Tema H1'

$f = x^3 - 60x^2 + 900x + 100, x \in [0, 31]$.

- Hamming distance 1 => bitstrings differ in exactly one bit position.

1) $x = 0 \Rightarrow x\_binary = 00000 \Rightarrow Neighbours_{x\_binary = 00000} = \{10000, 01000, 00100, 00010, 00001\} \overset{(*)}{\Rightarrow}$

Converting from binary to decimal:
Given a binary number $b_m b_{m-1} \dots b_1 b_0$, b is 0 or 1,
the decimal value Δ is given by:

$$\Delta = b_m \cdot 2^m + b_{m-1} \cdot 2^{m-1} + \dots + b_1 \cdot 2^1 + b_0 \cdot 2^0 \quad (*)$$

$\overset{(*)}{\Rightarrow} \Delta(Neighbours(00000)) = \{2^4 \cdot 1 + 2^3 \cdot 0 + 2^2 \cdot 0 + 2^1 \cdot 0 + 2^0 \cdot 0, 2^4 \cdot 0 + 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 0 + 2^0 \cdot 0, 2^4 \cdot 0 + 2^3 \cdot 0 + 2^2 \cdot 1 + 2^1 \cdot 0 + 2^0 \cdot 0, 2^4 \cdot 0 + 2^3 \cdot 0 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 0, 2^4 \cdot 0 + 2^3 \cdot 0 + 2^2 \cdot 0 + 2^1 \cdot 0 + 2^0 \cdot 1\}$

$= \{16, 8, 4, 2, 1\}$

2) $x = 1 \Rightarrow binary(1) = 00001 \Rightarrow Neighbours(00001) = \{10001, 01001, 00101, 00011, 00000\} \overset{(*)}{\Rightarrow} \Delta(Neighbours(00001)) = \{17, 9, 5, 3, 0\}$

3) $x = 2 \Rightarrow binary(2) = 00010 \Rightarrow Neighbours(00010) = \{10010, 01010, 00110, 00000, 00011\} \overset{(*)}{\Rightarrow} \Delta(Neighbours(00010)) = \{18, 10, 6, 0, 3\}$

4) $x = 3 \Rightarrow binary(3) = 00011 \Rightarrow Neighbours(00011) = \{10011, 01011, 00111, 00001, 00010\} \overset{(*)}{\Rightarrow} \Delta(Neighbours(00011)) = \{19, 11, 7, 1, 2\}$

5) $x = 4 \Rightarrow binary(4) = 00100 \Rightarrow Neighbours(00100) = \{10100, 01100, 00000, 00110, 00101\} \overset{(*)}{\Rightarrow} \Delta(Neighbours(00100)) = \{20, 12, 0, 6, 5\}$

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6) $x=5 \Rightarrow$ binary(5) = 00101 $\Rightarrow$ neighbours(00101) = {10101, 01101, 00001, 00111, 00100} $\overset{(x)}{\Rightarrow}$ D(neighbours(00101)) = {21, 13, 1, 7, 4}

7) $x=6 \Rightarrow$ binary(6) = 00110 $\Rightarrow$ neighbours(00110) = {10110, 01110, 00010, 00100, 00111} $\overset{(x)}{\Rightarrow}$ D(neighbours(00110)) = {22, 14, 2, 4, 7}

8) $x=8 \Rightarrow$ binary(8) = 01000 $\Rightarrow$ neighbours(01000) = {11000, 01100, 00000, 01010, 01001} = {24, 12, 10, 9}

8) $x=7 \Rightarrow$ binary(7) = 00111 $\Rightarrow$ neighbours(00111) = {10111, 01110, 00011, 00101, 00110} $\overset{(x)}{\Rightarrow}$ D(neighbours(00111)) = {23, 15, 3, 5, 6}

10) $x=9 \Rightarrow$ binary(9) = 01001 $\Rightarrow$ neighbours(01001) = {11001, 00001, 01101, 01011, 01000} $\overset{(x)}{\Rightarrow}$ D(neighbours(01001)) = {25, 1, 13, 11, 8}

11) $x=10 \Rightarrow$ binary(10) = 01010 $\Rightarrow$ neighbours(01010) = {11010, 00010, 01110, 01000, 01011} $\Rightarrow$ D(neighbours(01010)) = {26, 2, 14, 8, 11}

12) $x=11 \Rightarrow$ binary(11) = 01011 $\Rightarrow$ neighbours(01011) = {11011, 00011, 01111, 01001, 01010} $\Rightarrow$ D(neighbours(01011)) = {27, 3, 15, 9, 10}.

