# An Illustrated Guide to the

# *ANALYTIC HIERARCHY PROCESS*

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**http://www.boku.ac.at/mi/**

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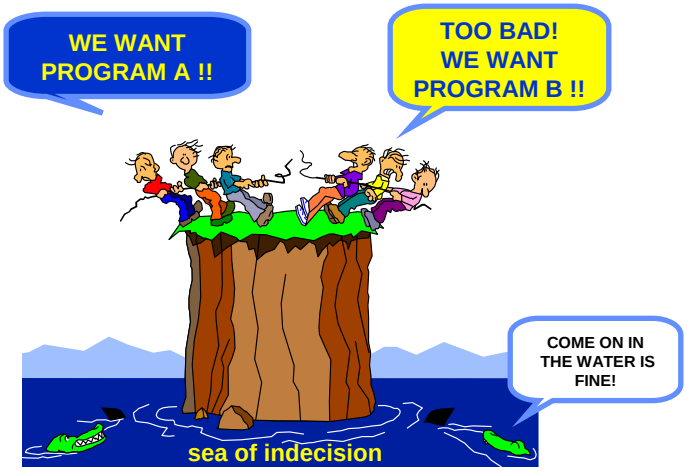
## Do your decision conferences turn out like this?




**or does this happen?**

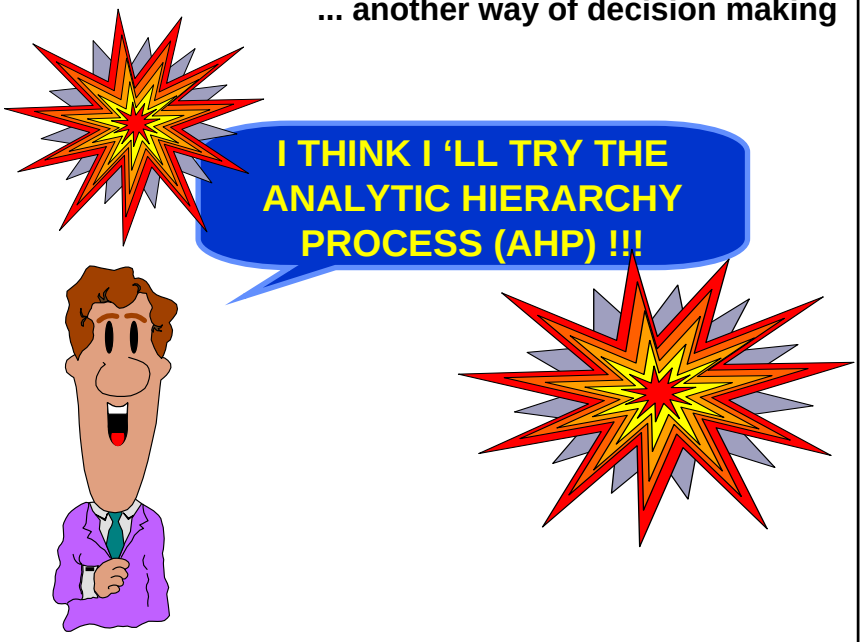
2

**DO YOUR RECOMMENDATIONS TURN OUT LIKE THIS?**


**MAYBE YOU NEED A NEW APPROACH**

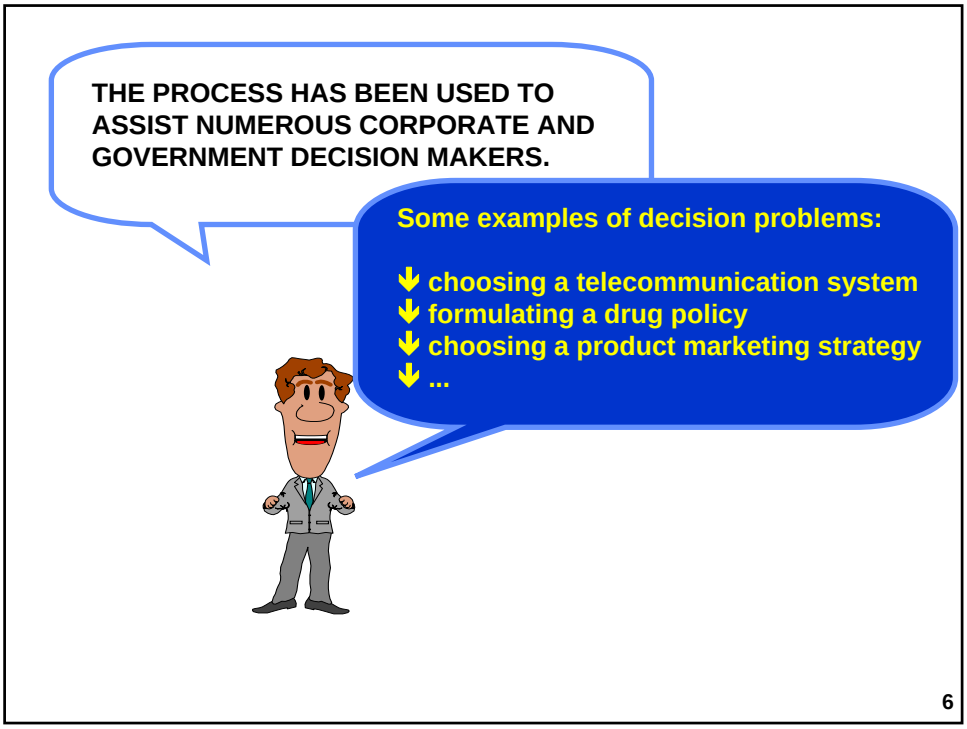
3

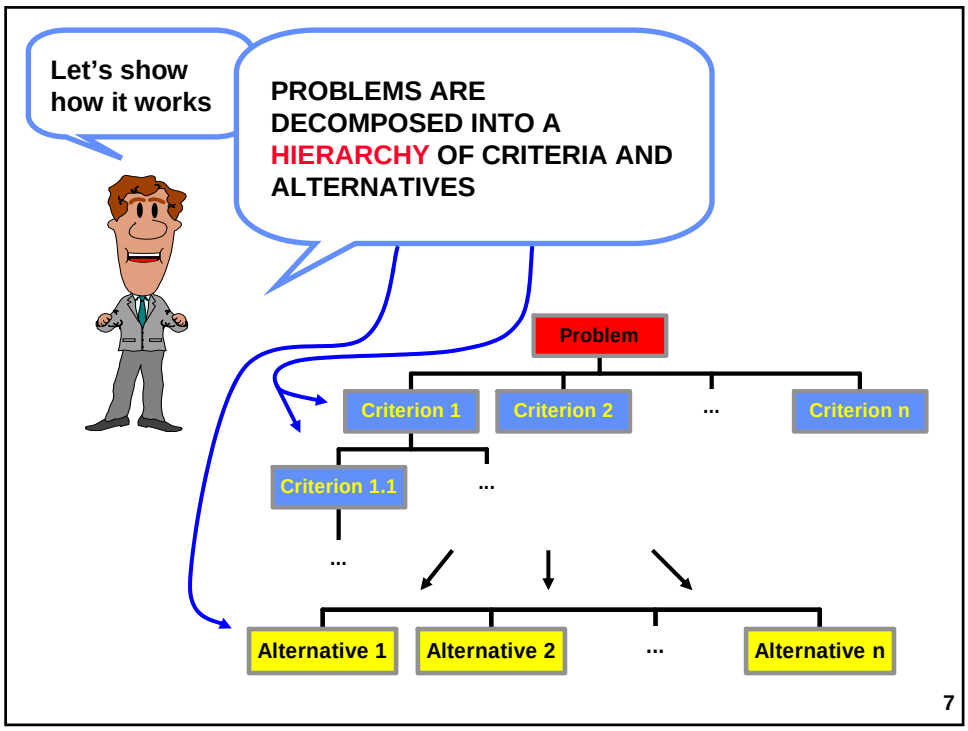
