Constraints on ψ-epistemic interpretations of quantum mechanics

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1 Introduction

Classifying the interpretations of quantum mechanics is a challenging endeavour, which becomes no easier as new interpretations are discovered. Nonetheless, it is an essential task in quantum foundations since understanding the underlying hypotheses common to a family of interpretations has allowed us to prove no-go theorems that constrain their validity. A consequence of these theorems is that quantum theory must unavoidably exhibit undesirable properties from a physicist's perspective, no matter how one interprets it. Thus, it is crucial to identify and state such properties carefully to have a prolific discussion.

The interpretations are classified in terms of the wave function's role. It is precisely the wave function that appears in Born's rule for calculating probabilities for measurement outcomes and is therefore at the root of the measurement problem. In particular, we can distinguish between the ψ-ontic and the ψ-epistemic interpretations. In the former, the wave function represents an actual state of the quantum system and can be made to be observer-independent. The latter posits that the wave function represents information that the observer has about the system. Ontic interpretations are required to propose a physically reasonable explanation for the measurement problem in terms of deeper degrees of freedom not probed by quantum mechanics. Conversely, the epistemicist does away with the problem by claiming that the wave function collapse occurs because the observer updates their description of the system by acquiring new information upon measurement.

To many physicists, ψ-ontic interpretations seem more compelling, but an ontic viewpoint of the wave function comes at a steep price. In 1964, John Bell proved that any ontic interpretation that reproduces the results of quantum mechanics must exhibit correlations that do not admit a local explanation [1]. Moreover, Lucien Hardy proved that a quantum state must contain an infinite amount of information [2]. Nevertheless, it seems puzzling that we cannot use the hidden degrees of freedom to transmit faster than light signals or encode infinite information. The non-locality is particularly troublesome, as it poses an insurmountable obstacle for extending the formalism to a relativistic regime (see, however, [3]).

It then comes as no surprise that a large proportion of people turn to the ψ-epistemic interpretations. However, are these exempt from the problems that plague the ontic frameworks? The purpose of this essay is to explore some recent developments in these lines, whereby strong no-go theorems restrict the properties that ψ-epistemic interpretations may have. We then explicitly state the properties that these theorems constrain and map out the road that physicists who argue for an epistemic wave function should follow.

2 The ontological models of quantum mechanics

To make the discussion more precise, we need to define what it means for an interpretation to be ψ-ontic or ψ-epistemic. A rigorous definition can be made within the ontological models of quantum theory [4], in which the wave function of a quantum system represents information about some deeper ontic state of the system. These are realistic models in the sense that external observer-independent entities are assumed to exist. However, the wave function does not necessarily represent complete information about such an ontic state. Moreover, these degrees of freedom are assumed to be unobservable; for this reason, some authors refer to these frameworks as 'hidden variables' models.

Within the ontological models, a ψ-ontic interpretation assigns a unique wave function to every ontic state of the system. Intuitively, this means that the wave function represents some coarse-grained information about the quantum system's state. Many ontic states could correspond to the same wave function, but all observers must assign a unique wave function given an ontic state. In contrast, the latter does not happen for a ψ-epistemic interpretation. The same ontic state could be assigned multiple different wave functions. Different observers could assign distinct quantum states to the same ontic state, depending on their information about the quantum system.

Regardless of the interpretation given to the quantum state, the minimum expectation for the ontological model of an interpretation is to reproduce the results of quantum mechanics. While a seemingly innocuous constraint, it has allowed for the proofs of several no-go theorems. Among these, Bell's theorem and Hardy's Excess Baggage Theorem explained above constrain the features of ψ-ontic interpretations in a seemingly unphysical way. The scientific community's hesitance to accept the conclusion of non-locality or a multiplicity of degrees of freedom boils down to an aversion to fine-tuning. Indeed, it seems unreasonable to expect, for example, that the non-local dynamics at the ontic level be precisely such that the non-locality disappears at the scales we can probe [5].

It is a common misconception, however, that ψ-epistemic interpretations are immune to such criticisms. For example, Bell's inequality can be derived from an assumption of preparation contextuality for mixed states, which states that the ontic state underlying a fixed quantum state depends on the preparation procedure. This assumption seems quite reasonable and is common to ψ-ontic models, but it is also present in some ψ-epistemic models. If non-locality is to be regarded as a powerful enough reason to discard the ψ-ontic interpretations, such epistemic models should be discarded alongside. Moreover, in the next section, we will discuss a theorem that only affects the epistemic interpretations, thus casting further doubts regarding the ontological model one should abide by.