13) $x=12 \Rightarrow$ binary(12) = 01100 $\Rightarrow$ neighbours(01100) = {11100, 00100, 01000, 01110, 01101} $\overset{(x)}{\Rightarrow}$ D(neighbours(01100)) = {28, 4, 8, 14, 13}

14) $x=13 \Rightarrow$ binary(13) = 01101 $\Rightarrow$ neighbours(01101) = {11101, 00101, 01001, 01111, 01100} $\overset{(x)}{\Rightarrow}$ D(neighbours(01101)) = {29, 5, 9, 15, 12}

15) $x=14 \Rightarrow$ binary(14) = 01110 $\Rightarrow$ neighbours(01110) = {11110, 00110, 01010, 01100, 01111} $\overset{(x)}{\Rightarrow}$ D(neighbours(01110)) = {30, 6, 10, 12, 15}

16) $x=15 \Rightarrow$ binary(15) = 01111 $\Rightarrow$ neighbours(01111) = {11111, 00111, 01011, 01101, 01110} $\overset{(x)}{\Rightarrow}$ D(neighbours(01111)) = {31, 7, 11, 13, 14}.

17) $x=16 \Rightarrow$ binary(16) = 10000 $\Rightarrow$ neighbours(10000) = {11000, 10100, 10010, 10001, 00000} $\overset{(x)}{\Rightarrow}$ D(neighbours(16)) = {24, 20, 18, 17}

18) $x=17 \Rightarrow$ binary(17) = 10001 $\Rightarrow$ neighbours(10001) = {00001, 11001, 10101, 10011, 10000} $\overset{(x)}{\Rightarrow}$ D(neighbours(10001)) = {1, 25, 21, 19, 16}

19) X=18 => binary(18)=10010 => Neighbours(10010) = {00010, 11010, 10110, 10000, 10011} (x) D(Neighbours(10010)) = {2, 26, 22, 16, 19}

20) X=19 => binary(19)=10011 => Neighbours(10011) = {00011, 11011, 10111, 10001, 10010} (x) D(Neighbours(10011)) = {3, 27, 23, 17, 18}

21) X=20 => binary(20) = 10100 => Neighbours(10100) = {00100, 11100, 10000, 10110, 10101} => D(Neighbours(10100)) = {4, 28, 16, 22, 21}

22) X=21 => binary(21) = 10101 => Neighbours(10101) = {00101, 11101, 10001, 10111, 10100} (x) D(Neighbours(10101)) = {5, 29, 17, 23, 20}

23) X=22 => binary(22) = 10110 => Neighbours(10110) = {00110, 11110, 10010, 10100, 10111} (x) D(Neighbours(10110)) = {6, 30, 18, 20, 23}

24) X=23 => binary(23) = 10111 => Neighbours(10111) = {00111, 11111, 10011, 10011, 10101, 10110} (x) D(Neigh.(.)) = {7, 31, 19, 21, 22}

25) X=24 => binary(24) = 11000 => Neighbours(11000) = {01000, 10000, 11100, 11010, 11001} (x) D(Neighbours(11000)) = {8, 16, 28, 26, 25}

26) X=25 => binary(25) = 11001 => Neighbours(11001) = {01001, 10001, 11101, 11011, 11000} (x) D(Neighbours(11001)) = {9, 17, 29, 27, 24}

27) X=26 => binary(26) = 11010 => Neighbours(11010) = {01010, 10010, 11110, 11000, 11011} (x) D(Neighbours(11010)) = {10, 18, 30, 24, 27}

28) X=27 => binary(27) = 11011 => Neighbours(11011) = {01011, 10011, 11111, 11001, 11010} (x) D(Neighbours(11011)) = {11, 19, 31, 25, 26}

29) X=28 => binary(28) = 11100 => Neighbours(11100) = {01100, 10100, 11000, 11110, 11101} (x) D(Neighbours(11100)) = {12, 20, 24, 30, 29}