**... another way of decision making**

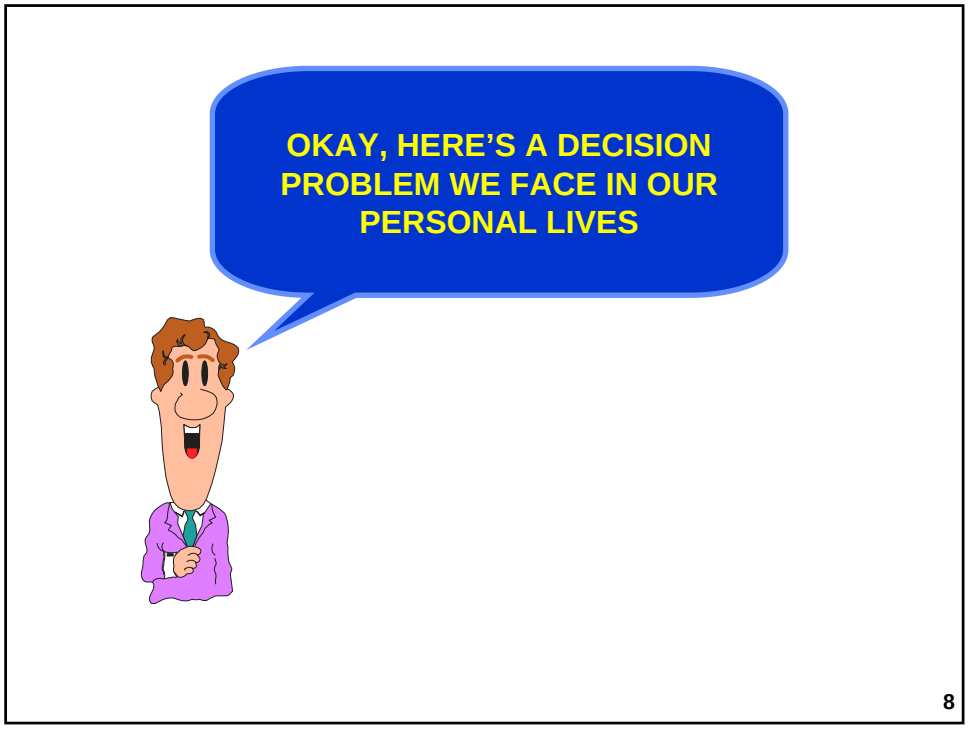



4


OKAY TELL US ABOUT AHP
DR THOMAS L. SAATY DEVELOPED THE PROCESS IN THE EARLY 1970'S AND...
5


THE PROCESS HAS BEEN USED TO ASSIST NUMEROUS CORPORATE AND GOVERNMENT DECISION MAKERS.
Some examples of decision problems:
choosing a telecommunication system
formulating a drug policy
choosing a product marketing strategy
...
6


Let's show how it works
PROBLEMS ARE DECOMPOSED INTO A HIERARCHY OF CRITERIA AND ALTERNATIVES
Problem
Criterion 1
Criterion 2
...
Criterion n
Criterion 1.1
...
...
Alternative 1
Alternative 2
...
Alternative n
7


OKAY, HERE'S A DECISION PROBLEM WE FACE IN OUR PERSONAL LIVES
8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AN IMPORTANT PART OF THE PROCESS IS TO ACCOMPLISH THESE THREE STEPS

- **STATE THE OBJECTIVE:**
  - SELECT A NEW CAR
- **DEFINE THE CRITERIA:**
  - STYLE, RELIABILITY, FUEL ECONOMY
- **PICK THE ALTERNATIVES:**
  - CIVIC COUPE, SATURN COUPE, FORD ESCORT, RENAULT CLIO

WHAT ABOUT COST?

SKEPTIC-GATOR

(BE QUIET, WE'LL TALK ABOUT THAT LATER)

