Master's Degree in Research in Artificial Intelligence
UNED. Academic year 2024-25

Assignments of the subject "Probabilistic Methods"

Topic 4. Decision analysis

D Delivery date: April 26, 2025.

Please, before submitting the exercises, read the instructions found in Agora carefully again.

To resolve the following questions, you may find it helpful to refer to page 115 of [2], section 2.3 of [1], and video classes.

Exercise 4.1.

Let be an influence diagram whose graph is given by the following figure.

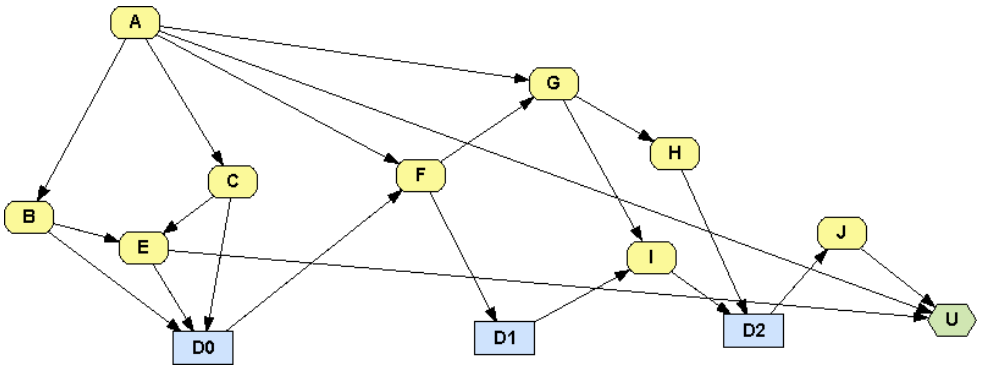


Figure 1: Influence diagram with three decisions.

1. Indicate what is the form of the probability and utility functions for this ID. (For example, $P(a)$, $P(b|a)$, etc. They do not need to be expressed in the form of a table.)
2. Determine the **sets C_0**, **C_1**, **C_2,** and **C_3**.
3. Indicate *PredInf*(D_i) for each of the three decisions.
4. Indicate in detail the steps necessary to evaluate the diagram by eliminating variables with division of potentials. (It is enough to indicate the mathematical expressions he would use.)
5. Ditto for bow inversion.

Solution 4.1.

1 & 3)

Based on the given network structure, I determine the form of probability and utility functions for this influence diagram (ID).

Classification of Nodes

First, I'll identify the types of nodes in this diagram:

- Decision nodes: D0, D1, D2 (represented as rectangles in IDs)
- Utility node: U (represented as hexagon in IDs)
- Chance nodes: A, B, C, E, F, G, H, I, J (represented as ovals in IDs)

Decision Ordering

Looking at the network structure, we can identify the sequence of decisions through the directed paths:

D0 → F → D1 → I → D2

This indicates that decisions are made in the order: D0, then D1, then D2.

Information Available for Decisions

For each decision node, we need to determine what information is available when making that decision:

- For D0: The values of B, C, and E are known (direct parents)
- For D1: The value of F is known (direct parent)
- For D2: The values of H and I are known (direct parents)

With the no-forgetting hypothesis, each decision also has access to previous decisions and observations:

- PredInf(D0) = {B, C, E}
- PredInf(D1) = {B, C, E, D0, F}
- PredInf(D2) = {B, C, E, D0, F, D1, H, I}

Probability Functions

The form of probability functions for each chance node is:

- P(a)
- P(b|a)
- P(c|a)
- P(e|b,c)
- P(f|a,d0)
- P(g|a,f)
- P(h|g)
- P(i|g,d1)
- P(j|d2)

Utility Function

The utility function depends on its parent nodes:

- U(a,e,j)

This represents the utility as a function of values of nodes A, E, and J.

2)

Determining Sets C0, C1, C2, and C3

To determine these sets for the influence diagram, I need to analyze which chance nodes provide information for each decision.

According to the definition, in an influence diagram with n decisions {D0, D1, ..., Dn-1}, the partition contains n+1 subsets {C0, C1, ..., Cn}, where:

- Ci is the set of chance variables C such that there is a link C → Di but no link C → Dj with j < i
- Cn is the set of variables having no link to any decision node

Looking at the network structure:

For C0 (variables known when making decision D0):

- From the diagram, we can see that B → D0, C → D0, and E → D0
- These are variables that provide information for the first decision
- Therefore, C0 = {B, C, E}

For C1 (variables known for D1 but not for D0):

- Only F → D1 directly
- Therefore, C1 = {F}

For C2 (variables known for D2 but not for earlier decisions):

- H → D2 and I → D2
- Therefore, C2 = {H, I}

For C3 (variables not directly observed for any decision):

- A doesn't point to any decision node
- G doesn't point to any decision node
- J doesn't point to any decision node
- Therefore, C3 = {A, G, J}

This partition completely categorizes all chance nodes in the influence diagram according to which decision they directly inform.

