

---

```

function [c,ceq] = funcNL(xVec,Mb,MG,Mc)
DB = xVec(1);
DC = xVec(2);

ceq = [];

Sut = 148000; %psi

% find Se
Cload = 1;
CsizeB = 0.869*DB^(-0.097);
CsizeG = CsizeB;
CsizeC = 0.869*DC^(-0.097);
Csurf = 2.7*148^-.265;
Ctemp = 1;
Creliab = 0.814;
Sep = 0.5*Sut;
SeB = Cload*CsizeB*Csurf*Ctemp*Creliab*Sep;
SeG = Cload*CsizeG*Csurf*Ctemp*Creliab*Sep;
SeC = Cload*CsizeC*Csurf*Ctemp*Creliab*Sep;

% find fatigue stress conecetration factors at G
[Kt_bending,Kt_torsion] = interpFatigue(DC,DB);
q = 0.862;
Kf_bending = 1+q*(Kt_bending-1);
Kf_torsion = 1+q*(Kt_torsion-1);
sigma_G = Kf_bending*((32*MG)/(pi*DB^3));
T = 600;
tau_torsion_G = Kf_torsion*((16*T)/(pi*DB^3));
sigma_m_G = sqrt(3)*tau_torsion_G;
sigma_m_B = 0;
sigma_m_C = (16*T*sqrt(3))/(pi*DC^3);

sigma_B = (32*Mb)/(pi*DB^3);
sigma_C = (32*Mc)/(pi*DC^3);

c(1) = -1/1.5 + sigma_m_B/Sut + sigma_B/SeB;
c(2) = -1/1.5 + sigma_m_G/Sut + sigma_G/SeG;
c(3) = -1/1.5 + sigma_m_C/Sut + sigma_C/SeC;
c(4) = DC/DB-2;
c(5) = 1.09-DC/DB;

end

```

---

```
function f = funObj(xVec)
f = xVec(1);

end
```

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---

```
function [Kt_bending,Kt_torsion] = interpFatigue(D,d)

DdTorsionVec = [2,1.33,1.2,1.09];
ATorsionVec = [0.86331, 0.84897, 0.83425, 0.90337];
bTorsionVec = [-.23865, -.23161, -.21649, -.12692];
DdBendingVec = [6,3,2,1.5,1.2,1.1,1.07,1.05,1.03,1.02,1.01];
ABendingVec =
    [0.87868,0.89334,0.90879,0.93836,0.97098,0.95120,0.97527,0.98137,0.98061,0.96048,
    bBendingVec =
    [-.33243,-.30860,-.28598,-.25759,-.21796,-.23757,-.20958,-.19653,-.18381,-.17711,
    r = 0.05;
    rd = r/d;
    Dd = [D/d];

ATorsion = interp1(DdTorsionVec,ATorsionVec,Dd);
bTorsion = interp1(DdTorsionVec,bTorsionVec,Dd);
ABending = interp1(DdBendingVec,ABendingVec,Dd);
bBending = interp1(DdBendingVec,bBendingVec,Dd);

Kt_torsion = ATorsion*rd^bTorsion;
Kt_bending = ABending*rd^bBending;

end
```

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restart :  
 with(DETools) :  
 with(LinearAlgebra) :

## ▼ Project 4 Maple Code for Statics

### ▼ Enter Knowns

$x_1 := 1.5625 :$   
 $x_2 := 2.375 :$   
 $x_3 := 3.125 :$   
 $x_4 := 0.6875 :$   
 $\phi_2 := 6 :$   
 $\phi_3 := 2 :$   
 $\phi_4 := 6 :$   
 $F := 900 :$   
 $L_1 := 2 :$

### ▼ Basic Level (No Radial Gear Forces)

$$\text{Equation}_1 := \frac{-\phi_4 \cdot G_{T4}}{2} + L_1 \cdot F = 0 :$$

$$\text{Equation}_2 := -x_1 \cdot G_{T4} - E_z \cdot x_2 = 0 :$$

$$\text{Equation}_3 := x_1 \cdot G_{R4} + E_y \cdot x_2 - F \cdot x_3 = 0 :$$

$$\text{Equation}_4 := H_y + G_{R4} + E_y - F = 0 :$$

$$\text{Equation}_5 := H_z + G_{T4} + E_z = 0 :$$

$$\text{Equation}_6 := \frac{-\phi_3 \cdot G_{T4}}{2} + \frac{\phi_2 \cdot G_{T2}}{2} = 0 :$$

