

LEARNING SEMINAR ON MANIFOLDS AND HOMOTOPY THEORY

ABSTRACT. As the goal of life of homotopy theorists is to compute the homotopy groups of spheres, geometric topologists on the other hand dream to compute the cohomology groups of the moduli of surfaces \mathcal{M}_g . There are tons of other families of groups (or spaces) whose homology is still unknown. Fortunately, the homology of some of these stabilize in a range: this phenomena is called homological stability. In particular, the groups $H^*(\mathcal{M}_g)$ are independent of g for degrees smaller than an affine function of g .

We plan to cover the cellular \mathbb{E}_k algebra method developed by Galatius-Kupers-Randal-Williams (see [GKR23]) to prove homological stability as well as the necessary homotopy-theoretic techniques. Depending on the participants' interests, we could either look into applications to arithmetic statistics (as in [LL25]). We could also look at the homology of general linear groups of fields and interactions with Algebraic K -theory.



FIGURE 1. *What freedom!*, 1903, Russian Museum, Saint Petersburg. The painting represents Geometric Topologists and Homotopy Theorists walking together, holding hands.

1. INTRODUCTION

Speaker: Azélie Picot

Abstract: Homological stability is a property of sequences of groups. In this introductory talk, I will define homological stability and state diverse examples that exhibit this phenomena. Then, we will discuss the current plan for the seminar and will decide among the different options we could explore.

2. QUILLEN'S SPECTRAL SEQUENCE ARGUMENT

Speaker: Azelie Picot**Abstract:**

3. THE SPLITTING COMPLEXES

Speaker:**Abstract:**4. MADSEN-WEISS THEOREM AND THE STABLE HOMOLOGY OF \mathcal{M}_g **Speaker:****Abstract:**5. WHAT ARE \mathbb{E}_k -ALGEBRAS?**Speaker:** Carlos Alvarado**Abstract:**6. POWER OPERATIONS AND FREE \mathbb{E}_k -ALGEBRAS**Speaker:** Sangmin Ko**Abstract:**

7. DERIVED INDECOMPOSABLES

Speaker: Azelie Picot**Abstract:**

8. SECONDARY STABILITY AND OTHER PHENOMENA

Speaker: Azelie Picot**Abstract:**

9. KOSZUL DUALITY

Speaker: Sangmin Ko**Abstract:**

10. APPLICATIONS

Possible topics include:

- (1) Homological stability for Hurwitz spaces and applications to arithmetic statistics: [\[LL25\]](#)
- (2) Homological stability for classical sequences of groups with coefficients being representations of arithmetic groups and applications to arithmetic statistics, see [\[Mil+25\]](#).
- (3) General linear groups of fields as in [\[GKR25\]](#) and [\[GKR25\]](#) and (unstable) algebraic K -theory (as in [\[Jan24\]](#)).

REFERENCES

- [GKR23] Soren Galatius, Alexander Kupers, and Oscar Randal-Williams. *Cellular E_k -algebras*. 2023. arXiv: [1805.07184 \[math.AT\]](https://arxiv.org/abs/1805.07184). URL: <https://arxiv.org/abs/1805.07184>.
- [GKR25] Soren Galatius, Alexander Kupers, and Oscar Randal-Williams. *E_∞ -cells and general linear groups of infinite fields*. 2025. arXiv: [2005.05620 \[math.AT\]](https://arxiv.org/abs/2005.05620). URL: <https://arxiv.org/abs/2005.05620>.
- [Jan24] Mikala Ørsnes Jansen. *Unstable algebraic K-theory: homological stability and other observations*. 2024. arXiv: [2405.02065 \[math.KT\]](https://arxiv.org/abs/2405.02065). URL: <https://arxiv.org/abs/2405.02065>.
- [LL25] Aaron Landesman and Ishan Levy. *Homological stability for Hurwitz spaces and applications*. 2025. arXiv: [2503.03861 \[math.AT\]](https://arxiv.org/abs/2503.03861). URL: <https://arxiv.org/abs/2503.03861>.
- [Mil+25] Jeremy Miller, Peter Patzt, Dan Petersen, and Oscar Randal-Williams. *Uniform twisted homological stability*. 2025. arXiv: [2402.00354 \[math.AT\]](https://arxiv.org/abs/2402.00354). URL: <https://arxiv.org/abs/2402.00354>.