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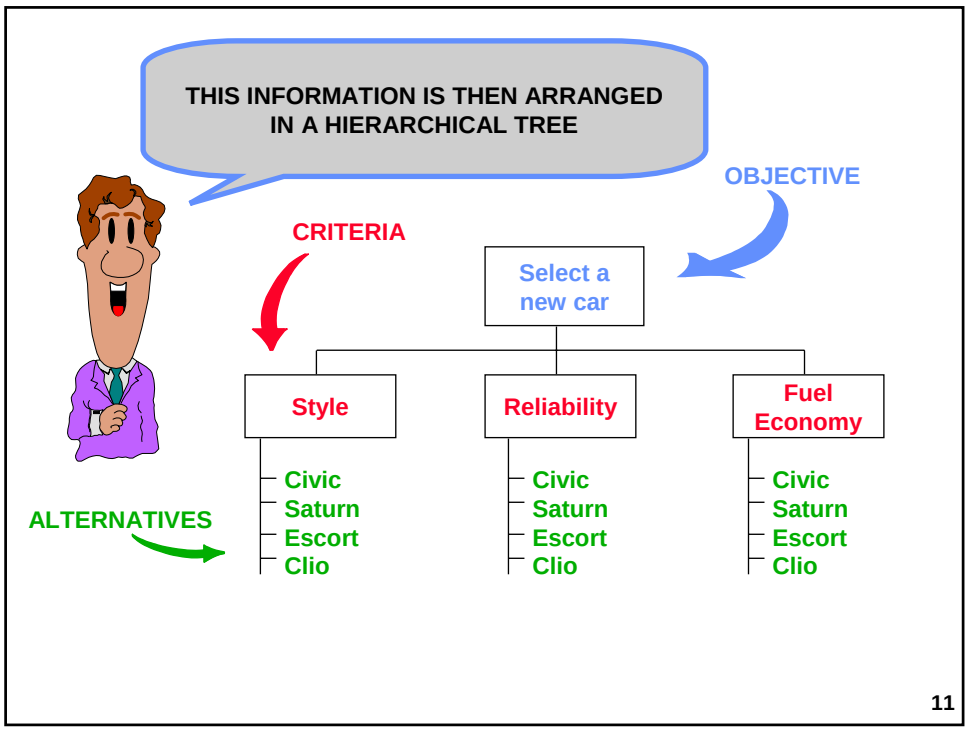
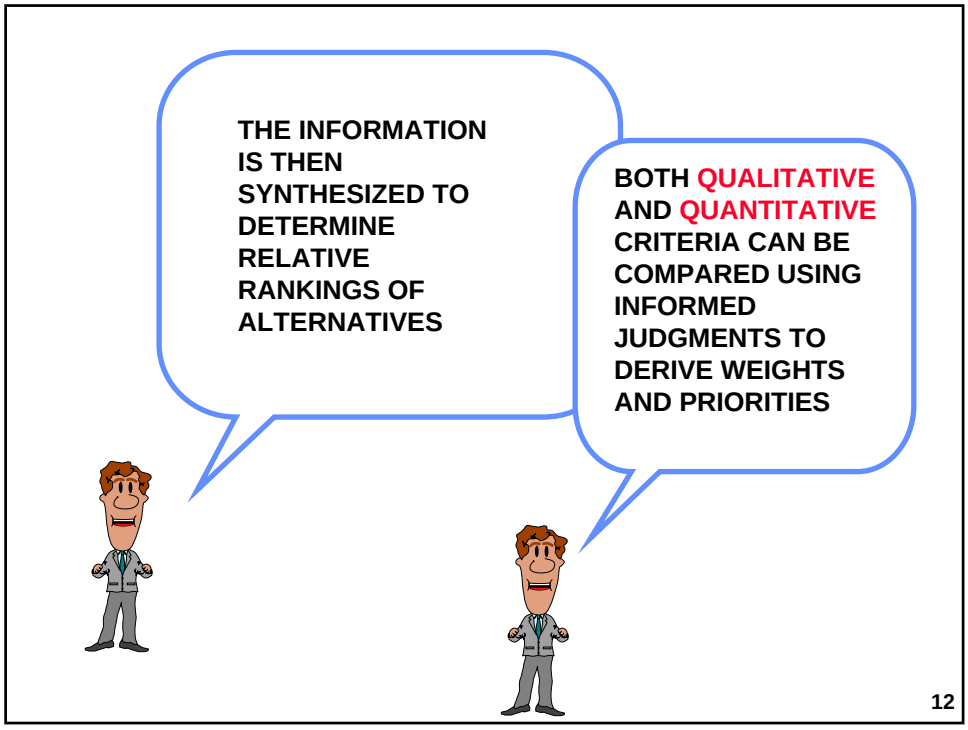

THIS INFORMATION IS THEN ARRANGED IN A HIERARCHICAL TREE
OBJECTIVE
CRITERIA
Select a new car
Style
Reliability
Fuel Economy
Civic
Saturn
Escort
Clio
Civic
Saturn
Escort
Clio
Civic
Saturn
Escort
Clio
ALTERNATIVES
11


THE INFORMATION IS THEN SYNTHESIZED TO DETERMINE RELATIVE RANKINGS OF ALTERNATIVES
BOTH QUALITATIVE AND QUANTITATIVE CRITERIA CAN BE COMPARED USING INFORMED JUDGMENTS TO DERIVE WEIGHTS AND PRIORITIES
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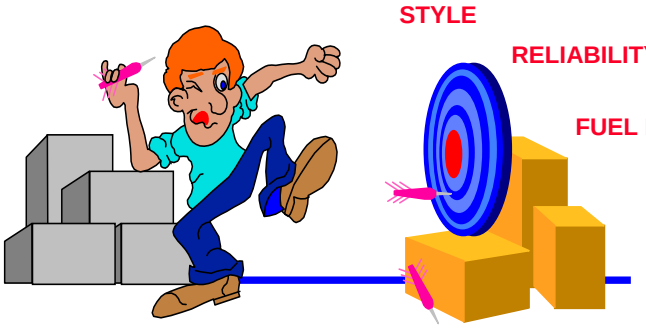
## HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