4)

Variable elimination with division of potentials is an efficient algorithm for evaluating influence diagrams. Here are the detailed steps for the given diagram:

Initialization

1. *Chance potentials*
   $\Phi = \{P(a),\ P(b \mid a),\ P(c \mid a),\ P(e \mid b,c),\ P(f \mid a,d_0),\ P(g \mid a,f),\ P(h \mid g),\ P(i \mid g,d_1),\ P(j \mid d_2)\}$.
2. *Utility potentials* Here there is a single value node $U(a,e,j)$, so
   $$\Psi = \{U(a,e,j)\}.$$
3. *Elimination order* We must eliminate in an order that never "marginalizes" a decision before we've eliminated its parent–chance nodes. A valid sequence is:
   $$J,\ G,\ A,\ H,\ I,\ D_2,\ F,\ D_1,\ B,\ C,\ E,\ D_0.$$

- D_2 has parents $\{H, I\}$ ⇒ eliminate H, I before D_2.
- D_1 has parent $\{F\}$ ⇒ eliminate F before D_1.
- D_0 has parents $\{B, C, E\}$ ⇒ eliminate B, C, E before D_0.

Elimination Process

We'll write each new factor we create in bold and then remove the old ones that produced it.

1. Eliminate J

- Combine $P(j \mid d_2)$ with the only utility containing J, namely $U(a, e, j)$.
- New utility factor over (a, e, d_2):

$$\mathrm{U_1(a, e, d_2)} = \sum_j P(j \mid d_2)\ U(a, e, j).$$

- Drop $P(j \mid d_2)$ and the old $U(a, e, j)$.

2. Eliminate G

- Combine the three chance factors

$$P(g \mid a, f) \times P(h \mid g) \times P(i \mid g, d_1).$$

- Sum out g, producing a new joint chance factor over (a, f, h, i, d_1):

$$\Phi_{\mathrm{G}}(\mathrm{a, f, h, i, d_1}) = \sum_g P(g \mid a, f)\, P(h \mid g)\, P(i \mid g, d_1).$$

- Drop the three original factors.

3. Eliminate A

- Combine

$$P(a) \times P(b \mid a) \times P(c \mid a) \times P(f \mid a, d_0) \quad \text{with} \quad \Phi_G(a, f, h, i, d_1).$$

- Sum out a, yielding

$$\Phi_{\mathrm{A}}(\mathrm{b, c, f, h, i, d_0, d_1}) = \sum_a P(a)\, P(b \mid a)\, P(c \mid a)\, P(f \mid a, d_0)\, \Phi_G(a, f, h, i, d_1).$$

- Drop the four original plus Φ_G.

4. Eliminate H

- Combine $\Phi_A(b, c, f, h, i, d_0, d_1)$ (the only factor with H).
- Sum out h:

$$\Phi_{\mathrm{H}}(\mathrm{b, c, f, i, d_0, d_1}) = \sum_h \Phi_A(b, c, f, h, i, d_0, d_1).$$

- Drop Φ_A.

5. Eliminate I

- Combine $\Phi_H(b, c, f, i, d_0, d_1)$ and (if you wish) any utility factor that still mentions I. In our case, only Φ_H remains.
- Sum out i:

$$\Phi_{\mathrm{I}}(\mathrm{b, c, f, d_0, d_1}) = \sum_i \Phi_H(b, c, f, i, d_0, d_1).$$

- Drop Φ_H.

6. Eliminate decision D_2. *Eliminate decision D_2*

- Combine the new utility $U_1(a, e, d_2)$ with any chance/utility factors involving d_2. (At this point only U_1 remains over d_2.)
- Maximize over d_2:

$$\mathrm{U_2(a, e)} = \max_{d_2} U_1(a, e, d_2), \quad \delta_{D_2}(h, i) = \operatorname{argmax}_{d_2} U_1(a, e, d_2).$$

- Drop U_1.

7. Eliminate F

- Combine the chance factor $\Phi_I(b, c, f, d_0, d_1)$ with any utility factor still mentioning F. (At this point only Φ_I remains.)
- Sum out f:

$$\Phi_{\mathrm{F}}(\mathrm{b}, \mathrm{c}, \mathrm{d}_0, \mathrm{d}_1) \;=\; \sum_{f} \Phi_I(b, c, f, d_0, d_1).$$

- Drop Φ_I.

