1 Introduction

The objective of this project is to model a wing using the Finite Element Method. Beam elements has been used to model stingers and flat shell elements has been used to model spars, ribs and the skin of the wing.

1.1 Problem definition

Consider the wing prototype with a NACA0018 airfoil depicted in Figure 1. The prototype wing is to be tested in a wind tunnel to analyze its structural response under aerodynamic loads. To do so, the root section is clamped.

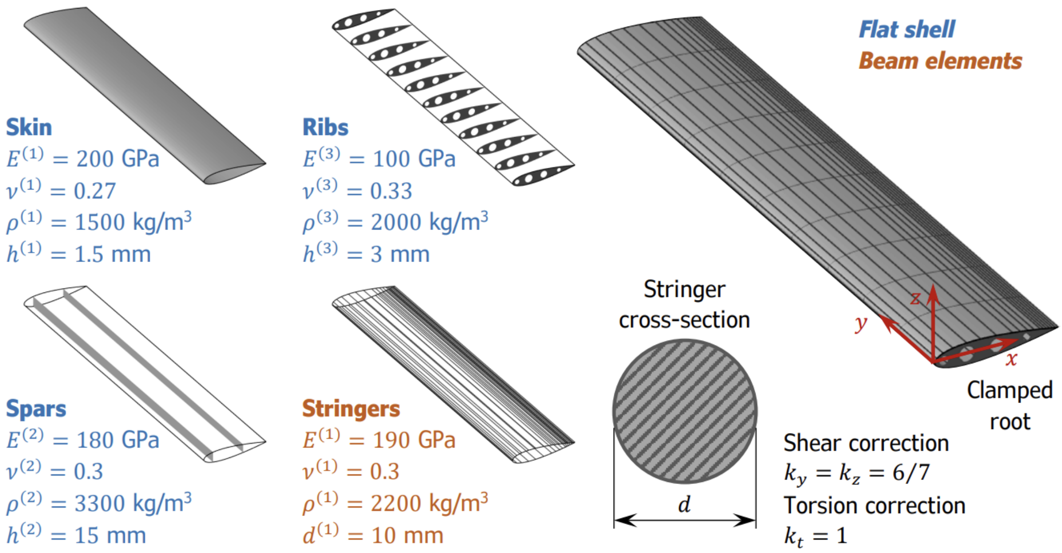
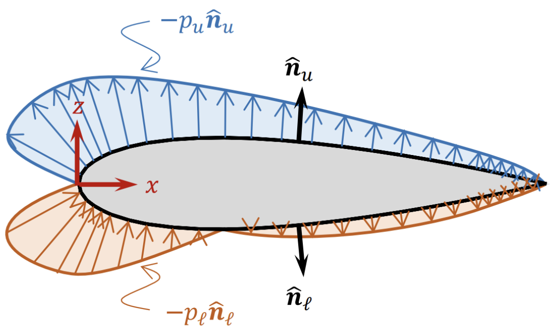


Figure 1: Wing in a wind tunnel setting.

The wing supports its own weight and a pressure load distributed over the upper and lower skin surfaces. This pressure load is modelled as:



$$p_u(x,y) = -4\alpha P(y)\left[\left(1-\frac{x}{c}\right)^4 + \sqrt{1-\frac{x}{c}}\right]$$

$$p_l(x,y) = 4\alpha P(y)\left[\left(1-\frac{x}{c}\right)^4 + \frac{1}{4}\sqrt{1-\frac{x}{c}}\right]$$

$$P(y) = \begin{cases} p_\infty & \text{for } y \leq 0.8b \\ p_\infty cos\left(\frac{\pi}{2}\frac{y-0.8b}{0.2b}\right) & \text{for } y > 0.8b \end{cases}$$

with α being the angle of attack (in rad), $c = 0.6$ m is the chord length, $b = 3$ m is the wingspan, and $p_\infty = 1.5625 \cdot 10^5$ Pa.