$$\text{Equation}_7 := +x_1 \cdot G_{T4} + x_4 \cdot G_{T2} - x_2 \cdot D_z = 0 :$$

$$\text{Equation}_8 := -x_1 \cdot G_{R4} + x_4 \cdot G_{R2} + x_2 \cdot D_y = 0 :$$

$$\text{Equation}_9 := A_y + G_{R2} - G_{R4} + D_y = 0 :$$

$$\text{Equation}_{10} := A_z - G_{T2} - G_{T4} + D_z = 0 :$$

$$\text{Equation}_{11} := G_{R4} = 0 :$$

$$\text{Equation}_{12} := G_{R2} = 0 :$$

$\text{dsolve}(\{ \text{Equation}_1, \text{Equation}_2, \text{Equation}_3, \text{Equation}_4, \text{Equation}_5, \text{Equation}_6, \text{Equation}_7, \text{Equation}_8, \\ \text{Equation}_9, \text{Equation}_{10}, \text{Equation}_{11}, \text{Equation}_{12} \}, \{ G_{T4}, E_z, G_{R4}, E_y, H_y, H_z, G_{T2}, D_z, G_{R2}, D_y, A_y, \\ A_z \})$

$$\left\{ A_y = 0, A_z = \frac{6600}{19}, D_y = 0, D_z = \frac{8600}{19}, E_y = \frac{22500}{19}, E_z = -\frac{7500}{19}, G_{R2} = 0, G_{R4} = 0, G_{T2} \right. \quad (1.2.1)$$

$$= 200, G_{T4} = 600, H_y = -\frac{5400}{19}, H_z = -\frac{3900}{19} \}$$

## Medium Level (Include Radial Gear Forces)

$$Equation_1 := \frac{-\phi_4 \cdot G_{T4}}{2} + L_1 \cdot F = 0 :$$

$$Equation_2 := -x_1 \cdot G_{T4} - E_z \cdot x_2 = 0 :$$

$$Equation_3 := x_1 \cdot G_{R4} + E_y \cdot x_2 - F \cdot x_3 = 0 :$$

$$Equation_4 := H_y + G_{R4} + E_y - F = 0 :$$

$$Equation_5 := H_z + G_{T4} + E_z = 0 :$$

$$Equation_6 := \frac{-\phi_3 \cdot G_{T4}}{2} + \frac{\phi_2 \cdot G_{T2}}{2} = 0 :$$

$$Equation_7 := +x_1 \cdot G_{T4} + x_4 \cdot G_{T2} - x_2 \cdot D_z = 0 :$$

$$Equation_8 := -x_1 \cdot G_{R4} + x_4 \cdot G_{R2} + x_2 \cdot D_y = 0 :$$

$$Equation_9 := A_y + G_{R2} - G_{R4} + D_y = 0 :$$

$$Equation_{10} := A_z - G_{T2} - G_{T4} + D_z = 0 :$$

$$Equation_{11} := G_{R4} = G_{T4} \cdot \tan\left(\frac{20 \cdot \text{Pi}}{180}\right) :$$

$$Equation_{12} := G_{R2} = G_{T2} \cdot \tan\left(\frac{20 \cdot \text{Pi}}{180}\right) :$$

$$\begin{aligned} & \text{evalf}\left(\text{dsolve}\left(\{Equation_1, Equation_2, Equation_3, Equation_4, Equation_5, Equation_6, Equation_7, \right. \right. \\ & \quad \left. Equation_8, Equation_9, Equation_{10}, Equation_{11}, Equation_{12}\}, \{G_{T4}, E_z, G_{R4}, E_y, H_y, H_z, G_{T2}, D_z, \right. \\ & \quad \left. G_{R2}, D_y, A_y, A_z\}\right) \end{aligned} \quad (1.3.1)$$

$$\begin{aligned} & \{A_y = 22.98759375, A_z = 347.3684211, D_y = 122.6005000, D_z = 452.6315789, E_y \\ & \quad = 1040.538065, E_z = -394.7368421, G_{R2} = 72.79404686, G_{R4} = 218.3821406, G_{T2} \\ & \quad = 200., G_{T4} = 600., H_y = -358.9202060, H_z = -205.2631579\} \end{aligned}$$