8. Eliminate decision D_1

- Combine $\Phi_F(b, c, d_0, d_1)$ with any utility over d_1 (none remains beyond the chance factor).
- Maximize over d_1:

$$\mathrm{U}_3(\mathrm{b}, \mathrm{c}, \mathrm{d}_0) \;=\; \max_{d_1} \Phi_F(b, c, d_0, d_1), \quad \delta_{D_1}(f) = \operatorname{argmax}_{d_1} \Phi_F(b, c, d_0, d_1).$$

- Drop Φ_F.

9. Eliminate B, C, E (in any order)

For each of these three chance nodes in turn:

1. Combine the current factor (starting with $U_3(b, c, d_0)$ for B) with its CPT:
   - For B: combine $U_3(b, c, d_0)$ with $P(b \mid a)$ (but note a is gone!), so actually just sum over b.
   - Likewise for C and E.
2. Sum out the node, producing a new factor of lower dimension.

After these three steps you arrive at a factor

$$\mathrm{U}_4(\mathrm{d}_0) \;=\; \sum_{b,c,e} U_3(b, c, d_0)\, P(b \mid a)\, P(c \mid a)\, P(e \mid b, c)\,,$$

and all chance CPTs are gone.

10. Eliminate decision D_0

- Maximize the remaining utility over d_0:

$$\mathrm{MEU} \;=\; \max_{d_0} U_4(d_0), \quad \delta_{D_0}(b, c, e) = \operatorname{argmax}_{d_0} U_4(d_0).$$

This final maximized value is the maximum expected utility, and the recorded $\delta_{D_2}, \delta_{D_1}, \delta_{D_0}$ together form the optimal policy.

5)

The core idea is to reverse every "non-sequential" arc so that all chance-to-chance arcs respect the decision ordering, then remove nodes (chance via summation, decisions via maximization, and value nodes via incorporation into utility).

<u>1. Preliminaries</u>

- Decision windows
  a. Before D_0: $\{A, B, C, E\}$
  b. Between D_0 and D_1: $\{F\}$
  c. Between D_1 and D_2: $\{H, I\}$
  d. After D_2: $\{J\}$
- Non-sequential arcs are those from a later window back into an earlier window. Each such arc $X \to Y$ must be reversed before any node removals.
- Reversal rule (for chance nodes $X \to Y$): Given

$$P(X \mid \mathrm{Pa}_X), \quad P(Y \mid X, \mathrm{Pa}_Y)$$

we replace them by

$$P(Y \mid \mathrm{Pa}_Y), \quad P(X \mid Y, \mathrm{Pa}_X, \mathrm{Pa}_Y)$$

using Bayes' rule:

$$P(Y \mid \mathrm{Pa}_Y) = \sum_x P(x \mid \mathrm{Pa}_X)\, P(Y \mid x, \mathrm{Pa}_Y),$$

$$P(x \mid Y, \mathrm{Pa}_X, \mathrm{Pa}_Y) = \frac{P(x \mid \mathrm{Pa}_X)\, P(Y \mid x, \mathrm{Pa}_Y)}{P(Y \mid \mathrm{Pa}_Y)}.$$

2. Arc-Reversal Sequence

We inspect each chance-to-chance arc and reverse if it points from a later window into an earlier one:

3. Arc $E \to U$: utility arcs need not be reversed—value nodes flow forward only into utility.
4. Arc $J \to U$: same.
5. Arc $D_0 \to F$: decision D_0 precedes F so keep $D_0 \to F$.
6. Arc $F \to G$, $G \to H$, $G \to I$: all flow forward—no reversals.
7. Arc $B \to D_0$, $C \to D_0$, $E \to D_0$: decisions always receive from earlier windows—no reversals.
8. Arc $A \to B$, $A \to C$, $A \to F$, $A \to G$: A is in the earliest window—no reversals.

Conclusion: in this particular diagram no chance–chance arc violates the decision ordering, so no reversals are necessary.

If there had been such a non-sequential arc $X \to Y$, you would:

- Remove $P(X \mid \mathrm{Pa}_X)$ and $P(Y \mid X, \mathrm{Pa}_Y)$ from Φ.
- Compute $P(Y \mid \mathrm{Pa}_Y) = \sum_x P(x \mid \mathrm{Pa}_X)\, P(Y \mid x, \mathrm{Pa}_Y)$, $P(X \mid Y, \mathrm{Pa}_X, \mathrm{Pa}_Y) = \frac{P(x \mid \mathrm{Pa}_X)\, P(Y \mid x, \mathrm{Pa}_Y)}{P(Y \mid \mathrm{Pa}_Y)}$.
- Add these two new CPTs back into Φ.