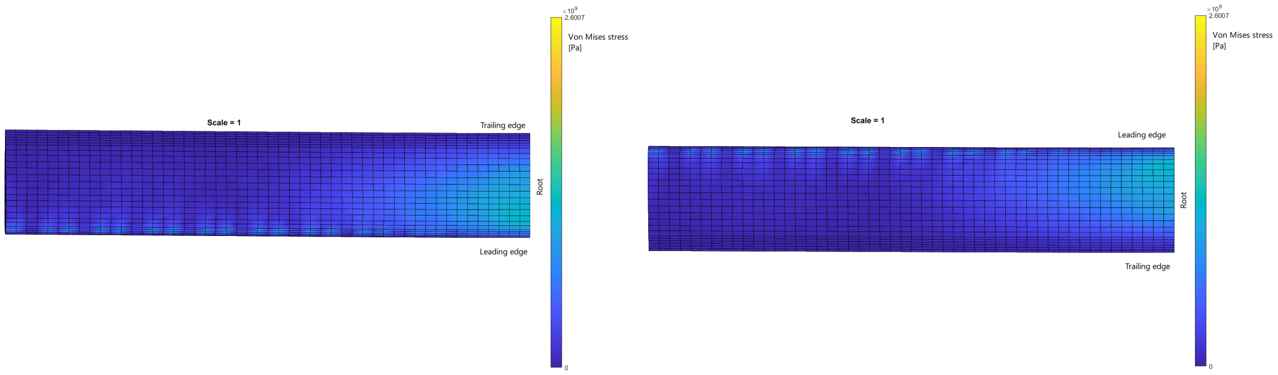
2 Methodology

To solve this problem, a Matlab code has been developed using the guidelines described in the documents *Algorithm for problems involving beam elements, Algorithm for problems involving flat shells* and *Algorithm for frequency analysis*, attached to the project statement.

3 Results

3.1 Section a)

For an angle of attack of $\alpha = 10°$ and $g = 9.81$ m/s^2, the deformation of the wing and the Von Misses stress has been calculated:



(a) Upper skin (b) Lower skin

Figure 2: Distribution of Von Misses stress over the wing skin

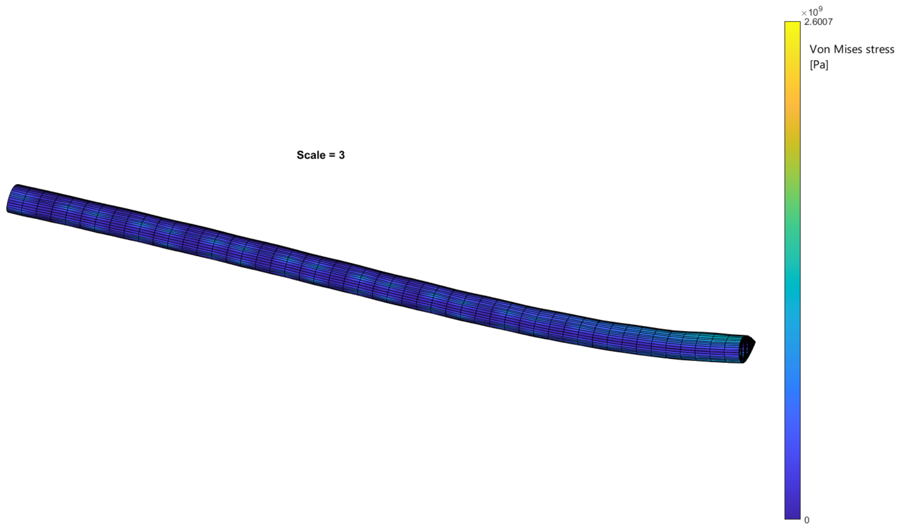


Figure 3: Wing deformation with a scale of x3.

This three figures are different angles of the same 3D plot, with the exception of Figure 3, which has a scale of x3 for a better distinction of the deformation. The Von Misses stress is higher at the root of the wing, where the deformation is bigger and the skin suffers higher compression/traction.

3.2 Section b)

The wing has the following frequencies and mode shapes, see Figure 4, characterized by the total mass matrix [**M**] and stiffness matrix [**K**]:

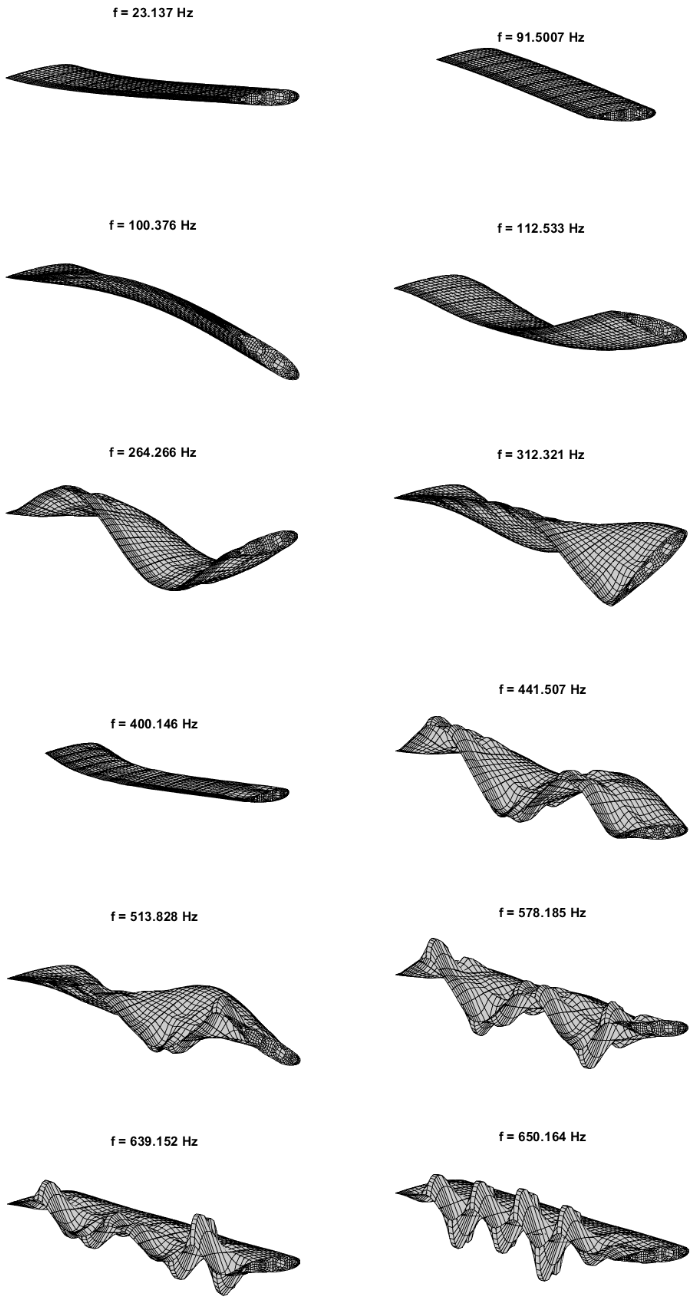


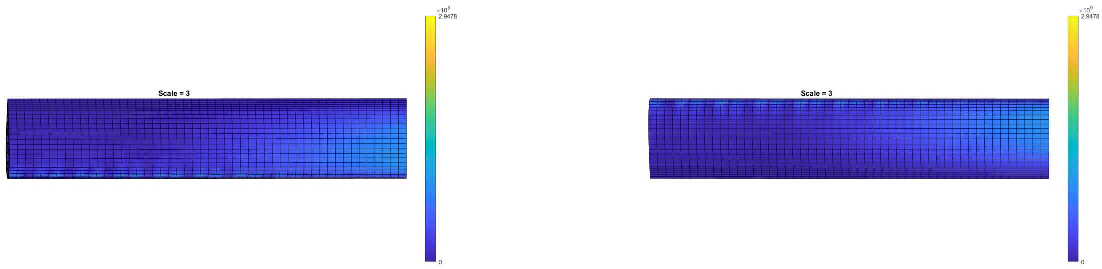
Figure 4: First 12 vibration modes

3.3 Section c)

The same static problem is solved again but this time by applying a model-order reduction scheme based on system projection on the most relevant modes for 2 and 6 modes approximations. Thus, the results obtained are the following ones;

3.3.1 Deformed state and stress distribution

The stress distribution is shown for 2 and 6 modes respectively as follows;