30) X=29 => binary(29) = 11101 => Neighbours(11101) = {01101, 10101, 11001, 11111, 11100} = {13, 21, 25, 31, 28}

31) X=30 => binary(30) = 11110 => Neighbours(11110) = {01110, 10110, 11010, 11100, 11111} (x) D(Neighbours(11110)) = {14, 22, 26, 28, 31}

$x=31 \Rightarrow$ binary(31) = 11111 $\Rightarrow$ neighbours(11111) = {01111, 10111, 11011, 11101, 11110} $\overset{(2)}{\Rightarrow}$ neighbours(neighbours(11111)) = {15, 23, 27, 29, 30}.

$f = x^3 - 60x^2 + 900x + 100, x \in [0, 31]$

f(0) = 100	f(8) = 3972	f(16) = 3236	f(24) = 964
f(1) = 941	f(9) = 4069	f(17) = 2973	f(25) = 725
f(2) = 1668	f(10) = 4100	f(18) = 2692	f(26) = 516
f(3) = 2287	f(11) = 4071	f(19) = 2399	f(27) = 343
f(4) = 2804	f(12) = 3988	f(20) = 2100	f(28) = 212
f(5) = 3225	f(13) = 3857	f(21) = 1801	f(29) = 129
f(6) = 3556	f(14) = 3684	f(22) = 1508	f(30) = 100
f(7) = 3803	f(15) = 3475	f(23) = 1227	f(31) = 131

(A) First improvement

As soon as a neighbour with a higher f(x) is found, move to that neighbour.

1) 0 $\xrightarrow[(f(16) > f(0))]{16, 8, 4, 2, 1}$ 16 $\xrightarrow{24, 20, 18, 17}$ 16

2) 1 $\xrightarrow[(f(17) > f(1))]{17, 9, 5, 3, 0}$ 17 $\xrightarrow[(f(16) > f(17))]{24, 20, 18,}$ 1, 25, 21, 19, 16 $\xrightarrow[16]{0, 24, 20, 18, 17}$ 16

3) 2 $\xrightarrow{18, 10, 6, 0, 3}$ ~~10 (maximum)~~ 18 $\xrightarrow[f(16) > f(18)]{2, 26, 22, 16, 19}$ $\xrightarrow[16]{0, 24, 20, 18, 17}$ 16

4) 3 $\xrightarrow[f(19) > f(3)]{19, 11, 7, 1, 2}$ 19 $\xrightarrow[f(17) > f(19)]{3, 27, 23, 17, 18}$ 17 $\xrightarrow[f(16) > f(17)]{1, 25, 21, 19, 16}$ 16 $\xrightarrow{0, 24, 20, 18, 17}$ 16

5) 4 $\xrightarrow[f(12) > f(14)]{20, 12, 0, 6, 5}$ 12 $\xrightarrow{28, 4, 8, 14, 13}$ 12

6) 5 $\xrightarrow[f(13) > f(5)]{21, 13, 1, 7, 4}$ 13 $\xrightarrow[f(9) > f(13)]{29, 5, 9, 15, 12}$ 9 $\xrightarrow[f(11) > f(9)]{25, 1, 13, 11, 8}$ 11 $\xrightarrow[f(10) > f(11)]{27, 3, 15, 9, 10}$ 10

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7) 6 $\xrightarrow[f(14)>f(6)]{22,14,2,4,7}$ 14 $\xrightarrow[f(10)>f(14)]{30,6,10,12,15}$ 10 $\xrightarrow{24,9,15,9}$ 10 (maxim)

8) 7 $\xrightarrow{23,15,3,5,6}$ 7

9) 8 $\xrightarrow[f(12)>f(8)]{24,0,12,10,9}$ 12 $\xrightarrow{28,4,8,14,13}$ 12

10) 9 $\xrightarrow[f(11)>f(9)]{25,1,13,11,8}$ 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

11) 10 $\longrightarrow$ 10 (maxim)

12) 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

13) 12 $\xrightarrow{28,4,8,14,13}$ 12

14) 13 $\xrightarrow[f(9)>f(13)]{29,5,9,15,12}$ 9 $\xrightarrow[f(11)>f(9)]{25,1,13,11,8}$ 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