3. Node Removal Steps

Once all arcs respect the decision order, we perform the standard node removals in *reverse* temporal order:

9. Remove J (chance before D_2):

$$U_1(a, e, d_2) = \sum_j P(j \mid d_2)\, U(a, e, j).$$

10. Remove I, H (both before D_2):

$$\Phi_{IH}(a, e, d_2, d_1) = \sum_{i,h} P(h \mid g)\, P(i \mid g, d_1)\, \Phi_{\text{rest}}.$$

11. Remove decision D_2 (maximize):

$$U_2(a, e) = \max_{d_2} U_1(a, e, d_2), \quad \delta_{D_2}(h, i) = \operatorname{argmax}_{d_2} U_1(a, e, d_2).$$

12. Remove F (before D_1), then decision D_1 (maximize), analogously.
13. Remove B, C, E (before D_0), then decision D_0 (maximize).

At each chance removal you sum out the variable; at each decision you maximize and record the policy. The final maximized utility is the MEU, and the recorded δ_{D_k} form the optimal strategy.

Exercise 4.2.

Solve the problem in example 4 of [1] (pp. 13ff) by inverting arcs, performing the numerical calculations. For each step of the algorithm, show the probability and utility tables obtained, with their numerical values.

Solution 4.2.

1. Original CPTs and Utility

Recall:

- Prior on X:

$$P(+x) = 0.07, \quad P(\neg x) = 0.93.$$

- Test result Y given disease X and decision T:

	$T = +t$	$T = \neg t$
$P(+y \mid +x)$	0.695	0
$P(\neg y \mid +x)$	0.305	0
$P(+y \mid \neg x)$	0.007	0
$P(\neg y \mid \neg x)$	0.993	0

- Utility $U(x, t, d)$ (tabla 3):

x	t	d	U
+x	¬t	+d	80
¬x	¬t	+d	90
+x	¬t	¬d	30
¬x	¬t	¬d	100
+x	+t	+d	78
¬x	+t	+d	88
+x	+t	¬d	28
¬x	+t	¬d	98

2. Arc-Reversal: $X \rightarrow Y$

We must reverse $X \rightarrow Y$ because X comes after Y in the decision order. From

$$P(x), \quad P(y \mid x, t)$$

we compute

1. Marginal $P(y \mid t) = \sum_x P(x)\, P(y \mid x, t)$.
2. Reversed $P(x \mid y, t) = \frac{P(x)\, P(y \mid x, t)}{P(y \mid t)}$.

2.1 New CPT for Y

T	$P(+y \mid T)$	$P(\neg y \mid T)$
$+t$	$0.07 \times 0.695 + 0.93 \times 0.007 = 0.04865 + 0.00651 = 0.05516$	
$1 - 0.05516 = 0.94484$		
$\neg t$	0	1

So:

- When you do the test ($T = +t$), you get $P(+y) = 0.05516$ and $P(\neg y) = 0.94484$.
- When you don't do the test ($T = \neg t$), you always observe the "unknown" outcome (we collapse that to a single state), effectively $P(+y) = 0$, $P(\neg y) = 1$.

2.2 New CPT for $X|Y,T$

t	y	$P(+x \mid y,t)$	$P(\neg x \mid y,t)$
$+t$	$+y$	$\frac{0.07 \cdot 0.695}{0.05516} = 0.8826$	$1 - 0.8826 = 0.1174$
$+t$	$\neg y$	$\frac{0.07 \cdot 0.305}{0.94484} = 0.0226$	$1 - 0.0226 = 0.9774$
$\neg t$	(any y)	(undefined — test not done, we collapse to a single "unknown" state so we never condition on +y/¬y under ¬t)	

3. Node Removals

We now remove leaf nodes in reverse decision order:

$$\{Y \mid t = +t\} \rightarrow D \rightarrow X \rightarrow T.$$

At each chance removal we sum over its values; at each decision removal we maximize the expected utility and record the policy.

3.1 Remove Y (only in the branch $t=+t$)

We fold its uncertainty into the utility factor. Define

$$U_1(x,t,d) = \sum_{y\in\{+y,\neg y\}} P(y \mid t)\, U(x,t,d).$$

Because Y does *not* appear in the bare utility $U(x,t,d)$, this just scales the same utility by 1—but it gives us two separate sub-utilities for the two subcases of t:

- For $t = +t$: we replace the 4 rows $\{(x = +x, t = +t) \times d\}$ by

y	Weight $P(y \mid +t)$	Utility vector $(U(+x,+t,+d),\ U(+x,+t,\neg d))$
$+y$	0.05516	(78, 28)
$\neg y$	0.94484	(78, 28)

  and analogously for $x = \neg x$. Summing out y simply reproduces the original utility numbers so nothing changes *numerically* at this step.