Before we delve deeper into these topics, we need to clarify that not all the interpretations of quantum mechanics admit an ontological model. Whether these interpretations can be deemed realist is up for discussion. Except for many-worlds, interpretations in this group are a subset of the ψ-epistemic interpretations, which we will call ψ-doxastic[1] [6]. These interpretations are immune to the aforementioned no-go theorems, and many physicists have ended up being pressed back to this type

[1] Many physicists have taken issue with their favourite interpretation being called neo-Copenhagen, which is the most common name given to these in the literature. Hopefully this nomenclature will please everyone.

of interpretation as more theorems are proved. Surprisingly, some recent results can constrain even ψ-doxastic interpretations. We will discuss these in detail in section 4.

3 The PBR theorem and its consequences

In 2012, Pussey, Rudolph, and Barrett published what would become known as the most important no-go theorem since Bell's [7]. It is a theorem that works only within the ontological models' framework, and it states that no ψ-epistemic model can fully reproduce the results of quantum mechanics. This result is shocking since physicists have built ψ-epistemic models that could reproduce large fragments of quantum mechanics ([8, 9, 10, 11]). In these, quantum phenomena long considered counter-intuitive became easy to understand. For these models, the consequence of the theorem is, at least in principle, that they could never be extended to account for the full quantum theory[2].

The intuition behind the PBR proof is, in fact, quite simple. The starting point is to consider two independent two-level quantum systems in four different states. Then it is possible to build a four outcome measurement such that the first state has zero probability of yielding the first outcome; the second state has zero probability of yielding the second outcome; and so on. It is not hard to see that, using the ψ-epistemic assumption, one can choose the ontic state for the product state so that, assuming independence, we get that all measurement outcomes have zero probability, thus contradicting the normalization of quantum states. This independence implies that the probability distribution over ontic states of the composite system, given the product quantum state is, in fact, the product of the distributions for each subsystem.

There is a critical assumption in the proof of the PBR theorem, which is the notion of independence at the ontic level. A product state in quantum theory represents a system of two (or more) particles that are not correlated with each other, at least at the level of quantum measurements. The assumption of independence extends this absence of correlations to the ontic states. Mathematically, this has two consequences: (i) product state separability, meaning that the ontic state of a product state should be modeled as a Cartesian pair of ontic states belonging to each of the subsystems; and (ii) product state factorization, which states that given quantum states for the subsystems, the probability distributions of the ontic states in both subsystems are independent [12].

If we were to convince ψ-epistemic leaning physicists to accept the consequences of the PBR theorem, then we must convince them that (i) and (ii) are non-negotiable. Firstly, we should understand what product state separability means physically. It is, in fact, a disguised notion of non-locality: we expect that two systems that were prepared very far away from each other and in isolation must have independent properties, and these uncorrelated properties should fully describe the composite system. Specifically, there should not be any global properties of the entire system independent of the local properties of the subsystems. The reason to deny such global properties boils down to a form of reductionism and the absence of fine-tuning. We cannot expect to lose properties such as non-locality by a mere coarse-graining. If we did, then the

[2]Spekkens claims to have found a ψ-epistemic ontological model for the full quantum theory as early as 2005 [12]. The model is convoluted, and for that reason, it remained unpublished, so it is unclear how it goes around the PBR theorem.

laws at the ontic level must conspire such that this is indeed what is observed.

In response to the above argument, the ψ-epistemicist may try to provide an ontological model where the loss of locality is well explained and not fine-tuned. At the risk of this being a Sisyphean task, a physicist may deem it wiser to accept product state separability, but not without bringing ψ-ontic interpretations down with their ship. For example, Spekkens argues that product state separability is motivated by the stronger notion of separability, which ψ-ontic models can never satisfy because entangled states are part of their ontology. Changing the assumption of product state separability for separability itself removes the need to make the model ψ-epistemic. This more restricted PBR theorem applies to every separable interpretation in the ontological models' framework. Furthermore, if we do not require that ψ-ontic models be separable, why should we require the same for the ψ-epistemic? The consequence is that, within the ontological models, non-locality is inevitable, independent of how one interprets the quantum state.

Notwithstanding these considerations for product state separability, we can still turn to question product state factorization. The argument against it is much simpler since both subsystems may have a shared past cause. As such, the probability distributions for the ontic states may not be independent after all. However, this counter-argument is still vulnerable to the fine-tuning problem since the probability distributions are independent at the detectable level. Moreover, if we impose locality in a sense that is as strong as in Bell's theorem, we can derive product state factorization. As was the case with separability, a similar problem plagues ψ-ontic interpretations.

