



# Master in Computer Vision *Barcelona*

Module: 3D Vision

Project: 3D recovery of urban scenes

Session 3

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# Session 3

**Goal:** compute the fundamental matrix that relates two images

**Algorithms:**

- Normalized 8-point algorithm (algebraic method).
- Robust normalized 8-point algorithm.

**Application:** Photo-sequencing.

# Session 3

## Mandatory tasks:

- Function that estimates the fundamental matrix  $F$  with the normalized 8-point algorithm.
- Compute the theoretical fundamental matrix that relates two images with corresponding camera matrices  $P = [I|0]$ , and  $P' = [R|t]$ .
- Function that robustly estimates  $F$  using the previous function and RANSAC (you can use as a basis the provided function in lab 2: 'Ransac\_DLT\_homography').

The inliers are obtained with a threshold on the first order approximation of the geometric error: **Sampson distance**,

$$\frac{(x_i'^T F x_i)^2}{(F x_i)_1^2 + (F x_i)_2^2 + (F^T x_i')_1^2 + (F^T x_i')_2^2}$$

- Compute the epipolar lines of the matching points in both images.
- Apply the theoretical concepts to do photo-sequencing.

# Session 3

## Geometric distance

(used for determining the inliers in the RANSAC function)

$$d([x_i], [\hat{x}_i])^2 + d([x'_i], [\hat{x}'_i])^2 \text{ s. t. } \hat{x}'_i^T F \hat{x}_i = 0 \quad \forall i$$

where the different matchings  $x_i \longleftrightarrow x'_i$  are the data,  
[.] is the projection operator to Euclidean coordinates.

# Session 3

## Geometric distance

A variant is (we use the distance of a point to a line  $d(x, l) = |x^T l| / ||l||$ ):

$$\begin{aligned} & d(x'_i, Fx_i)^2 + d(x_i, F^T x'_i)^2 \\ &= (x'_i{}^T Fx_i)^2 \left( \frac{1}{(Fx_i)_1^2 + (Fx_i)_2^2} + \frac{1}{(F^T x'_i)_1^2 + (F^T x'_i)_2^2} \right) \end{aligned}$$

We will use the **Sampson error**

(1st order approx. of the geometric distance)

$$\frac{(x'_i{}^T Fx_i)^2}{(Fx_i)_1^2 + (Fx_i)_2^2 + (F^T x'_i)_1^2 + (F^T x'_i)_2^2}$$

# Photo Sequencing

Int J Comput Vis (2014) 110:275–289  
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## Photo Sequencing

Tali Dekel (Basha) · Yael Moses · Shai Avidan

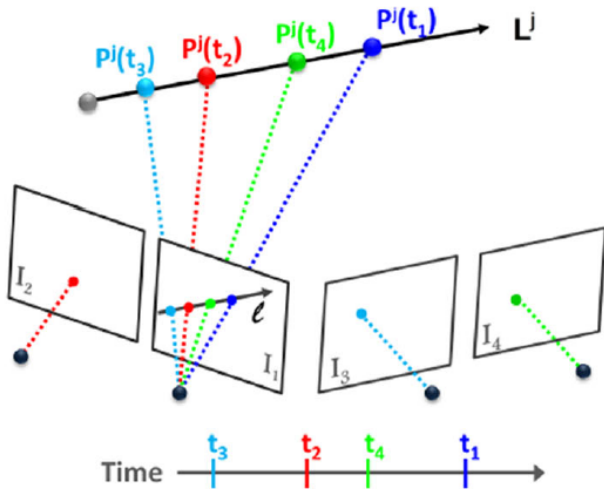
Given a set of images of a dynamic scene taken at different viewpoints and different time instants, the photo-sequencing algorithm establishes an ordering of the images according to the time they were taken.



There are two underlying hypothesis:

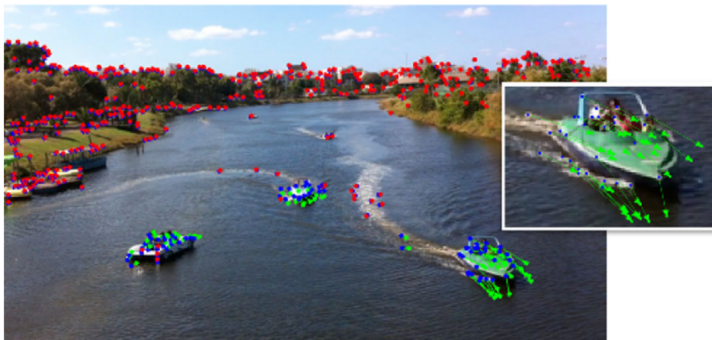
- Object trajectories can be approximated by straight lines.
- Two of the images are taken from approximately the same position.