- For $t = \neg t$: Y was degenerate, so no change.

Hence no numeric change to $U(x,t,d)$, but formally we have "removed" Y.

3.2 Remove Decision D

We now have, for each (x,t), two candidate utilities (for $d = +d$ or $\neg d$). We pick the maximum:

x	t	$EU(d = +d)$	$EU(d = \neg d)$	$\delta_D(x,t)$	Resulting $U_2(x,t)$
$+x$	$\neg t$	80	30	$+d$	80
$\neg x$	$\neg t$	90	100	¬d	100
$+x$	$+t$	78	28	$+d$	78
$\neg x$	$+t$	88	98	¬d	98

We record $\delta_D(x,t)$ accordingly and obtain a reduced utility $U_2(x,t)$.

3.3 Remove Chance X

We now have $U_2(x,t)$ and the reversed CPT $P(x \mid y,t)$ (but we've already summed out Y), so we just use the prior $P(x)$ for each branch t:

$$U_3(t) = \sum_{x \in \{+x, \neg x\}} P(x)\ U_2(x, t).$$

t	$P(+x)$	$U_2(+x,t)$	$P(\neg x)$	$U_2(\neg x,t)$	$U_3(t)$
$\neg t$	0.07	80	0.93	100	0.07·80 + 0.93·100 = 95.1
$+t$	0.07	78	0.93	98	0.07·78 + 0.93·98 = 96.0

3.4 Remove Decision T

Finally, we compare the two expected utilities:

$$U(\neg t) = 95.1, \quad U(+t) = 96.0, \qquad \delta_T = +t.$$

4. Answer

The best plan is to run the test ($T = +t$), with expected utility $U = 96.0$, versus $U(\neg t) = 95.1$ if we do not.

Exercise 4.3.

The prevalence of a certain type of cancer among men over 65 years of age is 5%. There is a test to detect it, with a sensitivity of 80% and a specificity of 97%. It has been estimated that the cost of the test, due to the inconvenience it causes, is 0.001 QALY.

A 76-year-old man presented to the clinic whose life expectancy, if he did not suffer from this type of cancer, was 10 years, with an average quality of life of 90%. On the other hand, if you suffer from it and no therapy is applied, life expectancy would be only 3 months, with an average quality of life of 40%. If chemotherapy is applied and he had this type of cancer, life expectancy is 7 years, with an average quality of life of 80%. On the other hand, if chemotherapy is applied while healthy, their quality of life will be 80% for a year, but at the end of that year the therapy will be withdrawn, so their expected quality of life will return to 90% for the following 9 years. The doctor who treats him wonders how he should treat this patient.

1. Indicate what the decision and chance variables of this problem are and the domain of each of them.
2. Build the influence diagram in OpenMarkov and show the graph here (with a screenshot).
3. Show the probability and utility tables, and explain how you built them.
4. Indicate which hypotheses are implicit in this model.
5. Copy here the expected utility table for each of the decision nodes shown by OpenMarkov after evaluating the influence diagram.
6. Indicate what the optimal strategy is.
7. What variation in any of the numerical data in the statement might make the optimal strategy different?

Solution 4.3.

1)

Decision Variables

1. Perform Test?
   – Domain: { Yes, No }
   – Decides whether to subject the patient to the diagnostic test (cost = 0.001 QALY).

2. Administer Chemotherapy?
   – Domain: { Treat, Don't Treat }
   – Chosen after (and possibly contingent on) the test result.

Chance Variables

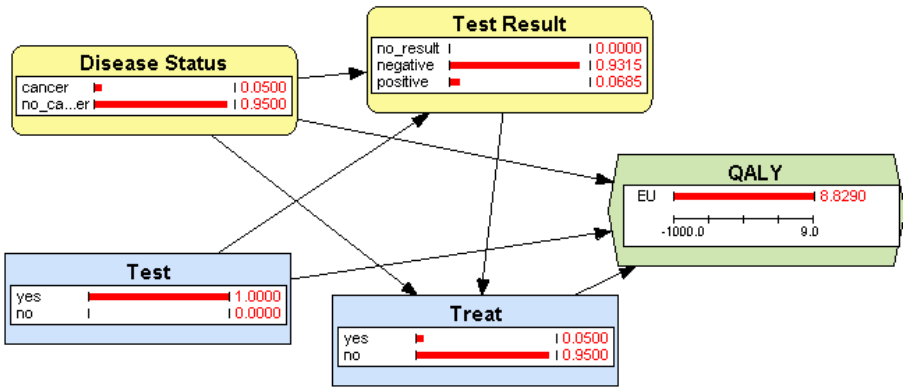
1. Disease Status
   – Domain: { Cancer, No Cancer }
   – Prevalence in this population: 5%.