---

## Table of Contents

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```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Sam Alvares, Ian Landwehr, Sam Ridgely
% ME480-03: Machine Component Design
% Dr. Constans
%
% Project IV: Fatigue Analysis of Input Shaft
% Due: April 23th, 2021
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

## Given from Problem Statement

```
clc; clear variables; close all;
% Given Measurements          % Units
x1 = 1.5625;                  % in
x2 = 2.375;                   % in
x3 = 3.125;                   % in
x4 = 0.6875;                  % in
L_input = 2.375;              % in
N = L_input/.0001+1;          % Determine # points so that points at
    at 0.001 increments
Sut = 148000;                 % psi
```

## Basic

Results of statics analysis (from Maple)

```
Ay = 0;
Az = 6600/19;
Dy = 0;
Dz = 8600/19;
Ey = 22500/19;
Ez = -7500/19;
GR2 = 0;
GR4 = 0;
GT2 = 200;
GT4 = 600;
Hy = -5400/19;
Hz = -3900/19;
```

---

```

% Find Bending Momemnt: y-direction
[x,My,Mz] = deal(zeros(N,1));
for i = 1:N
    x(i) = (i-1)*L_input/(N-1);
    My(i) = Ay*heaviside(x(i))*x(i)+GR2*heaviside(x(i)-x4)*(x(i)-x4)...
            -GR4*heaviside(x(i)-x1)*(x(i)-x1)+Dy*heaviside(x(i)-
x2)*(x(i)-x2);
end

% z-direction
for i = 1:N
    Mz(i) = Az*heaviside(x(i))*x(i)-GT2*heaviside(x(i)-x4)*(x(i)-x4)...
            -GT4*heaviside(x(i)-x1)*(x(i)-x1)+Dz*heaviside(x(i)-
x2)*(x(i)-x2);
end

% Resultant
MR = sqrt(My.^2+Mz.^2);

% Find resultant moment at key locations
xb = x4;
xG = 1.0625;
xc = x1;
for i = 1:length(x)
    xCheck = x(i);
    if xCheck == xb
        Ib = i;
    elseif xCheck == xG
        IG = i;
    elseif xCheck == xc
        Ic = i;
    end
end

Mb = MR(Ib);
MG = MR(IG);
Mc = MR(Ic);

% Create bending moment diagrams as subplot, label points b,G,c
figure(1)
subplot(3,1,1), plot(x,My,'g','LineWidth',1), grid on
title('Basic: Bending Moment Diagrams','FontSize',20)
ylabel('M_{y} (lb_{f}-in)','FontSize',12)
xlabel('x (in)','FontSize',12)
subplot(3,1,2), plot(x,Mz,'g','LineWidth',1), grid on
ylabel('M_{z} (lb_{f}-in)','FontSize',12)
xlabel('Position (in)','FontSize',12)
subplot(3,1,3), plot(x,MR,'g','LineWidth',1), grid on
ylabel('M_{R} (lb_{f}-in)','FontSize',12)
xlabel('Position (in)','FontSize',12)
hold on
plot(xb,Mb,'ro')
label = 'B';
text(xb,Mb,label,'VerticalAlignment','top','HorizontalAlignment','left')