**Here's one way !**

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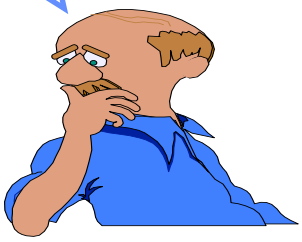
**HERE'S ANOTHER WAY**

Hmm, I think reliability is the most important followed by style and fuel economy is least importeant so I will make the following judgements ....



## USING JUDGMENTS TO DETERMINE THE RANKING OF THE CRITERIA

1. RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE
2. STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY
3. RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY

he's not very consistent here ... that's o.k.

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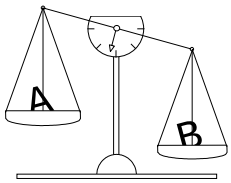
## Pairwise Comparisons




**USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED**

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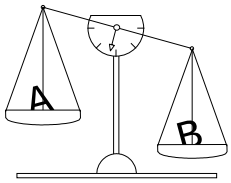
## Pairwise Comparisons




**USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED**

**1 equal 3 moderate 5 strong 7 very strong 9 extreme**

| | STYLE | RELIABILITY | FUEL ECONOMY |
|---|---|---|---|
| STYLE | 1/1 | 1/2 | 3/1 |
| RELIABILITY | | 1/1 | 4/1 |
| FUEL ECONOMY | | | 1/1 |

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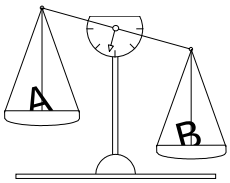
## Pairwise Comparisons




**USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED**

**1** equal **3** moderate **5** strong **7** very strong **9** extreme

| | STYLE | RELIABILITY | FUEL ECONOMY |
|---|---|---|---|
| STYLE | 1/1 | 1/2 | 3/1 |
| RELIABILITY | 2/1 | 1/1 | 4/1 |
| FUEL ECONOMY | 1/3 | 1/4 | 1/1 |

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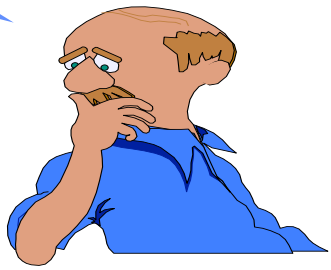
**How do you turn this MATRIX into ranking of criteria?**



| | STYLE | RELIABILITY | FUEL ECONOMY |
|---|---|---|---|
| STYLE | 1/1 | 1/2 | 3/1 |
| RELIABILITY | 2/1 | 1/1 | 4/1 |
| FUEL ECONOMY | 1/3 | 1/4 | 1/1 |

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**HOW DO YOU GET A RANKING OF PRIORITIES FROM A PAIRWISE MATRIX?**


ACTUALLY...

DR THOMAS L. SAATY, CURRENTLY WITH THE UNIVERSITY OF PITTSBURGH, DEMONSTRATED MATHEMATICALLY THAT THE EIGENVECTOR SOLUTION WAS THE BEST APPROACH.

REFERENCE : THE ANALYTIC HIERARCHY PROCESS, 1990, THOMAS L. SAATY

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## HERE'S HOW TO SOLVE FOR THE EIGENVECTOR:

1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.
2. THE ROW SUMS ARE THEN CALCULATED AND NORMALIZED.
3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.


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IT'S MATRIX ALGEBRA TIME !!!

| | STYLE | RELIABILITY | FUEL ECONOMY |
|---|---|---|---|
| STYLE | 1/1 | 1/2 | 3/1 |
| RELIABILITY | 2/1 | 1/1 | 4/1 |
| FUEL ECONOMY | 1/3 | 1/4 | 1/1 |

FOR NOW, LET'S REMOVE THE NAMES AND CONVERT THE FRACTIONS TO DECIMALS :

| | | |
|---|---|---|
| 1.0000 | 0.5000 | 3.0000 |
| 2.0000 | 1.0000 | 4.0000 |
| 0.3333 | 0.2500 | 1.0000 |

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STEP 1: SQUARING THE MATRIX

THIS TIMES

| | | |
|---|---|---|
| 1.0000 | 0.5000 | 3.0000 |
| 2.0000 | 1.0000 | 4.0000 |
| 0.3333 | 0.2500 | 1.0000 |