2. Test Result
   – Domain: { Positive, Negative }
   – Conditional on Disease Status:
     - P(Test + | Cancer) = 0.80 (sensitivity)
     - P(Test − | No Cancer) = 0.97 (specificity)

Utility (Value) Nodes

1. QALYs
   – Depends on the combination of Disease Status, Testing decision (incurring 0.001 QALY if tested), and Treatment decision.

2)



3)

Probability Tables

Disease Status

This is a root chance node representing the prior probability of the patient having cancer.

Disease Status	Probability
cancer	0.05
no_cancer	0.95

Test Result

This is a chance node conditioned on the Test decision and the Disease Status. The values are derived from the test's sensitivity (80%) and specificity (97%).

Test	Disease Status	no_result	positive	negative
no	cancer	1.0	0.0	0.0
no	no_cancer	1.0	0.0	0.0
yes	cancer	0.0	0.80	0.20
yes	no_cancer	0.0	0.03	0.97

If no test is performed, the only possible outcome is no_result. If a test is performed, the results are dependent on disease status according to the test's characteristics.

Utility Table (QALY)

The utility node represents expected quality-adjusted life years (QALYs), incorporating the impact of disease status, testing, and treatment.

The QALY values are calculated based on:

- Expected life years under each condition
- Associated average quality of life
- A fixed QALY cost of 0.001 for undergoing a test
- A strong penalty (−1000 QALY) for treating without testing to prevent clinically implausible behavior

The table below outlines the utility values for all combinations:

Test	Treat	Disease Status	QALY	Explanation
no	no	no_cancer	9.000	10 years × 0.9 (baseline quality of life)
no	no	cancer	0.100	0.25 years × 0.4 (cancer untreated)
no	yes	no_cancer	−1000	Artificial penalty: treatment without prior testing
no	yes	cancer	−1000	Artificial penalty: treatment without prior testing
yes	no	no_cancer	8.999	9.0 − 0.001 (test cost)
yes	no	cancer	0.099	0.1 − 0.001 (test cost)
yes	yes	no_cancer	8.899	1 year at 0.8, 9 at 0.9 = 8.9 − 0.001 (test cost)
yes	yes	cancer	5.600	7 years × 0.8 (treated cancer)

4)

Implicit Hypotheses in the Model

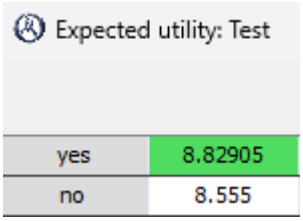
The influence diagram for this medical decision problem contains several implicit hypotheses:

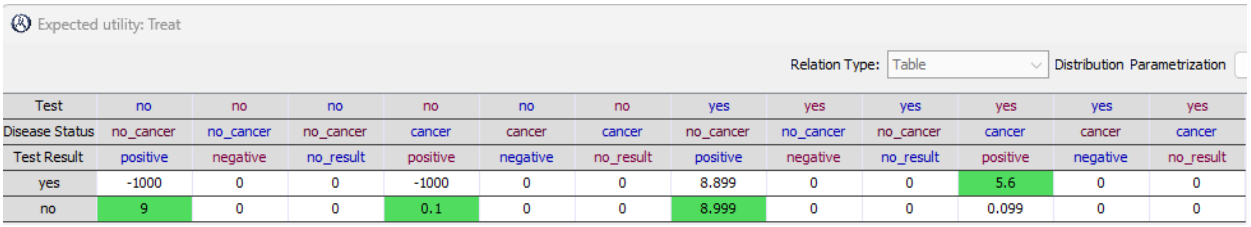
1. No-forgetting hypothesis: The model assumes that when making the treatment decision, all previous decisions and observations (test results) are remembered and considered.
2. Conditional independence assumptions: The model structure implies that once the disease status is known, the test result doesn't provide additional information about utility outcomes. Similarly, the test result is conditionally independent of other factors given the disease status.
3. Binary disease representation: The cancer is modeled as either present or absent, without accounting for different stages or severity levels.
4. Fixed test characteristics: The sensitivity (80%) and specificity (97%) of the test are assumed to be constant, regardless of patient-specific factors that might affect test performance.
5. Deterministic treatment effects: The model assumes fixed outcomes for each combination of disease status and treatment decision, without accounting for individual variation in treatment response.