```

---

---

```

plot(xG,MG,'ro')
label = 'G';
text(xG,MG,label,'VerticalAlignment','top','HorizontalAlignment','left')
plot(xc,Mc,'ro')
label = 'C';
text(xc,Mc,label,'VerticalAlignment','top','HorizontalAlignment','left')

% Run optimization problem
x0 = [7/16;5/8];
A = [-1 0;
      0 -1
      1 -1];
b = [0;0;-1/16];

xSol = fmincon(@funObj,x0,A,b,[],[],[],
[],@(xVec)funcNL(xVec,Mb,MG,Mc));

% Solve for results with optimized value - rounded up to 1/16
increment
DB = ceil(xSol(1) * 16) / 16;
DC = ceil(xSol(2) * 16) / 16;
Cload = 1;
CsizeB = 0.869*DB^(-0.097);
CsizeG = CsizeB;
CsizeC = 0.869*DC^(-0.097);
Csurf = 2.7*148^-.265;
Ctemp = 1;
Creliab = 0.814;
Sep = 0.5*Sut;
SeB = Cload*CsizeB*Csurf*Ctemp*Creliab*Sep;
SeG = Cload*CsizeG*Csurf*Ctemp*Creliab*Sep;
SeC = Cload*CsizeC*Csurf*Ctemp*Creliab*Sep;

% find fatigue stress conecetration factors at G
[Kt_bending,Kt_torsion] = interpFatigue(DC,DB);
q = 0.862;
Kf_bending = 1+q*(Kt_bending-1);
Kf_torsion = 1+q*(Kt_torsion-1);
sigma_G = Kf_bending*((32*MG)/(pi*DB^3));
T = 600;
tau_torsion_G = Kf_torsion*((16*T)/(pi*DB^3));
sigma_m_G = sqrt(3)*tau_torsion_G;
sigma_m_B = 0;
sigma_m_C = (16*T*sqrt(3))/(pi*DC^3);
sigma_B = (32*Mb)/(pi*DB^3);
sigma_C = (32*Mc)/(pi*DC^3);

FOSB = 1/(sigma_m_B/Sut + sigma_B/SeB);
FOSG = 1/(sigma_m_G/Sut + sigma_G/SeG);
FOSC = 1/(sigma_m_C/Sut + sigma_C/SeC);

fprintf('\n----- BASIC -----')
fprintf('\nOptimization Solution: DB = %1.4f, DC = %1.4f\n',xSol(1),xSol(2))

```

---



---

```
fprintf('Round up to nearest 1/16": DB = %1.4f, DC = %1.4f\n',DB,DC)
fprintf('FOSB = %1.4f\nFOSG = %1.4f\nFOSC = %1.4f\n',FOSB,FOSG,FOSC)
```

*Local minimum found that satisfies the constraints.*

*Optimization completed because the objective function is non-decreasing in feasible directions, to within the value of the optimality tolerance, and constraints are satisfied to within the value of the constraint tolerance.*

----- BASIC -----

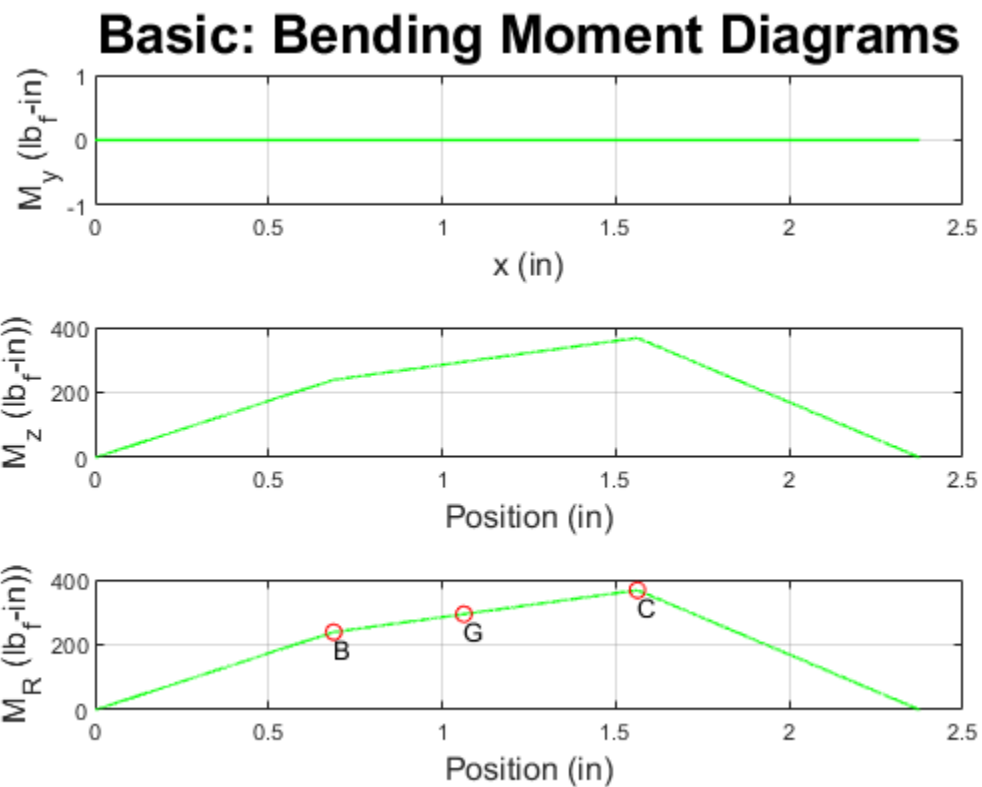
Optimization Solution: DB = 0.6320, DC = 0.6945

