1 Introduction

Robustness of an estimator is a multifaceted statistical concept that does not have universal definition. We will refer to an estimator as robust if it is not too sensitive to outliers which means that the estimator draws a valid conclusion about underlying population parameters even in the presence of outliers in the dataset.

A class of minimum density power divergence estimators was introduced by Basu et al. (1998) for robust estimation in general parametric models. They defined the divergence functional $d_\alpha(f,g)$ between two p-variate probability density functions $g(\mathbf{x})$ and $f(\mathbf{x})$ as

$$d_\alpha(f,g) := \int_{\mathbb{R}^p} \left(f^{1+\alpha}(\mathbf{x}) - \left(1 + \frac{1}{\alpha}\right) g(\mathbf{x}) f^{\alpha}(\mathbf{x}) + \frac{1}{\alpha} g^{1+\alpha}(\mathbf{x}) \right) d\mathbf{x} \tag{1}$$

For $\alpha > 0$, on the strength of Basu et al. (1998), $d_\alpha(\cdot,\cdot)$ is a divergence in the sense that it is a premetric satisfying the identity of indiscernibles.

Assuming $\mathbf{x}_1, \ldots, \mathbf{x}_n \overset{i.i.d.}{\sim} g(\mathbf{x})$ (with $g(\mathbf{x})$ being unknown) and invoking the law of large numbers, we get

$$\int_{\mathbb{R}^p} f^{\alpha}(\mathbf{x}) g(\mathbf{x})\, d\mathbf{x} \approx \frac{1}{n} \sum_{i=1}^{n} f^{\alpha}(\mathbf{x}_i). \tag{2}$$

2 Simulation and Comparison

We empirically verify the breakdown point of the resulting algorithmic implementation of the MDPD estimator.

n	Nominal bdp	MCD	MDPD	MCD	MDPD
30	8	9	10	9	10
50	13	15	16	15	16
100	27	28	31	28	31
200	54	56	61	55	61
300	81	85	91	83	91
400	108	112	121	110	121

Table 1: Empirical breakdown point comparisons.

3 Example

Figure 1 suggests the variability in the dataset changes across the recorded time frame. As the number of chlamydia cases increase, volatility increases and vice versa.

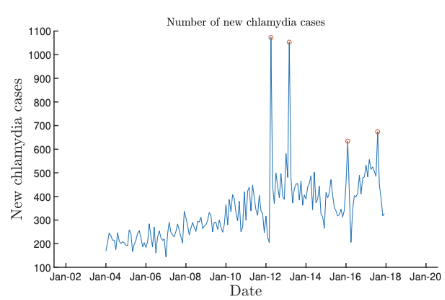

Figure 1: Reported monthly new chlamydia cases from 2004 to 2017.

References

Basu, A., Harris, I. R., Hjort, N. L., and Jones, M. (1998). Robust and efficient estimation by minimising a density power divergence. *Biometrika*, 85(3):549–559.