NEWTONIAN TIDAL EFFECTS AND NEUTRON STAR INSPIRAL

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1 Problem

Suppose that a neutron star binary (each with mass $M = 1.4\ M_\odot$) is in a stable, Keplerian orbit (*i.e.* with no loss of energy). Figure 1 illustrates this system.

1. Compute the separation (in km) between the stars in terms of the gravitational wave frequency.
2. Compute the gravitational force between star 1 and a chunk of mass on the near side of star 2, and again for a chunk on the far side of star 2. Compare this, quantitatively, to the force binding those two chunks to their own star 2. Make a plot of these quantities, vs the gravitational wave frequency.

Solution

1. From Kepler's third law, we can relate the orbital period (P_{orbital}) and the orbital separation (a):

$$\frac{P_{\text{orbital}}^2}{a^3} = \frac{4\pi^2}{2GM}.$$

We also know that gravitational waves from this system radiate at twice the orbital frequency, $f_{\text{GW}} = 2f_{\text{orbital}} = 2/P_{\text{orbital}}$, so we can solve for a with a bit of algebra:

$$a = \left[\frac{2GM}{(\pi f_{\text{GW}})^2}\right]^{1/3}. \tag{1}$$

That effectively solves the first part.

2. From Newton's law of universal gravitation, we know that the force felt by star 2 due to star 1 on the near side is

$$F_{\text{near}} = -\frac{GM^2}{(a-R)^2} \tag{2}$$

where R is the neutron star radius. Similarly, the force felt on the far side is

$$F_{\text{far}} = -\frac{GM^2}{(a+R)^2}. \tag{3}$$

The difference between these is the tidal force:

$$F_{\text{tidal}} = F_{\text{near}} - F_{\text{far}} = GM^2\left(\frac{1}{(a-R)^2} - \frac{1}{(a+R)^2}\right) \tag{4}$$

When the tidal force overcomes the gravitational self-force of the neutron star,

$$F_{\text{self}} = \frac{GM^2}{R^2}, \tag{5}$$

we know the star will start to be destroyed. Figure 2 illustrates this process for different neutron star radii, in terms of the gravitational wave frequency.

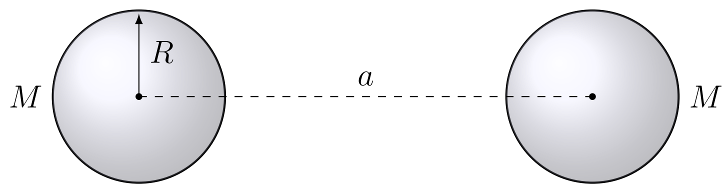

Figure 1: Diagram of the neutron star binary, showing its orbital separation (a) and the radius (R) and masses (M) of the individual neutron stars.

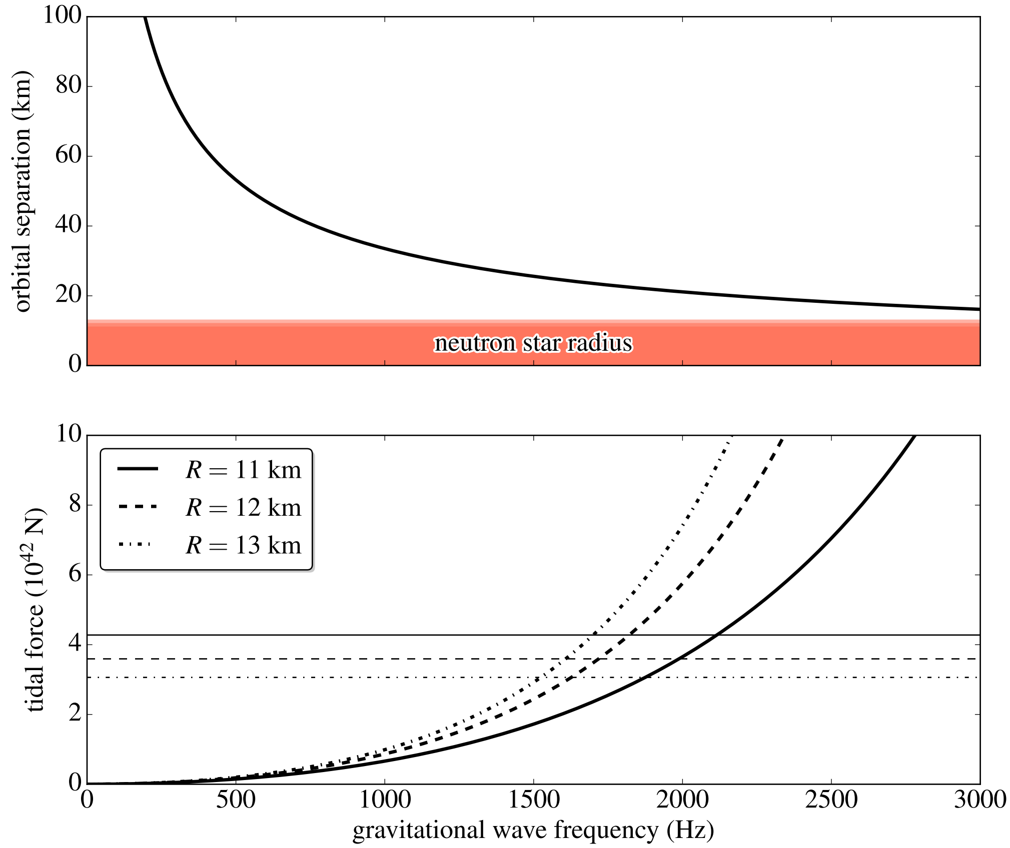

Figure 2: Top: the orbital separation (in km) between two neutron stars as a function of gravitational wave frequency. Bottom: The tidal forces (in Newtons) on one neutron star due to the other, compared at different neutron star radii. The horizontal lines show the neutron star's total binding force – when the tidal force exceeds this, the neutron star will be ripped to smitherines in an absolutely epic lightshow.