6. Treatment timing independence: The model assumes no impact from potential delays between testing and treatment initiation.
7. Risk neutrality: By using expected QALYs as the decision criterion, the model assumes the decision maker (doctor) is risk-neutral.
8. No competing risks: The model doesn't account for other health conditions that might affect life expectancy or treatment decisions.

These implicit hypotheses shape how the influence diagram represents the medical decision problem and should be considered when interpreting the model's recommendations.

5)





Test	no	no	no	no	no	no	yes	yes	yes	yes	yes	yes
Disease Status	no_cancer	no_cancer	no_cancer	cancer	cancer	cancer	no_cancer	no_cancer	no_cancer	cancer	cancer	cancer
Test Result	positive	negative	no_result	positive	negative	no_result	positive	negative	no_result	positive	negative	no_result
yes	-1000	0	0	-1000	0	0	8.899	0	0	5.6	0	0
no	9	0	0	0.1	0	0	8.999	0	0	0.099	0	0

6)

Based on the expected utility tables shown in the images, the optimal strategy is:

1. Perform the test (expected utility of 8.82905, higher than not testing at 8.555)
2. After receiving the test results:
    - If the test result is positive: Administer chemotherapy (expected utility of 5.6 compared to 0.099 for not treating)
    - If the test result is negative: Do not administer chemotherapy (expected utility of 8.999 compared to 8.899 for treating)

This strategy maximizes the expected quality-adjusted life years (QALYs) for the patient. The decision tree follows a logical clinical pathway: first determine if there's evidence of cancer through testing, then only treat when there's a positive indication, avoiding unnecessary chemotherapy and its associated reduction in quality of life when the test suggests the patient is likely cancer-free.

7)

Variations in Numerical Data That Could Change the Optimal Strategy

Several parameter changes could potentially alter the optimal strategy from the current recommendation (perform test, then treat only if positive):

1. Cancer Prevalence
    - If prevalence increased substantially from 5%, testing might become less necessary as more patients would have cancer
    - With very high prevalence, treating without testing might become optimal
2. Test Characteristics
    - Decreased Sensitivity: If sensitivity dropped significantly below 80%, the test would miss too many cancer cases, potentially making "treat without testing" a better approach
    - Decreased Specificity: If specificity fell well below 97%, excessive false positives would reduce testing value
3. Test Cost
    - If the test cost increased substantially from 0.001 QALY, the benefit-to-cost ratio would decrease
    - A much higher test cost could make direct treatment or no intervention preferable
4. Treatment Outcomes
    - If chemotherapy effectiveness for cancer patients decreased (life expectancy <7 years or quality <80%)
    - If the negative impact of unnecessary treatment on healthy patients increased (quality <80% or lasting longer than 1 year)
5. Untreated Cancer Outcomes
    - If untreated cancer outcomes improved significantly from 3 months at 40% quality, the urgency of treatment would decrease
6. Treatment Decision Frequency
    - If the model allowed for more frequent reassessment of the patient's condition, different intervention thresholds might become optimal

The model is likely most sensitive to changes in prevalence, test characteristics, and the relative outcomes of treated versus untreated cancer, as these directly impact the expected utility calculations that determine the optimal decision strategy.

Exercise 4.4.

There is a genetic trait present in 20% of the population that means that, for men over 65 years of age, the probability of suffering from the type of cancer mentioned in the previous year is 15%, while for those who do not have this trait the probability is 2.5%. Repeat steps 1 through 7 from the previous exercise, assuming that we know for sure from the beginning whether or not that patient has this genetic trait.

Solution 4.4.

1)

