bnv & amv+fnz \\ cms+dnw & cmt+dnx & cmu+dnv & cmv+dnz \\ ems+fnw & emt+fnx & emu+fnv & emv+fnz \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^2 d_{ii} a_i b_i^T &= (a_1 \ d_{11} \ b_1^T) + (a_2 \ d_{22} \ b_2^T) \\ &= \left( \begin{bmatrix} a \\ c \\ e \end{bmatrix} m \begin{bmatrix} s & t & u & v \end{bmatrix} \right) + \left( \begin{bmatrix} b \\ d \\ f \end{bmatrix} n \begin{bmatrix} w & x & y & z \end{bmatrix} \right) \\ &= \left( \begin{bmatrix} am \\ cm \\ em \end{bmatrix} \begin{bmatrix} s & t & u & v \end{bmatrix} \right) + \left( \begin{bmatrix} bn \\ dn \\ fn \end{bmatrix} \begin{bmatrix} w & x & y & z \end{bmatrix} \right) \\ &= \begin{bmatrix} amst+bnw & amt+bnx & amu+bnv & amv+fnz \\ cms+dnw & cmt+dnx & cmu+dnv & cmv+dnz \\ ems+fnw & emt+fnx & emu+fnv & emv+fnz \end{bmatrix} \end{aligned}$$

Thus,  $E = ADB = \sum_{i=1}^2 d_{ii} a_i b_i^T$  holds true