Solutions to exercises on Fundamentals

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1 Exam exercises

Exercise 1 *June 2018*

The execution of a program has two parts. The first part requires 20% of the total execution time and the second part the remaining 80%. The second part can be parallelized using OpenMP in a multiprocessor. Assume that the speedup is perfect for any number of processors. What would be the overall speedup of the application when 10 processors are used?

Solution 1

It is enough with the application of Amdahl's Law with values 0.2 for the sequential part and 0.8 for the parallel part.

$$S = \frac{1}{0.2 + \frac{0.8}{10}} = \frac{1}{0.2 + 0.08} = \frac{1}{0.28} = 3.57$$

Exercise 2 *June 2015.*

An application allows to process a very high resolution image where a certain fraction can be parallelized, while other part must be run sequentially.

Assume that there is no upper bound to the number of processes that can be used for the parallelization. We need to get a global speedup of at least 10 for the parallel version.

Express the fraction of code that must be parallelized as a function of the parallelism degree (number of processes running in parallel).

Solution 2

$$S = \frac{1}{(1-F) + \dfrac{F}{n}}$$

$$S \times \left((1-F) + \frac{F}{n}\right) = 1$$

$$S - S \times F + \frac{S \times F}{n} = 1$$

$$n \times S - n \times S \times F + S \times F = n$$

$$n \times S - n = n \times S \times F - S \times F$$

$$S \times F \times (n-1) = n \times (S-1)$$

$$F = \frac{n \times (S-1)}{S \times (n-1)}$$

For the case of $S = 10$

$$F = \frac{n \times (10-1)}{10 \times (n-1)} = \frac{9 \times n}{10 \times n - 10}$$

As F must be less or equal than 1:

$$F \leq 1$$

$$\frac{9 \times n}{10 \times n - 10} \leq 1$$

$$9 \times n \leq 10 \times n - 10$$

$$n \geq 10$$

Exercise 3 *June 2019*

An application for generating 3D video is run currently in a computer with 4 cores. The application has a portion that is not parallelized and takes 5% of total execution time. The application spends 50% of the total time in memory processing and is totally parallelized. The remaining 25% of the total time is dedicated to read/write operations on disks.

Two options are being considered.

- Replacing the current disks for a new storage technology that requires one fourth of the time for input/output.
- Replace the processor by a processor with 32 cores.

You are asked:

1. Determine the speedup obtained when replacing the disks.
2. Determine the speedup when replacing the processor.
3. Determine the number of cores that would give the same speedup than replacing the disks.

Solution 3

Section 1 If the disk is replaced, the improvement fraction F is 0.25. The improvement speedup $S(m)$ will be 4. Consequently, applying Amdahl's Law, we get:

$$S = \frac{1}{0.75 + \frac{0.25}{4}} = \frac{1}{0.8125} = 1.2308$$

Section 2 If the processor is replaced, the improvement speedup is given by the ratio between the ne number of cores n_n and the old number of colres n_o.

$$S_m = \frac{n_n}{n_o} = \frac{32}{4} = 8$$

The improvement fraction F is in this case 0.5.

$$S = \frac{1}{0.5 + \frac{0.5}{8}} = \frac{1}{0.5 + 0.0625} = \frac{1}{0.5625} = 1.7778$$

Section 3 The improvement speedup for a given number of cores n, is:

$$S_i = \frac{n}{4}$$

Consequently, the total speedup is:

$$S = \frac{1}{0.5 + \frac{0.5}{\frac{n}{4}}} = \frac{1}{0.5 + \frac{2}{n}}$$

If we make this value equal to $\frac{1}{0.8125}$, we get:

$$\frac{1}{0.8125} = \frac{1}{0.5 + \frac{2}{n}}$$

$$0.8125 = 0.5 + \frac{2}{n}$$

$$\frac{2}{n} = 0.8125 - 0.5$$

$$n = \frac{2}{0.3125} = 6.4$$

Consequently a minimum of 7 cores are needed.

Exercise 4 *October 2013.*

In your organization, there is an application with the following characteristics:

- The application spends 80% of time executing instructions and 20% of the remaining time waiting for disk operations.
- The time that the application spends executing instructions is distributed in 20% for floating point instruction (which require 8 CPI) and 80% for the rest of instructions (which require 6 CPI).

You are evaluating the migration to a new machine in which instructions require 25% more of CPI but whose clock frequency is double.

What is the global speedup for the application?

