

# LEARNING SEMINAR ON MANIFOLDS AND HOMOTOPY THEORY

ABSTRACT. As the goal of life of homotopy theorists is to compute the homotopy groups of spheres, geometric topologists on the other hand dream to compute the cohomology groups of the moduli of surfaces  $\mathcal{M}_g$ . There are tons of other families of groups (or spaces) whose homology is still unknown. Fortunately, the homology of some of these stabilize in a range: this phenomena is called homological stability. In particular, the groups  $H^*(\mathcal{M}_g)$  are independent of  $g$  for degrees smaller than an affine function of  $g$ .

We plan to cover the cellular  $E_k$  algebra method developed by Galatius-Kupers-Randal-Williams as well as the necessary homotopy-theoretic techniques. As an application, we will discuss the cohomology of general linear groups of fields. From this point, we could look into more surprising interactions between algebraic K-theory and Geometric Topology. Depending on the participants' interests, we may also discuss applications to arithmetic statistics or representation theory.



FIGURE 1. *What freedom!*, 1903, Russian Museum, Saint Petersburg. The painting represents Geometric Topologists and Homotopy Theorists walking together, holding hands.

## 1. TOPIC IDEAS

This is a non-exhaustive list of topics we could cover:

- (1) Introduction: kinda announce main results and characters we want to discuss in this seminar
- (2) What is  $M_g$ ? From AG to AT
- (3) Stable homology of moduli spaces of surfaces
- (4) Proof of Dundas-McCarthy Theorem and Goodwillie Calculus
- (5) Homological Stability and algebraic K-theory

- (6) Power Operations on  $E_k$ -algebras
- (7) Stable  $h$ -cobordisms and  $K$ -theory
- (8) Dwyer-Weiss-Williams stuff (always wanted to read this, something with K theory and manifolds)
- (9) Some spectral sequences
- (10) (G)-stuff: unstable equivariant homotopy, genuine, naive
- (11) Rognes filtration of  $K$ -theory, as in: <https://arxiv.org/pdf/2512.19128.pdf> and Rognes's connectivity conjecture

## 2. INTRODUCTION

Introduce topics we wanna discuss in seminar and how they should connect together.

### 3. ALL YOU NEED TO KNOW ABOUT $\mathcal{M}_g$

- 3.1. Curves, surfaces, mapping class groups.
- 3.2. Stable homology of  $\mathcal{M}_g$ .
- 3.3. Homological Stability for mapping class groups.

### 4. A GENERAL METHOD TO PROVE HOMOLOGICAL STABILITY

- 4.1. What are  $E_k$ -algebras?
- 4.2.
- 4.3.
- 4.4. Power Operations.
- 4.5. Examples in Number Theory.

### 5. GENERAL LINEAR GROUPS AND ALGEBRAIC $K$ -THEORY

- 5.1.
- 5.2.
- 5.3.

### 6. MORE ON ALGEBRAIC $K$ -THEORY

- 6.1. Another type of stability result: the parametrized  $h$ -cobordism theorem.
- 6.2. Dwyer-Weiss Williams paper.
- 6.3. The Dundas-McCarthy Theorem and its proof.