(a) Upper skin

(b) Lower skin

Figure 5: 2 modes projected distribution of Von Misses stress over the wing skin

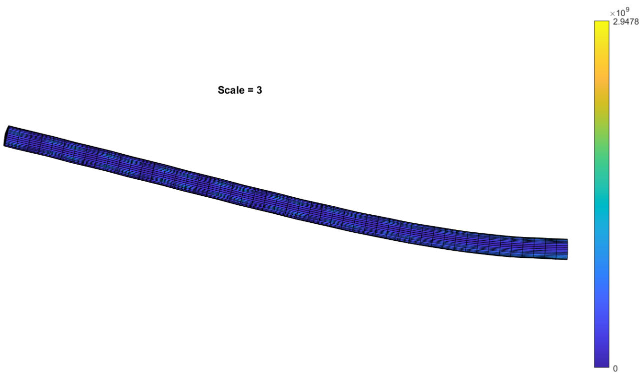
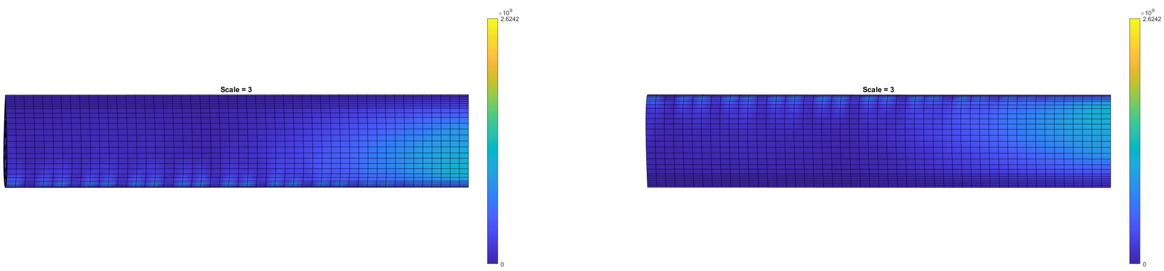


Figure 6: Deformation with 2 modes



(a) Upper skin

(b) Lower skin

Figure 7: 6 modes projected distribution of Von Misses stress over the wing skin

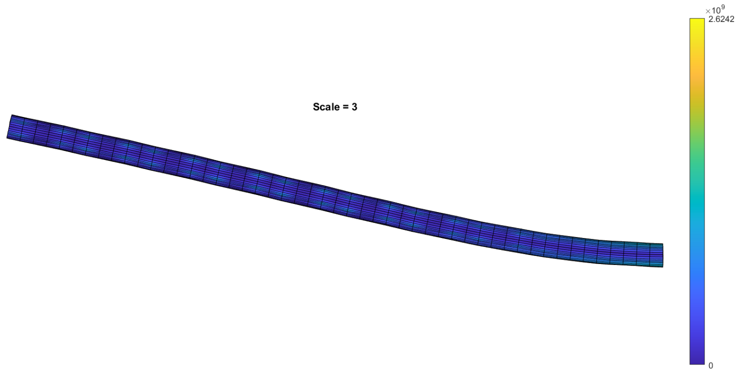


Figure 8: Deformation with 6 modes

The previous images show that there is little difference in the deformation of the wing for 2 and 6 projected modes with respect to the original solution, but is harder to verify the Von Misses stresses with the current range of colours given by Matlab plot.

3.3.2 Leading and trailing edges vertical displacements along the wingspan

The vertical displacements u_z are plotted at the leading and trailing edges along the wingspan for both cases in order to compare their solutions to the original case one.