Solution 4

$$T_{inst}(orig) = 0.2 \cdot 8 \cdot IC \cdot P + 0.8 \cdot 6 \cdot IC \cdot P = (1.6 + 4.8) \cdot IC \cdot P = 6.4 \cdot IC \cdot P$$

$$T_{inst}(mej) = 0.2 \cdot 10 \cdot IC \cdot \frac{P}{2} + 0.8 \cdot 7.5 IC \cdot \frac{P}{2} = (1 + 3) \cdot IC \cdot P = 4 \cdot IC \cdot P$$

$$S_{inst} = \frac{6.4}{4} = 1.6$$

$$S = \frac{1}{0.2 + \frac{0.8}{1.6}} = \frac{1}{0.2 + 0.5} = \frac{1}{0.7} = 1,42$$

Exercise 5 *January 2014.*

A single core computer runs a finance risk assessment application. The application is computation intensive (computations take 90% of total execution time). The remaining 10% is devoted for waiting for I/O operations.

The time that the applications is running computation instructions is divided into 75% for floating point operations and 25% for other instructions. Executing a floating point operation requires, on average, 12 CPI. The rest of instructions require, on average, 4 CPI.

Migrating this application to a new machine is being evaluated. The following alternatives are considered. In both alternatives, there is no improvement in the I/O time for disk.

- **Alternative A**: A single-core process with clock frequency 50% higher than the original machine, where floating point instructions require 10% more cycles per instruction and the rest of instructions require 25% more cycles per instruction.
- **Alternative B**: A four-core process with a clock frequency 50% lower than the original machine, where floating point instructions require 20% less cycles per instruction and the rest of instructions the same number of cycles per instructions.

State the following questions giving an appropriate reasoning:

1. Which is the global speedup/slowdown for the application in case A?
2. Which is the global speedup/slowdown for the application in case B, assuming that the computation part can be fully parallelized while the I/O part cannot be improved at all?

Solution 5

Time for executing instructions in the original computer will be:

$$T_{orig} = 0.75 \times 12 \times IC \times P + 0.25 \times 4 \times IC \times P = (9 + 1) \times IC \times P \tag{1}$$

Alternative A Time for executing instructions in computer A will be:

$$T_A = (0.75 \times (1,1 \times 12) + 0.25 \times (1.25 \times 4)) \times IC \times \frac{P}{1.5} = \frac{(9.9 + 1.25) \times IC \times P}{1.5} = \frac{11.15}{1.5} \times IC * P \tag{2}$$

$$T_A = 7.433 \times IC \times P \tag{3}$$

Speedup due to instructions will be:

$$S_A^I = \frac{T_{orig}}{T_A} = \frac{10}{7.433} = 1.345 \tag{4}$$

Applying Amdahl's law, the global speedup will be:

$$S_A = \frac{1}{0.1 + \frac{0.9}{1.345}} = 1.3 \tag{5}$$

Alternative B In this case, assuming complete parallelization for the computing part, we may consider the number of instructions to be executed in each core is one fourth from the original.

$$T_B = (0.75 \times 0.8 \times 12 + 0.25 \times 4) \times \frac{IC}{4} \times \frac{P}{0.5} = (7.2 + 1) \times \frac{2}{4} \times IC \times P = \tag{6}$$

$$T_B = 4.1 \times IC \times P \tag{7}$$

The speedup due to instructions will be:

$$S_B^I = \frac{T_{orig}}{T_B} = \frac{10}{4.1} = 2.439 \tag{8}$$

Applying Amdahl's law, the global speedup will be:

$$S_B = \frac{1}{0.1 + \frac{0.9}{2.439}} = 2.132 \tag{9}$$

Exercise 6 *October 2013.*

Given a processor consuming a dynamic power **P**, there are two alternatives for reducing the consumed dynamic power.

1. Decrease voltage to the half keeping the same value for clock frequency.
2. Decrease frequency to the half keeping the same value for the voltage.

Reason which of them gets a higher dynamic power reduction and quantify the value for this reduction.

Solution 6

If we decrease voltage to the half we have:

$$\frac{P_{new}}{P_{old}} = \frac{\frac{1}{2} \cdot X_c \cdot (V \cdot 0.5)^2 \cdot f}{\frac{1}{2} \cdot X_c \cdot V^2 \cdot f} = \frac{0.25 \cdot V^2 \cdot f}{V^2 \cdot f} = 0.25$$

If we decrease frequency to the half we have:

$$\frac{P_{new}}{P_{old}} = \frac{\frac{1}{2} \cdot X_c \cdot V^2 \cdot f \cdot 0.5}{\frac{1}{2} \cdot X_c \cdot V^2 \cdot f} = 0.5$$

Consequently, we get a higher reduction for the dynamic power in the case of reducing voltage to the half.