THIS

| | | |
|---|---|---|
| 1.0000 | 0.5000 | 3.0000 |
| 2.0000 | 1.0000 | 4.0000 |
| 0.3333 | 0.2500 | 1.0000 |

I.E. (1.0000 * 1.0000) + (0.5000 * 2.0000) +(3.0000 * 0.3333) = 3.0000

RESULTS IN THIS

| | | |
|---|---|---|
| 3.0000 | 1.7500 | 8.0000 |
| 5.3332 | 3.0000 | 14.0000 |
| 1.1666 | 0.6667 | 3.0000 |

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**STEP 2 : NOW, LET'S COMPUTE OUR FIRST EIGENVECTOR (TO FOUR DECIMAL PLACES)**

**FIRST, WE SUM THE ROWS**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3.0000 | + | 1.7500 | + | 8.0000 | = | 12.7500 | 0.3194 |
| 5.3332 | + | 3.0000 | + | 14.0000 | = | 22.3332 | 0.5595 |
| 1.1666 | + | 0.6667 | + | 3.0000 | = | 4.8333 | 0.1211 |
| | | | | | | 39.9165 | 1.0000 |

**SECOND, WE SUM THE ROW TOTALS** →

**FINALLY, WE NORMALIZE BY DIVIDING THE ROW SUM BY THE ROW TOTALS (I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)**

**THE RESULT IS OUR EIGENVECTOR ( A LATER SLIDE WILL EXPLAIN THE MEANING IN TERMS OF OUR EXAMPLE)** →

| |
|---|
| 0.3194 |
| 0.5595 |
| 0.1211 |

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**THIS PROCESS MUST BE ITERATED UNTIL THE EIGENVECTOR SOLUTION DOES NOT CHANGE FROM THE PREVIOUS ITERATION (REMEMBER TO FOUR DECIMAL PLACES IN OUR EXAMPLE)**

**CONTINUING OUR EXAMPLE,**
**AGAIN, STEP 1: WE SQUARE THIS MATRIX**

| | | |
|---|---|---|
| 3.0000 | 1.7500 | 8.0000 |
| 5.3332 | 3.0000 | 14.0000 |
| 1.1666 | 0.6667 | 3.0000 |

**WITH THIS RESULT** →

| | | |
|---|---|---|
| 27.6653 | 15.8330 | 72.4984 |
| 48.3311 | 27.6662 | 126.6642 |
| 10.5547 | 6.0414 | 27.6653 |

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**AGAIN STEP 2 : COMPUTE THE EIGENVECTOR (TO FOUR DECIMAL PLACES)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 27.6653 | + | 15.8330 | + | 72.4984 | = | 115.9967 | 0.3196 |
| 48.3311 | + | 27.6662 | + | 126.6642 | = | 202.6615 | 0.5584 |
| 10.5547 | + | 6.0414 | + | 27.6653 | = | 44.2614 | 0.1220 |
| | | | | | TOTALS | 362.9196 | 1.0000 |

**COMPUTE THE DIFFERENCE OF THE PREVIOUS COMPUTED EIGENVECTOR TO THIS ONE:**

| | | | | |
|---|---|---|---|---|
| 0.3194 | − | 0.3196 | = | - 0.0002 |
| 0.5595 | − | 0.5584 | = | 0.0011 |
| 0.1211 | − | 0.1220 | = | - 0.0009 |