We can conclude from the above discussion that the PBR theorem does not really forbid ψ-epistemic interpretations of quantum theory within the ontological models. However, it does bring to light some reasonable assumptions that ψ-epistemic models must give up to reproduce quantum mechanics. Having similar constraints set by Bell on the ontic models, we can say that the PBR theorem levelled the playing field: ψ-epistemic interpretations do not solve all of our problems and must also be non-local at the ontic scale. There is, however, an area where the epistemic models are still weaker: thus far, there is no model that reproduces quantum mechanics entirely. These models should violate the product separability and factorization axioms as a consequence of the PBR theorem. Some such frameworks have been proposed; however, many seem to have problems reproducing the state update rule of quantum mechanics.

4 Constraints for doxastic interpretations

The combined results of Bell and PBR seem to imply that the ontological models must be highly fine-tuned to reproduce quantum theory, which has led many physicists to abandon them altogether. The consequence is that physics and philosophy have been brought closer together than they have been in centuries since the notion of physical reality must be challenged for these interpretations to work out. They propose novel ways to look at physics, from pure instrumentalism to structural realism, outside of orthodox scientific realism. In this realm of interpretations lying beyond the ontological models, the distinction between ψ-ontic and ψ-epistemic still exists, even if not in a precise mathematical way. Our focus will be on the ψ-epistemic subset of these, known as ψ-doxastic. Examples of these are quantum bayesianism, relational quantum mechanics, and informational interpretations.

One of the reasons why many physicists adopt the ψ-epistemic perspective is the thought experiment proposed by Wigner [13], now called the 'Wigner's Friend experiment.' The setup is simple: an observer F (the friend) is in a lab L isolated from an observer W (Wigner). Suppose that W and F know of the presence of a spin state in the laboratory. If F measures it without informing W, the spin state is updated for F but not for W. Wigner does not have the information that allows him to do so. Instead, Wigner will model the friend's state and the spin as evolving unitarily, leading to an apparent contradiction. If the friend and spin system's wave function were updated for Wigner, this would contradict the assumption that the laboratory is isolated. It must then follow that the state represents information for different observers: they may assign different wave functions to a quantum system simultaneously. Wigner's experiment is too idealized to be carried out in real life, so such arguments are generally utilized to challenge the solutions to the measurement problem proposed by a given interpretation of quantum mechanics. In general, it is expected that they give a satisfactory explanation for the apparent paradox.

Surprisingly, arguments analogous to Wigner's thought experiment can be used to place constraints in doxastic frameworks. In 2016, Frauchiger and Renner proved that the following three assumptions about observers using quantum mechanics are incompatible [14]: (a) if an observer O uses quantum mechanics to predict they will get or have gotten the value $x = \xi$ upon measurement of an observable X with probability 1, then O is certain that $x = \xi$; (b) if observer P that uses quantum mechanics to describe an observer O is sure that observer O is certain that $x = \xi$, then P is certain that $x = \xi$; and (c) if observer O is certain that $x = \xi$, then O cannot simultaneously establish that they are confident that $x \neq \xi$. Let us emphasize that these are not statements of any ontic state of the system; they are statements about an observer's degrees of knowledge or belief that uses quantum mechanics.

The specific details of the Frauchiger-Renner (FR) thought experiment are beyond the scope of this essay, but we can still describe it roughly. It is nothing but a generalization of Wigner's setup, but instead, there are two outside observers W and $\bar{W}$ and two friends F and $\bar{F}$ performing measurements on some quantum systems. The experimenters are isolated from the outside world, but they can communicate through a private channel. The outside observers can measure the state of the lab as a whole, which depends on the evolution of the spin state. In summary, F can communicate the result of her measurement of $\bar{F}$, the external observers W and $\bar{W}$ can measure the states of the respective labs, and W and $\bar{W}$ can then communicate with each other. If they use quantum mechanics and the rules (a), (b), and (c) to make their predictions, they will get a contradiction.

The salient feature of this theorem is that it does not assume an ontological model; instead, it talks about the states of belief or knowledge of agents using quantum mechanics to make predictions. In this sense, not only does it apply to any interpretation of quantum mechanics, but also it directly constrains the doxastic interpretations. All the doxastic interpretations make a crucial assumption: quantum theory can be applied to make predictions at any scale. This means, among other things, that one observer can, in principle, describe another observer using a superposition of quantum states. We observe that this is a hidden assumption in the FR theorem, but it is a weak premise since it is a shared feature of the interpretations it intends to target in the first place.

Within the doxastic interpretations, we may distinguish between objective and perspectival interpretations. In terms of the above, objective interpretations assume (b), while perspectival ones do not. The traditional Copenhagen interpretation happens to satisfy all of (a), (b), and (c), so this theorem deals a decisive blow to Copenhagenists and partisans of doxastic objective interpretations. Attempts have been made to break down further assumptions that go into the FR argument [15], or even to disprove the theorem [16], in an attempt to save objectivity. This discussion is still ongoing, but for the most part, the quantum foundations community agrees that this result favours perspectival interpretations. A way to solve the problem could be to decide on a canonical way to place the cut between classical and quantum systems so that observers cannot be described using quantum theory. Such a threshold is unappealing for this anti-ψ-ontological position since defining it could be arbitrary or impossible to define appropriately.