Round up to nearest 1/16": DB = 0.6875, DC = 0.7500

FOSB = 5.2079

FOSG = 1.9098

FOSC = 3.1800



## Medium

Results of statics analysis (from Maple)

A<sub>y</sub> = 22.98759375;

A<sub>z</sub> = 347.3684211;

---

```

Dy = 122.6005000;
Dz = 452.6315789;
Ey = 1040.538065;
Ez = -394.7368421;
GR2 = 72.79404686;
GR4 = 218.3821406;
GT2 = 200.;
GT4 = 600.;
Hy = -358.9202060;
Hz = -205.2631579;

% Find Bending Momemnt: y-direction
[x,My,Mz] = deal(zeros(N,1));
for i = 1:N
    x(i) = (i-1)*L_input/(N-1);
    My(i) = Ay*heaviside(x(i))*x(i)+GR2*heaviside(x(i)-x4)*(x(i)-x4)...
            -GR4*heaviside(x(i)-x1)*(x(i)-x1)+Dy*heaviside(x(i)-
x2)*(x(i)-x2);
end

% z-direction
for i = 1:N
    Mz(i) = Az*heaviside(x(i))*x(i)-GT2*heaviside(x(i)-x4)*(x(i)-x4)...
            -GT4*heaviside(x(i)-x1)*(x(i)-x1)+Dz*heaviside(x(i)-
x2)*(x(i)-x2);
end

% Resultant
MR = sqrt(My.^2+Mz.^2);

% Find resultant moment at key locations
xb = x4;
xG = 1.0625;
xc = x1;
for i = 1:length(x)
    xCheck = x(i);
    if xCheck == xb
        Ib = i;
    elseif xCheck == xG
        IG = i;
    elseif xCheck == xc
        Ic = i;
    end
end

Mb = MR(Ib);
MG = MR(IG);
Mc = MR(Ic);

% Create bending moment diagrams as subplot, label points b,G,c
figure(2)
subplot(3,1,1), plot(x,My,'g','LineWidth',1), grid on
title('Medium: Bending Moment Diagrams','FontSize',20)
ylabel('M_{y} (lb_{f}-in)','FontSize',12)

```

---

---

```

xlabel('x (in)','FontSize',12)
subplot(3,1,2), plot(x,Mz,'g','LineWidth',1), grid on
ylabel('M_{z} (lb_{f}-in)','FontSize',12)
xlabel('Position (in)','FontSize',12)
subplot(3,1,3), plot(x,MR,'g','LineWidth',1), grid on
ylabel('M_{R} (lb_{f}-in)','FontSize',12)
xlabel('Position (in)','FontSize',12)
hold on
plot(xb,Mb,'ro')
label = 'B';
text(xb,Mb,label,'VerticalAlignment','top','HorizontalAlignment','left')
plot(xG,MG,'ro')
label = 'G';
text(xG,MG,label,'VerticalAlignment','top','HorizontalAlignment','left')
plot(xc,Mc,'ro')
label = 'C';
text(xc,Mc,label,'VerticalAlignment','top','HorizontalAlignment','left')

% Run optimization problem
x0 = [7/16;5/8];
A = [-1 0;
      0 -1
      1 -1];
b = [0;0;-1/16];

xSol = fmincon(@funObj,x0,A,b,[],[],[],
[],@(xVec)funcNL(xVec,Mb,MG,Mc));

% Solve for results with optimized value - rounded up to 1/16
increment
DB = ceil(xSol(1) * 16) / 16;
DC = ceil(xSol(2) * 16) / 16;
Cload = 1;
CsizeB = 0.869*DB^(-0.097);
CsizeG = CsizeB;
CsizeC = 0.869*DC^(-0.097);
Csurf = 2.7*148^-.265;
Ctemp = 1;
Creliab = 0.814;
Sep = 0.5*Sut;
SeB = Cload*CsizeB*Csurf*Ctemp*Creliab*Sep;
SeG = Cload*CsizeG*Csurf*Ctemp*Creliab*Sep;
SeC = Cload*CsizeC*Csurf*Ctemp*Creliab*Sep;

% find fatigue stress conecetration factors at G
[Kt_bending,Kt_torsion] = interpFatigue(DC,DB);
q = 0.862;
Kf_bending = 1+q*(Kt_bending-1);
Kf_torsion = 1+q*(Kt_torsion-1);
sigma_G = Kf_bending*((32*MG)/(pi*DB^3));
T = 600;
tau_torsion_G = Kf_torsion*((16*T)/(pi*DB^3));
sigma_m_G = sqrt(3)*tau_torsion_G;
sigma_m_B = 0;