**TO FOUR DECIMAL PLACES THERE'S NOT MUCH DIFFERENCE**
**HOW ABOUT ONE MORE ITERATION?**

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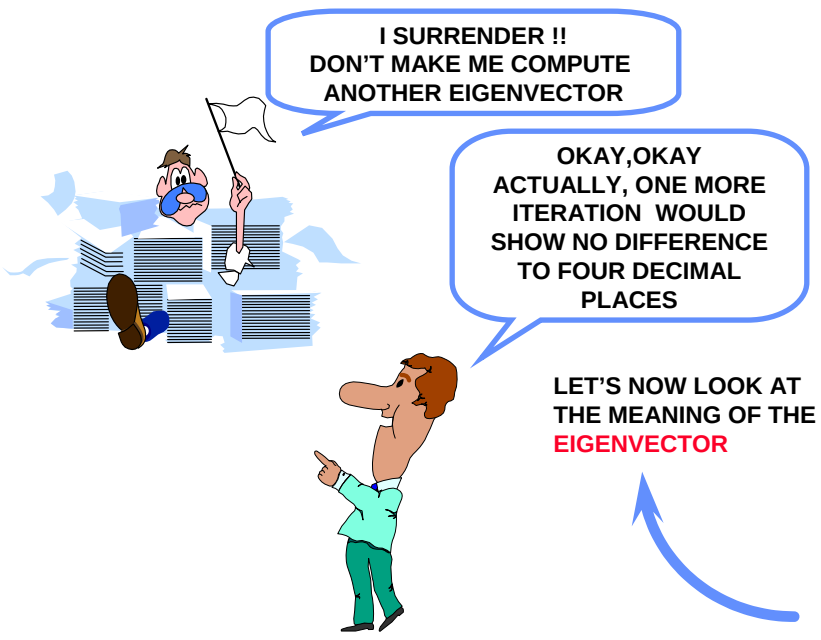


**LET'S NOW LOOK AT THE MEANING OF THE EIGENVECTOR**

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HERE'S OUR PAIRWISE MATRIX WITH THE NAMES

| | STYLE | RELIABILITY | FUEL ECONOMY |
|---|---|---|---|
| STYLE | 1/1 | 1/2 | 3/1 |
| RELIABILITY | 2/1 | 1/1 | 4/1 |
| FUEL ECONOMY | 1/3 | 1/4 | 1/1 |

AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

| | | |
|---|---|---|
| STYLE | 0.3196 | THE SECOND MOST IMPORTANT CRITERION |
| RELIABILITY | 0.5584 | THE MOST IMPORTANT CRITERION |
| FUEL ECONOMY | 0.1220 | THE LEAST IMPORTANT CRITERION |

NOW BACK TO THE HIEARCHICAL TREE...

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HERE'S THE TREE WITH THE CRITERIA WEIGHTS




WHAT ABOUT THE ALTERNATIVES?

SKEPTIC-GATOR

I'M GLAD YOU ASKED...

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IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

STYLE

| | CIVIC | SATURN | ESCORT | CLIO |
|---|---|---|---|---|
| CIVIC | 1/1 | 1/4 | 4/1 | 1/6 |
| SATURN | 4/1 | 1/1 | 4/1 | 1/4 |
| ESCORT | 1/4 | 1/4 | 1/1 | 1/5 |
| CLIO | 6/1 | 4/1 | 5/1 | 1/1 |

AND...

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IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

RELIABILITY

| | CIVIC | SATURN | ESCORT | CLIO |
|---|---|---|---|---|
| CIVIC | 1/1 | 2/1 | 5/1 | 1/1 |
| SATURN | 1/2 | 1/1 | 3/1 | 2/1 |
| ESCORT | 1/5 | 1/3 | 1/1 | 1/4 |
| CLIO | 1/1 | 1/2 | 4/1 | 1/1 |

ITS MATRIX ALGEBRA TIME!!!

30

COMPUTING THE EIGENVECTOR DETERMINES THE RELATIVE RANKING OF ATERNATIVES UNDER EACH CRITERION

| RANKING | | STYLE |
|---|---|---|
| 3 | CIVIC | .1160 |
| 2 | SATURN | .2470 |
| 4 | ESCORT | .0600 |
| 1 | CLIO | .5770 |

| RANKING | | RELIABILITY |
|---|---|---|
| 1 | CIVIC | .3790 |
| 2 | SATURN | .2900 |
| 4 | ESCORT | .0740 |
| 3 | CLIO | .2570 |

WHAT ABOUT FUEL ECONOMY?

SKEPTIC-GATOR

ANOTHER GOOD QUESTION...

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AS STATED EARLIER, AHP CAN COMBINE BOTH QUALITATIVE AND QUANITATIVE INFORMATION

FUEL ECONOMY INFORMATION IS OBTAINED FOR EACH ALTERNATIVE:

| | FUEL ECONOMY (MILES/GALLON) | | |
|---|---|---|---|
| CIVIC | 34 | 34 / 113 = | .3010 |
| SATURN | 27 | 27 / 113 = | .2390 |
| ESCORT | 24 | 24 / 113 = | .2120 |
| CLIO | 28 | 28 / 113 = | .2480 |
| | 113 | | 1.0000 |

NORMALIZING THE FUEL ECONOMY INFO ALLOWS US TO USE IT WITH OTHER RANKINGS

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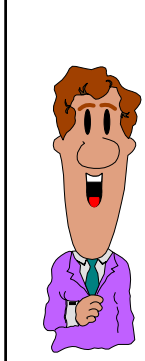

