COMPACT BINARY INSPIRAL AND GW170817

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An Introduction to LIGO Astronomy

Suppose that two neutron stars with masses $m_1 \geq m_2$ are in a stable circular orbit (see Figure 1). As we have recently observed in the transient signal GW170817, this system will radiate away its potential energy over time in the form of gravitational waves. In this group problem set, we'll first write down a few observables to flesh out some details of how this happens. Then, we'll look at some very real examples of astrophysics that can be learned through gravitational wave (GW) and electromagnetic (EM) observations.

In group discussion we'll build an intuition by tinkering with some signal processing techniques, imagining how to observe this system with the ground-based LIGO detectors. This will make use of data obtained from the LIGO Open Science Center, a service of LIGO Laboratory, the LIGO Scientific Collaboration, and the Virgo Collaboration. LIGO is funded by the U.S. National Science Foundation. Virgo is funded by the French Centre National de Recherche Scientifique (CNRS), the Italian Istituto Nazionale della Fisica Nucleare (INFN) and the Dutch Nikhef, with contributions by Polish and Hungarian institutes.

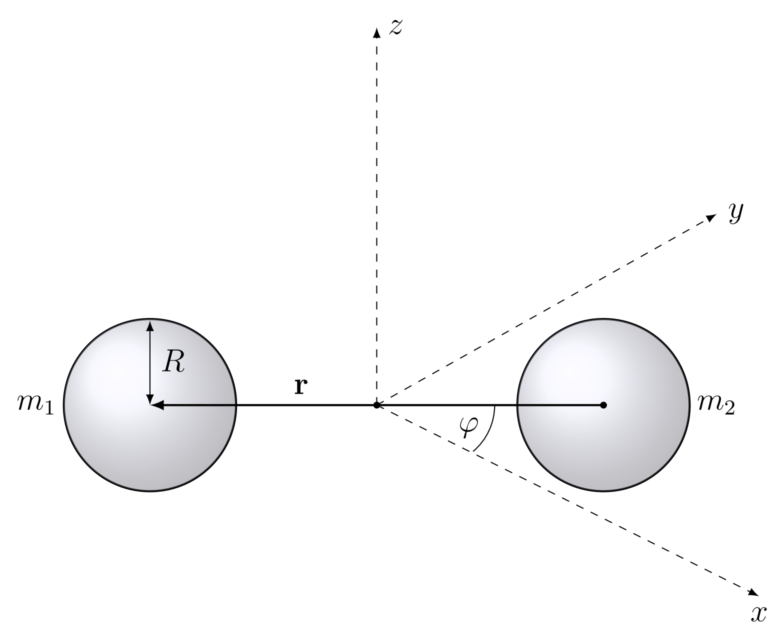

Figure 1: Diagram of a neutron star binary in the center of mass frame, showing its orbital separation vector ($\mathbf{r}$) and the radius (R) and masses ($m_1 \geq m_2$) of the individual neutron stars. The orbital phase angle φ is also indicated.

Note: In what follows, $M = m_1 + m_2$ is the total mass and $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the binary.

1. Recall that according to general relativity, it's very difficult to extract energy from this system once it reaches the innermost stable circular orbit (ISCO), which has a separation of roughly
$$r_{\rm ISCO} = \frac{6GM}{c^2}. \tag{1}$$
Given that the GW frequency is twice the orbital frequency, use Kepler's third law to write down a rough scaling law for the observed frequency $f_{\rm ISCO}$ at ISCO in terms of M.

2. What is $f_{\rm ISCO}$ in Hz if $M = 2.74\,M_\odot$?

3. Recall that the orbital separation, r, shrinks over time as potential energy is lost to gravitational waves. Consequently, the frequency sweeps up. In a previous problem set we approximated this process as
$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{r^3}. \tag{2}$$
Without solving any differential equations, estimate how long it takes the system to evolve from an observed frequency of f_0 up through merger. First, find the characteristic timescale of Eq. 2 starting from some initial distance r_0; then, use Kepler's third law to relate r_0 to f_0.

4. Once the signal crosses 20 Hz, how long do we have until merger (in seconds) if $m_1 = 1.57\,M_\odot$ and $m_2 = 1.17\,M_\odot$? How does this compare with the observed duration ($\sim$100 s) of transient signal GW170817 in LIGO Livingston's sensitive frequency band?

5. **Event counting.** The U.S.-based LIGO detectors in Hanford, WA, and Livingston, LA, recorded data for about 49 days from September 2015 – January 2016, and again for about 117 days from November 2016 – August 2017. While the noise floor in each detector fluctuated greatly during these observing periods, we can estimate the detectors' average sensitive distance for binary neutron star mergers as 70 Mpc. In other words, as we'll discuss in group, mergers of two neutron stars out to about this distance would produce strong enough gravitational waves to be detected by LIGO.

   Given that we have observed exactly one such merger so far, what is the inferred rate of binary neutron star merger events in Gpc^{-3} yr^{-1}?

6. What is the same rate, per *galaxy* per year? (Assume one galaxy takes up roughly 1 Mpc^3.)

7. **Striking gold!** From EM observations[1] we have learned that some $\sim$0.05 $M_\odot$ of ejecta were released during an explosive kilonova after the merger event. It has been suspected for some time that explosions like these are responsible for producing most of the elements heavier than iron (such as gold, silver, and platinum) in the universe. Using the measured ejecta mass and your estimate of the neutron star merger rate per galaxy, can you comment on how this compares to the estimated rate of heavy element synthesis of 10^{-7} $M_\odot$ yr^{-1} in the Milky Way?

8. What uncertainties do you think there are in these measurements?

[1] Cowperthwaite *et al.*, *Astrophysical Journal Letters* **848**, 2 (2017).