15) 14 $\xrightarrow[f(10)>f(14)]{30,6,10,12,15}$ 10 $\longrightarrow$ 10 (maxim)

16) 15 $\xrightarrow[f(7)>f(15)]{31,7,11,13,14}$ 7 $\xrightarrow{23,15,3,5,6}$ 7

17) 16 $\xrightarrow{24,20,18,17}$ 16

18) 17 $\xrightarrow[f(16)>f(17)]{1,25,21,19,16}$ 16 $\xrightarrow{0,24,20,18,17}$ 16

19) 18 $\xrightarrow[f(16)>f(18)]{2,26,22,16,19}$ 16 $\xrightarrow{0,24,20,18,17}$ 16

20) 19 $\xrightarrow[f(17)>f(19)]{3,27,23,17,18}$ 17 $\xrightarrow[f(16)>f(17)]{1,25,21,19,16}$ 16

21) 20 $\xrightarrow[f(4)>f(20)]{4,28,16,22,21}$ 4 $\xrightarrow[f(12)>f(4)]{20,12,0,6,5}$ 12 $\xrightarrow{28,4,8,14,13}$ 12

22) 21 $\xrightarrow[f(4)>f(21)]{5,29,17,23,20}$ 5 $\xrightarrow[f(12)>f(4)]{20,12,0,6,5}$ 13 $\xrightarrow[f(9)>f(13)]{29,5,9,15,12}$ 9 $\xrightarrow[f(11)>f(9)]{25,1,13,11,8}$ 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

23) 22 $\xrightarrow[f(6)>f(22)]{6,30,18,20,23}$ 6 $\xrightarrow[f(14)>f(6)]{22,14,2,4,7}$ 14 $\xrightarrow[f(10)>f(14)]{30,6,10,12,15}$ 10 $\longrightarrow$ 10

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24) 23 $\xrightarrow[f(7)>f(23)]{7,31,19,21,22}$ 7 $\xrightarrow{23,15,13,5,6}$ 7

25) 24 $\xrightarrow[f(8)>f(24)]{8,16,28,26,25}$ 8 $\xrightarrow[f(12)>f(8)]{24,0,12,10,9}$ 12 $\xrightarrow{28,4,8,14,13}$ 12

26) 25 $\xrightarrow[f(9)>f(25)]{9,17,29,27,24}$ 9 $\xrightarrow[f(11)>f(9)]{25,11,13,1,8}$ 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

27) 26 $\xrightarrow[f(10)>f(26)]{10,18,30,24,27}$ 10 $\longrightarrow$ 10

28) 27 $\xrightarrow[f(11)>f(27)]{11,19,31,25,26}$ 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

29) 28 $\xrightarrow[f(12)>f(28)]{12,20,24,30,29}$ 12 $\xrightarrow{28,4,8,14,13}$ 12

30) 29 $\xrightarrow[f(13)>f(29)]{13,21,25,31,28}$ 13 $\xrightarrow[f(9)>f(13)]{29,5,9,15,12}$ 9 $\xrightarrow[f(11)>f(9)]{25,1,13,11,8}$ 11 $\xrightarrow[f(10)>f(11)]{27,3,15,9,10}$ 10

31) 30 $\xrightarrow[f(14)>f(30)]{14,22,26,28,31}$ 14 $\xrightarrow[f(10)>f(14)]{30,6,10,12,15}$ 10 $\longrightarrow$ 10

32) 31 $\xrightarrow[f(15)>f(31)]{15,23,27,29,30}$ 15 $\xrightarrow[f(7)>f(15)]{31,7,11,13,4}$ 7 $\longrightarrow$ 7

maximum	Attraction basin
16	{0, 1, 2, 3, 16, 17, 18, 19}
12	{4, 8, 12, 20, 24, 28}
10	{5, 6, 9, 10, 11, 13, 21, 22, 25, 26, 27, 29, 30}
7	{7, 15, 23, 31}