HERE'S THE TREE WITH ALL THE WEIGHTS

OBJECTIVE

CRITERIA

Select a new car
1.00

| Style .3196 | | Reliability .5584 | | Fuel Economy .1220 | |
|---|---|---|---|---|---|
| Civic | .1160 | Civic | .3790 | Civic | .3010 |
| Saturn | .2470 | Saturn | .2900 | Saturn | .2390 |
| Escort | .0600 | Escort | .0740 | Escort | .2120 |
| Clio | .5770 | Clio | .2570 | Clio | .2480 |

ALTERNATIVES

OKAY, NOW WHAT ? I THINK WE'RE READY FOR THE ANSWER...

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A LITTLE MORE MATRIX ALGEBRA GIVES US THE SOLUTION:

| | STYLE | RELI-ABILITY | FUEL ECONOMY | | CRITERIA RANKING | |
|---|---|---|---|---|---|---|
| CIVIC | .1160 | .3790 | .3010 | * | 0.3196 | STYLE |
| SATURN | .2470 | .2900 | .2390 | | 0.5584 | RELIABILITY |
| ESCORT | .0600 | .0740 | .2120 | | | |
| CLIO | .5770 | .2570 | .2480 | | 0.1220 | FUEL ECONOMY |

I.E. FOR THE CIVIC (.1160 * .3196) + (.3790 * .5584) + (.3010 * .1220) = .3060

| | | |
|---|---|---|
| Civic | .3060 | |
| Saturn | .2720 | |
| Escort | .0940 | |
| Clio | .3280 | |

AND THE WINNER IS !!!

THE CLIO IS THE HIGHEST RANKED CAR

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IN SUMMARY, THE ANALYTIC HIERARCHY PROCESS PROVIDES A LOGICAL FRAMEWORK TO DETERMINE THE BENEFITS OF EACH ALTERNATIVE

| | |
|---|---|
| 1. Clio | .3280 |
| 2. Civic | .3060 |
| 3. Saturn | .2720 |
| 4. Escort | .0940 |

WHAT ABOUT COSTS?

SKEPTIC-GATOR

WELL, I'LL TELL YOU...

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ALTHOUGH COSTS COULD HAVE BEEN INCLUDED, IN MANY COMPLEX DECISIONS, COSTS SHOULD BE SET ASIDE UNTIL THE BENEFITS OF THE ALTERNATIVES ARE EVALUATED

OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I DON'T CARE ABOUT ITS BENEFITS

DISCUSSING COSTS TOGETHER WITH BENEFITS CAN SOMETIMES BRING FORTH MANY POLITICAL AND EMOTIONAL RESPONSES

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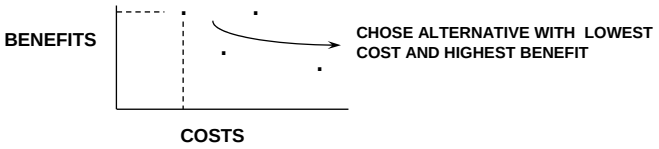
**WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:**

1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE




2. BENEFIT TO COST RATIOS

3. LINEAR PROGRAMMING

4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS

**IN OUR EXAMPLE...**

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**LET'S USE BENEFIT TO COST RATIOS**

| | COST $ | NORMALIZED COSTS | BENEFIT - COST RATIOS |
|---|---|---|---|
| 1. CLIO | 18,000 | .3333 | .3280 / .3333 = .9840 |
| 2. CIVIC | 12,000 | .2222 | .3060 / .2222 = 1.3771 |
| 3. SATURN | 15,000 | .2778 | .2720 / .2778 = .9791 |
| 4. ESCORT | 9,000 | .1667 | .0940 / .1667 = .5639 |
| | 54,000 | 1.0000 | |

(REMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)

**AND...**

**THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO**

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AHP CAN BE USED FOR VERY COMPLEX DECISIONS

MANY LEVELS OF CRITERIA AND SUBCRITERIA CAN BE INCLUDED




HERE'S SOME EXAMPLES

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AHP CAN BE USED FOR A WIDE VARIETY OF APPLICATIONS

STRATEGIC PLANNING

RESOURCE ALLOCATION

SOURCE SELECTION

BUSINESS/PUBLIC POLICY

PROGAM SELECTION

AND MUCH MUCH MORE...

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