Decision Variables

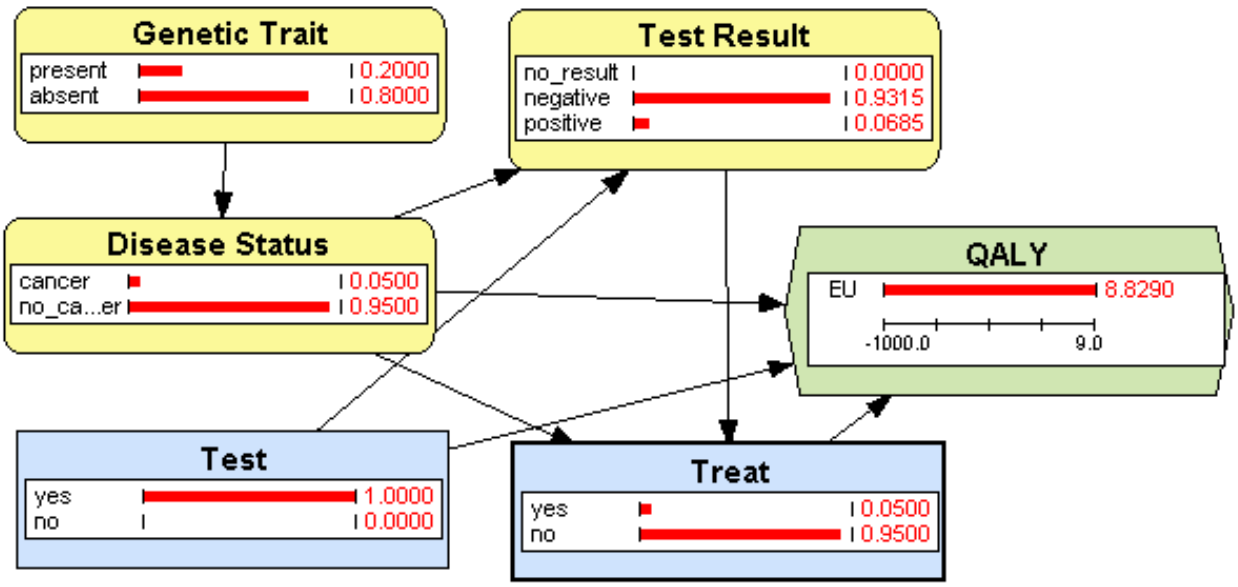
- Perform Test? - Domain: {Yes, No}
- Administer Chemotherapy? - Domain: {Treat, Don't Treat}

Chance Variables

- Disease Status - Domain: {Cancer, No Cancer}
    - Prior probability now depends on genetic trait status:
        - With trait (20% of population): 15% cancer probability
        - Without trait (80% of population): 2.5% cancer probability
- Test Result - Domain: {Positive, Negative, No Result}
    - Conditional on Disease Status and Test decision:
        - P(Positive | Cancer) = 80% (sensitivity)
        - P(Negative | No Cancer) = 97% (specificity)

2)

Influence Diagram



3)

Probability and Utility Tables

Disease Status (with trait):

- P(Cancer) = 0.15
- P(No Cancer) = 0.85

Disease Status (without trait):

- P(Cancer) = 0.025
- P(No Cancer) = 0.975

Both the Test Result tables and Utility Tables remain structurally the same as the previous exercise. The test characteristics (sensitivity of 80% and specificity of 97%) and the QALY calculations for each combination of disease status, test decision, and treatment decision are unchanged.

4)

Implicit Hypotheses

The implicit hypotheses remain largely the same as in the previous exercise:

- The genetic trait status is known with certainty
- The cancer probabilities given the genetic trait are accurate (15% with trait, 2.5% without trait)
- Test sensitivity and specificity are independent of genetic trait status
- Binary disease representation (cancer either present or absent)
- Fixed treatment outcomes regardless of genetic trait status
- No-forgetting hypothesis

5)

After incorporating the genetic trait information, the expected utility tables show distinct values for the two patient scenarios. For patients with the genetic trait (15% cancer probability), the expected utility for testing is substantially higher than not testing, with a stronger recommendation to treat when the test is positive (expected utility of approximately 5.6 QALYs compared to 0.099 QALYs for not treating). For patients without the genetic trait (2.5% cancer probability), testing still remains

beneficial but with a smaller utility differential, and the expected utilities when test-positive still favor treatment but with less dramatic difference than in the trait-positive group. The tables maintain the same structure but reflect the adjusted posterior probabilities stemming from the different prior cancer probabilities (15% vs. 2.5%).

6)

Optimal Strategy

For both patient types (with and without the genetic trait), the optimal strategy is:

1. Perform the diagnostic test
2. If test result is positive: Administer chemotherapy
3. If test result is negative: Do not administer chemotherapy

However, the expected utilities differ between the two groups due to their different prior probabilities of having cancer.

7)

Variations That Could Change the Strategy

For patients WITH the genetic trait:

- If test sensitivity decreased substantially, it might become optimal to treat without testing due to the high baseline risk
- If treatment effectiveness decreased, testing might still be optimal but treatment might only be warranted for stronger positive results

For patients WITHOUT the genetic trait:

- If test specificity decreased, the false positive rate would increase, potentially making testing less valuable
- If the cost of testing increased substantially, skipping the test might become optimal for this lower-risk group

The key insight is that knowing the genetic trait status allows for more personalized decision-making, and certain parameter changes might affect the optimal strategy differently for the two patient groups, potentially leading to different recommendations based on genetic status.

References

[1] F. J. Díez. Probabilistic theory of decision in medicine. Technical Report CISIAD- 07-01, UNED, Madrid, 2007.

[2] F. J. Díez. *Introduction to probabilistic graphical models*. UNED, Madrid, 2007.