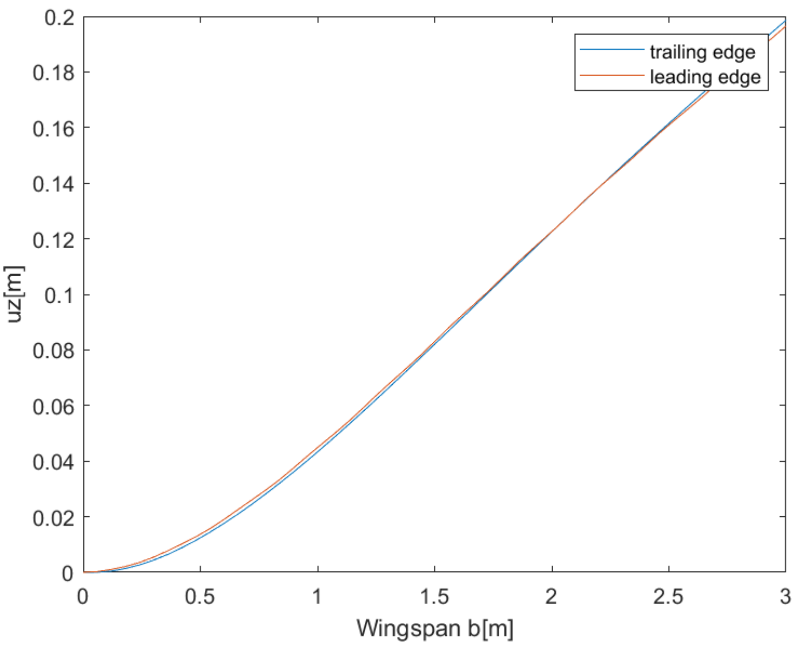


Figure 9: Trailing and leading edges vertical displacement of the original case

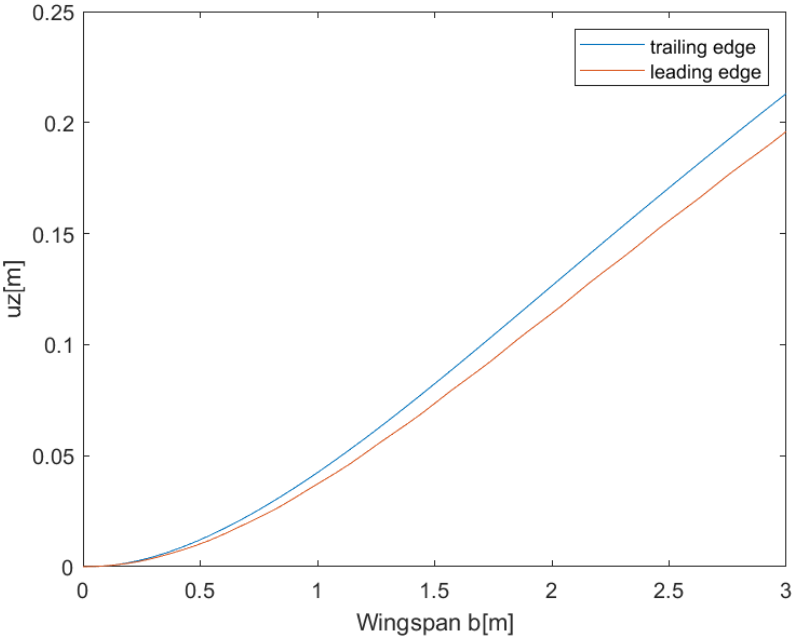


Figure 10: Trailing and leading edges vertical displacement for 2 modes projection case

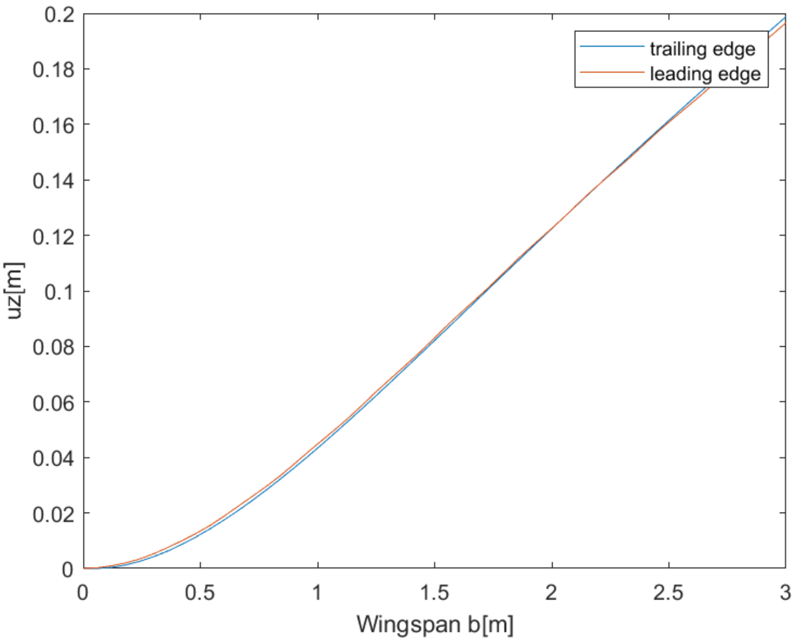


Figure 11: Trailing and leading edges vertical displacement for 6 modes projection case

The previous plots show that the trailing and leading edges vertical displacement should be similar and that they are deformed at the same time. On the other hand, it can be observed that for 2 modes projection, the solution is not well reconstructed, but with 6 modes is feasible since the solutions are very similar to the original case.