(B) Best Improvement

Choose the neighbour with the highest f(x) among all the neighbours

1) 0 16,8,4,2,1 → 8 (f(8) > f(0), f(1)...) 24,9,12,10,9 → 10 26,2,14,8,11 → 10

2) 1 17,9,5,3,0 → 9 25,1,13,11,8 → 11 9,10,15,3,27 → 10 26,2,14,8,11 → 10

3) 2 18,10,6,0,3 → 10 → 10

4) 4 20,12,8,6,5 → 12 28,4,8,14,13 → 12

5) ~~3~~ 19,11,7,1,2 → 11 27,3,15,9,10 → 10 → 10

6) 5 21,13,1,7,4 → 13 29,5,9,15,12 → 9 25,1,13,11,8 → 11 27,3,15,9,10 → 10 → 10

7) 6 22,14,2,4,7 → 7 23,15,3,5,6 → 7

8) 7 23,15,3,5,6 → 7

9) 8 24,9,12,10,9 → 10 26,2,14,8,11 → 10

10) 9 25,1,13,11,8 → 11 9,10,15,3,27 → 10 26,2,14,8,11 → 10

11) 10 26,2,14,8,11 → 10

12) 11 27,3,15,9,10 → 10 26,2,14,8,11 → 10

13) 12 28,4,8,14,13 → 12

14) 13 29,5,9,15,12 → 9 25,1,13,11,8 → 11 9,10,15,3,27 → 10 → 10

15) 14 30 6,10,12,15 → 10 → 10

16) 15 31,7,11,13,14 → 11 27,3,15,9,10 → 10 → 10

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17) 16 $\xrightarrow{0,24,20,18,17}$ 16

18) 17 $\xrightarrow{1,25,21,19,16}$ 16 $\xrightarrow{0,24,20,18,17}$ 16

19) 18 $\xrightarrow{2,26,22,16,19}$ 16 $\xrightarrow{0,24,20,18,17}$ 16

20) 19 $\xrightarrow{3,24,23,17,18}$ 17 $\xrightarrow{1,25,21,19,16}$ 16 $\xrightarrow{0,24,20,18,17}$ 16

21) 20 $\xrightarrow{4,28,16,22,21}$ 16 $\xrightarrow{0,24,20,18,17}$ 16

22) 21 $\xrightarrow{5,29,17,23,20}$ 5 $\xrightarrow{21,13,1,7,4}$ 13 $\xrightarrow{29,5,9,15,12}$ 9 $\xrightarrow{25,1,13,11,8}$ 11 $\xrightarrow{27,3,15,9,10}$ 10 $\longrightarrow$ 10

23) 22 $\xrightarrow{6,30,18,20,23}$ 6 $\xrightarrow{22,14,2,4,7}$ 7 $\xrightarrow{23,15,3,5,6}$ 7

24) 23 $\xrightarrow{7,31,19,21,22}$ 7

25) 24 $\xrightarrow{8,16,28,26,25}$ 8 $\xrightarrow{24,0,12,10,9}$ 10 $\longrightarrow$ 10

26) 25 $\xrightarrow{9,17,29,27,24}$ 9 $\xrightarrow{25,1,13,11,8}$ 11 $\longrightarrow$ 10 $\longrightarrow$ 10

27) 26 $\xrightarrow{10,18,30,24,27}$ 10 $\longrightarrow$ 10

28) 27 $\xrightarrow{11,19,31,25,26}$ 11 $\xrightarrow{27,3,15,9,10}$ 10 $\longrightarrow$ 10

29) 28 $\xrightarrow{12,20,24,30,29}$ 12 $\xrightarrow{28,4,8,14,13}$ 12

30) 29 $\xrightarrow{13,21,25,31,28}$ 13 $\xrightarrow{29,5,9,15,12}$ 9 $\xrightarrow{25,1,13,11,8}$ 11 $\longrightarrow$ 10 $\longrightarrow$ 10

31) 30 $\xrightarrow{14,22,26,28,31}$ 14 $\xrightarrow{30,6,10,12,15}$ 10 $\longrightarrow$ 10

32) 31 $\xrightarrow{15,23,27,29,30}$ 15 $\xrightarrow{31,7,11,13,14}$ 11 $\xrightarrow{27,3,15,9,10}$ 10 $\longrightarrow$ 10

maximum	Attraction Basin
10	{0, 1, 2, 3, 5, 8, 9, 10, 11, 13, 14, 15, 21, 24, 25, 26, 27, 29, 30, 31}
12	{4, 12, 28}
7	{6, 7, 22, 23}
16	{16, 17, 18, 19, 20}

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