5 Experimental tests for reality

The Frauchiger-Renner thought experiment was an inspiration for a proposal of an actual experiment that can be realized with our current technology. Brukner's proposal [17] is a combination of the FR and Bell setups and concludes that one cannot have locality, observer-independent facts, and independently chosen measurements if a proposed inequality is violated. The assumption that quantum mechanics can be used at any scale (i.e., the Heisenberg cut is arbitrary) is also implicit.

Given that interpretations that admit an ontological model have no choice but to accept a form of non-locality, they will remain unaffected by any conclusions derived from a realization of Brukner's experiment. The experiment was promptly carried out after its proposal [18], and the result is that the inequalities are violated, thus concluding that one of Brukner's proposed conditions must be false. This conclusion resulted in sundry tabloid-worthy headlines claiming that an experiment had proven that reality does not exist.

Nonetheless, performing this experiment is a significant feat, given the technological unfeasibility of scenarios such as FR. The conclusion is that nature agrees with the conclusions derived from the thought experiment; after all, FR managed to discard objective interpretations using a purely deductive approach: objective doxastic interpretations are losing strength on mathematical and empirical grounds.

The experiment is not without loopholes, however. The observers in the experiment are qubits (photons), and it is debatable whether they would qualify as observers. An interpretation where they do is relational quantum mechanics. However, according to Leifer [19], considering such quantum systems as observer runs into a basis problem that can only be solved by a decoherence argument, in the same way as the many-worlds interpretation. That is, observers must be sufficiently decohered to qualify as such. Indeed, so long as the measurement problem is unsolved, or as long as we cannot perform interference experiments on human beings[3], there will be severe ambiguities in the conclusions we can draw from such Wigner Friend-like tests (see [20] for an in-depth discussion from different points of view).

[3] which will probably be deemed unethical.

6 Where do we go from here?

We have learned that the last few decades have seen many constraints on what we can consider as valid interpretations of quantum mechanics. In the more scientific-realist realm, the ontological models must be non-local to reproduce quantum mechanics fully. The requirement of reproducing quantum probabilities seems to be a constraint so strong that it can only be achieved by fine-tuning our theories. While an unpleasant consequence, it is worthwhile investigating whether any mechanisms would make locality emergent. At least, we should attempt to explain how this weaker non-locality does not influence large-scale physics.

The key feature of all these proposals is that quantum mechanics can be applied as a description at all levels, including macroscopic observers. On the less realist side, fairly logical assumptions about different observers' degrees of knowledge or belief seem to lead to logical and experimental contradictions. However, the fact that large objects are decohered, so they cannot interfere, tells us that there is indeed a difference between observers and quantum systems, one that we might only be able to understand from a theory beyond quantum mechanics. There must be an upper bound to the scales where we can apply quantum mechanics, and careful placement of this cut is needed to make sense of some interpretations of quantum mechanics. Similarly, the absence of a theory of quantum gravity provides evidence that quantum mechanics has a lower bound in its scale of applicability.

A common problem seems to be non-locality, as even Brukner's inequality could be taken as evidence of an ultimately non-local theory. However, it is unclear how a theory that does not reference a universal speed limit should be required to manifest a feature such as Bell locality. The interpretations of quantum mechanics could only give us a picture of what happens in the universe if the theory were to be precisely accurate. However, we know it is not. We have already developed quantum field theory, which is compatible with special relativity. The only problem is that quantum field theory is still far from being mathematically well understood. The naive intuition that physicists gain when calculating with it is nothing but an intuition about perturbative results. If non-locality is an issue for a physicist or philosopher, their time would be well invested in understanding the best formal axiomatic systems of quantum field theories that we have obtained so far and then interpreting such results.

Another way to deal with non-locality is to consider local interpretations only, but then we need to give up some other hidden assumptions within the ontological models. One such proposal is superdeterminism, which states that measurement settings cannot be chosen independently from each other or the state preparation. For a long time, it was thought that such models had to be fine-tuned to reproduce the exact results of quantum mechanics and, in particular, the expected correlations for entangled states. Some recent work by Donadi and Hossenfelder [21] has shown that this fine-tuning might not be necessary after all, but it has the dire consequence that free will might not exist.

In the end, the interpretation that one abides by is a judgement of value: one can give up locality, weaken the concept of objective reality, give up free will, among many other options. As long as we have physicists and philosophers exploring all these paths, we can rest assured that progress in physics will come.

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