```

---

---

```

sigma_m_C = (16*T*sqrt(3))/(pi*DC^3);
sigma_B = (32*Mb)/(pi*DB^3);
sigma_C = (32*Mc)/(pi*DC^3);

FOSB = 1/(sigma_m_B/Sut + sigma_B/SeB);
FOSG = 1/(sigma_m_G/Sut + sigma_G/SeG);
FOSC = 1/(sigma_m_C/Sut + sigma_C/SeC);

fprintf('\n----- MEDIUM -----')
fprintf('\nOptimization Solution: DB = %1.4f, DC = %1.4f\n',xSol(1),xSol(2))
fprintf('Round up to nearest 1/16": DB = %1.4f, DC = %1.4f\n',DB,DC)
fprintf('FOSB = %1.4f\nFOSG = %1.4f\nFOSC = %1.4f\n',FOSB,FOSG,FOSC)

```

*Local minimum found that satisfies the constraints.*

*Optimization completed because the objective function is non-decreasing in feasible directions, to within the value of the optimality tolerance, and constraints are satisfied to within the value of the constraint tolerance.*

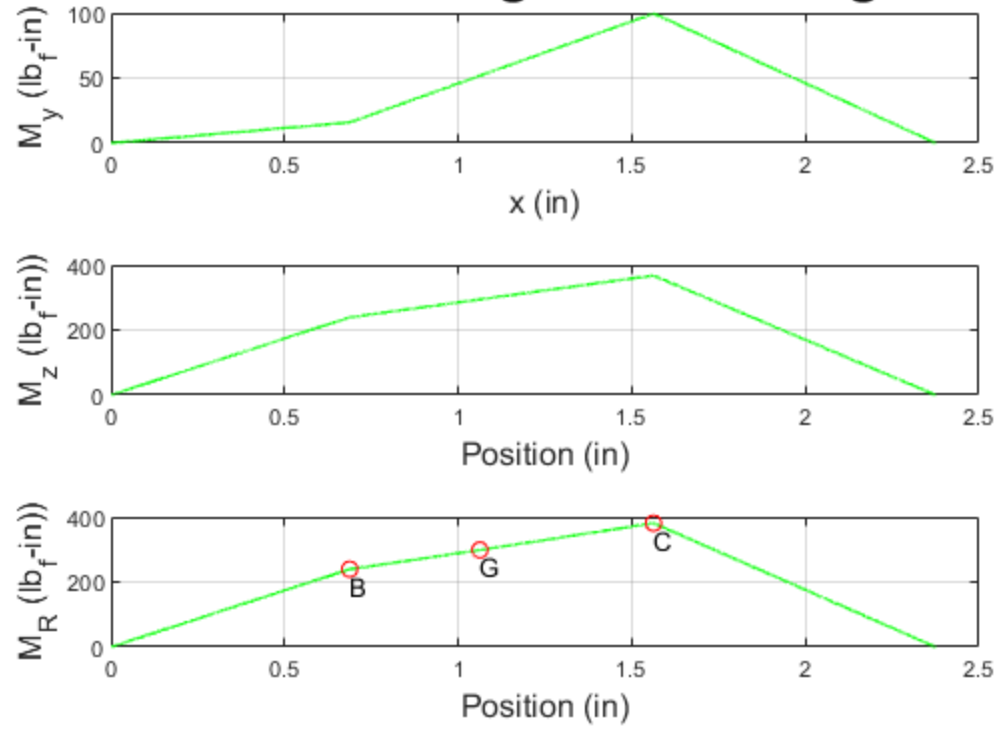
```

----- MEDIUM -----
Optimization Solution: DB = 0.6345, DC = 0.6970
Round up to nearest 1/16": DB = 0.6875, DC = 0.7500
FOSB = 5.1965
FOSG = 1.8882
FOSC = 3.0985

```

---

## Medium: Bending Moment Diagrams



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