# Photo Sequencing



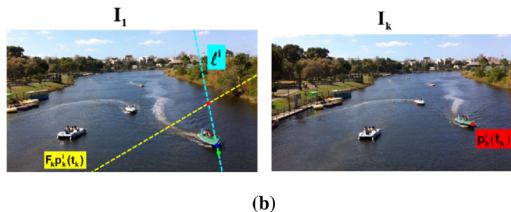
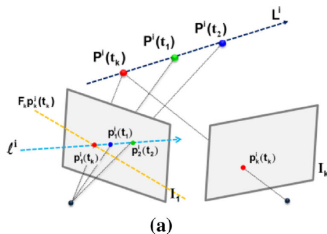
# Photo Sequencing

Computing static and dynamic features (thanks to hypothesis 2)





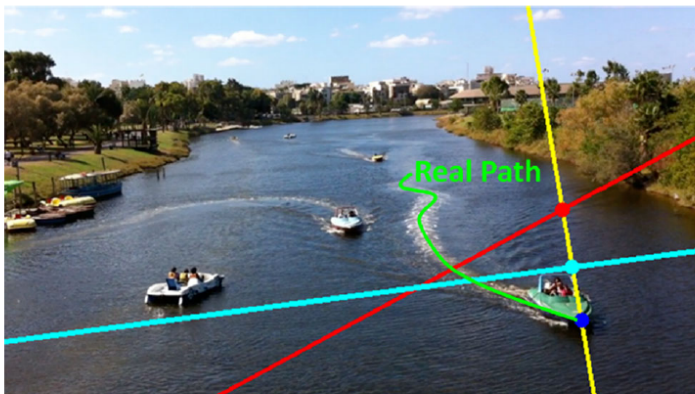
# Photo Sequencing



**Fig. 5** (a) The projection of the trajectory,  $L^i$ , of the point  $P^i$ , forms the line  $\ell^i$  on image  $I_1$ . The feature points  $p_1^i(t_1)$ ,  $p_2^i(t_2)$ , in image  $I_1$ , and  $p_k^i(t_k)$  in image  $I_k$ , are corresponding dynamic features. The line  $\ell^i$  intersects the *epipolar line* (in yellow), which corresponds to  $p_k^i$ . The intersection point,  $p_1^i(t_k)$ , is the projection of  $P^i$  onto  $I_1$  at time

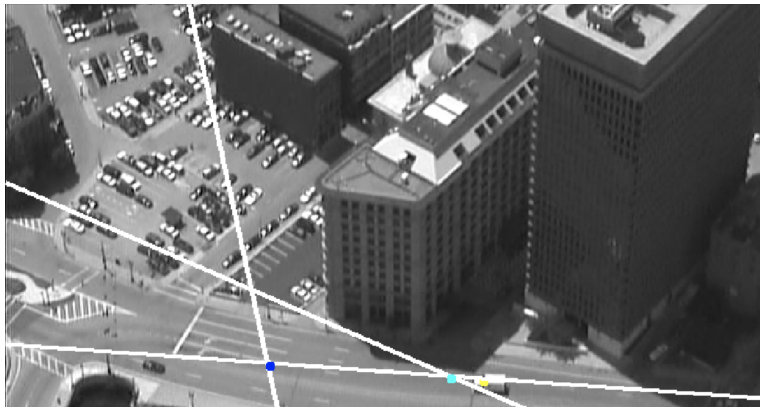
step  $t_k$ . The spatial order of  $p_1^i(t_1)$ ,  $p_2^i(t_2)$ , and  $p_k^i(t_k)$ , along  $\ell^i$ , defines the temporal order between  $I_1$ ,  $I_2$  and  $I_k$ . (b) The computation on real images: the projected trajectory,  $\ell^i$ , in cyan; the *epipolar line* in yellow; the intersection in red

# Photo Sequencing



**Fig. 6** Linear Motion Assumptions: In *green*, the real path of the green boat; in *yellow*, the approximated 2D image line. The epipolar lines intersect both the real path and the 2D image line. The spatial order of both sets of intersections is the same

# Photo Sequencing



# Session 3

**Language:** PYTHON

## To Do:

- Complete the code in lab3.ipynb as indicated in the same file (e.g. ground-truth  $F$ , epipolar lines)
- Write the function `fundamental_matrix`
- Write the function `Ransac_fundamental_matrix`
- Complete the code on photo-sequencing, dynamic feature given

# Evaluation

To deliver **before 11am of the day before** the next lab session:

- **Code deliverable:**
  - READY TO BE LAUNCHED on the provided images
- **Short document:**
  - Results
  - Problems and comments, conclusions

# Evaluation

## Grading:

- Report: **2 points**
- Normalized 8-point algorithm: **2 points**
- F from P1 and P2: **1 point**
- Robust 8-point algorithm (RANSAC): **2 points**
- Epipolar lines: **1 point**
- Photo-sequencing: **2 points**