new results and new treatments by the author. This volume can be highly recommended to those who are working in the field of Markov processes or who wish to learn more about the subject.

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Characteristic Functions. By E. LUKACS. Hafner Publ. Co., New York, 1960. 216 pp. $6.50.

The characteristic function, which is the Fourier transform of the distribution function, is one of the most powerful tools available in probability theory and statistics. During the past few decades there has been extensive research on characteristic functions, but the results are widely scattered over many journals. This monograph brings together these mathematical results, many for the first time, and does so in a well organized and very readable form. A second volume in the same series by E. Lukacs and R. G. Laha will treat applications of characteristic functions to statistics. In the present volume most of the theorems are given with detailed proofs so that the book is essentially self-contained, thus making it very valuable for students and those without prior training in these methods. The first four chapters (77 pages) give the classical results for characteristic functions; for example, the uniqueness theorem, inversion formulae, the convolution theorem, the theorems of Helly, the continuity theorem, and criteria for characteristic functions. The last five chapters (121 pages) deal with various factorization problems, and include very clear and complete accounts of infinitely divisible characteristic functions, indecomposable characteristic functions, and stable characteristic functions. This volume would be a welcome addition to the libraries of those working in probability and statistics, and it would also be a very useful book to all in applied mathematics who use Fourier techniques.

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Contributions to the Theory of Nonlinear Oscillations, Vol. 5. (Annals of Math. Studies, No. 45.) Edited by L. CESARI, J. LASALLE, and S. LEFSCHETZ. Princeton University Press, Princeton, N. J., 1960. 286 pp. $4.50.

This volume, the fifth of a worthy series, consists of 13 papers covering a variety of new results in ordinary differential equations of primary interest to research workers in this and related areas. Included also is a brief preface by the editors and an appendix by E. Pinney consisting of errata to his earlier paper in Vol. III of this series. Because of the variety of topics covered the review must necessarily consist of only a brief remark about the subject of each paper.

J. P. LASALLE extends previous work on the "bang-bang" principle by obtaining new results for the time optimal problem for control systems which are linear in the sense that the elements being controlled are linear and as a function of time the steering enters linearly.

J. Jarník and J. Kurzweil continue recent work by the second author on generalized differential equations with a study of continuous dependence on a parameter. There is an unfortunate omission and misprint in the main condition of the paper ((6), p. 26), which the reviewer believes should read

$$\sum_{i=1}^{\infty} 2^i \Psi(\sigma/2^i) < \infty.$$

Jane Cronin studies the problem of existence of periodic solutions of perturbed systems of nonlinear equations (both nonautonomous and autonomous) by combining the classical method of Poincaré with the use of the notion of topological degree. This yields a new approach (and also more general results) to the problem in the "degenerate case" (the relevant variational equation has a periodic solution) with various degrees of degeneracy.

In the first of two papers J. K. Hale extends the method of successive approximations developed by Cesari, Hale, Gambill and studies the behaviour of solutions of linear second order periodic systems of the form $y_j'' + \sigma_j^2 y_j = \epsilon \sum_{k=1}^{n} \Phi_{jk}(t) y_k (j = 1, 2, \cdots, n)$, where σ_j are positive constants, ϵ is a small parameter, and Φ_{jk} are periodic functions of period $2\pi/\omega$, near "resonance points" ($2\sigma_j = s\omega$, $\sigma_j \pm \sigma_k = s\omega$, $j \neq k$, for some integer s).

In his second contribution Hale employs the method mentioned above to deduce criteria for asymptotic stability and asymptotic orbital stability (in case of autonomous systems) of periodic solutions of weakly nonlinear systems of second order equations.

In the next paper L. Cesari uses functional analysis techniques (in particular the Schauder, Banach, and also the Brouwer fixed point theorems) to obtain the existence of periodic solutions, in some cases families of such solutions, of nonlinear systems containing a small parameter under very weak assumptions (continuity or at most Lipschitz conditions are required). This paper sheds considerable light and at the same time extends the method of successive approximations referred to in the preceeding two papers.

A. Stokes uses the Tychonov fixed point theorem to obtain in a simple way a number of recent known results on global existence, boundedness, and stability of solutions of nonlinear systems with, however, slightly more stringent hypotheses ($G(t, r)$ monotonic in r for each fixed t).

Departing from the usual geometric approaches to study qualitative behaviour, L. Markus uses a strictly algebraic approach—namely, nonassociative but commutative real linear algebras—to classify real systems of quadratic differential equations in the plane and (partly) in higher dimensions.

G. Reeb considers an interesting property of the totality of bounded solutions of certain dynamical systems.

P. Mendelson studies Lagrange stable motions in a neighborhood of critical points of real nonlinear autonomous systems by utilizing methods of topological dynamics due primarily to Ważewski.

The joint paper by J. ANDRE and P. SEIBERT considers the local theory of nonlinear systems involving functions which are only piecewise continuous. This study, motivated by problems of automatic control, also considers certain systems with a time lag due to switching delay.

C. COLEMAN studies the problem of asymptotic stability in 3-space for nonlinear systems, where the leading terms are homogeneous functions of degree $m > 1$. When $m = 1$ his results reduce to the classical theorems of Lyapunov-Perron.

The monograph concludes with a paper by FIGUEIREDO on the existence and uniqueness of periodic solutions of the general Liénard type scalar equation

$$\ddot{x} + R(\dot{x}) + x = 0.$$

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Elements of the Theory of Functions and Functional Analysis. By A. N. KOLMOGOROV and S. V. FOMIN. Vol. I. *Metric and Normed Spaces.* (Translated from the first Russian edition, 1954, by L. F. BORON.) Graylock Press, Rochester, New York, 1957. ix + 129 pp. $3.95. and Vol. II. *Measure—The Lebesgue Integral—Hilbert Space.* (Translated from the first Russian edition, 1960, by H. KAMEL and H. KOMM.) Graylock Press, Albany, New York, 1961. ix + 128 pp. $4.00.

The appearance of the second volume makes it possible to review the entire work at this time. The translators and publisher are to be congratulated for making these volumes available in English so soon after publication of the original Russian editions, the translations and printing being models of clarity. A good deal of the material covered would be classified in this country as theory of real variables, namely: Vol. I., Chap. 1. Fundamentals of Set Theory, Chap. 2. Metric Spaces; Vol. II., Chap. 5. Measure Theory, Chap. 6. Measurable Functions, and Chap. 7. The Lebesgue Integral. The approach to measure and integration is fairly standard and straightforward. The properties of the real number system are assumed known to the reader. These concepts are illustrated and expanded in the chapters dealing with functional analysis proper, which are: Vol. I., Chap. 3. Normed Linear Spaces, Chap. 4. Linear Operator Equations; Vol. II. Chap. 8. Square Integrable Functions, Chap. 9. Abstract Hilbert Space. Integral Equations with Symmetric Kernel.

Vol. I. also contains a supplement to Chap. 3, which gives a brief introduction to generalized functions, which are defined to be "kernels" for linear functionals on the space of everywhere differentiable functions which vanish outside some interval. Vol. I. concludes with lists of symbols, definitions, and theorems, and a bibliography and index. Exercises have been added to Vol. II. by one of the translators, which enhances its usefulness as a text. Vol. II. closes with a supplement containing corrections to Vol. I and an index.

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