UNIVERSIDAD DE SEVILLA

INSEGUS

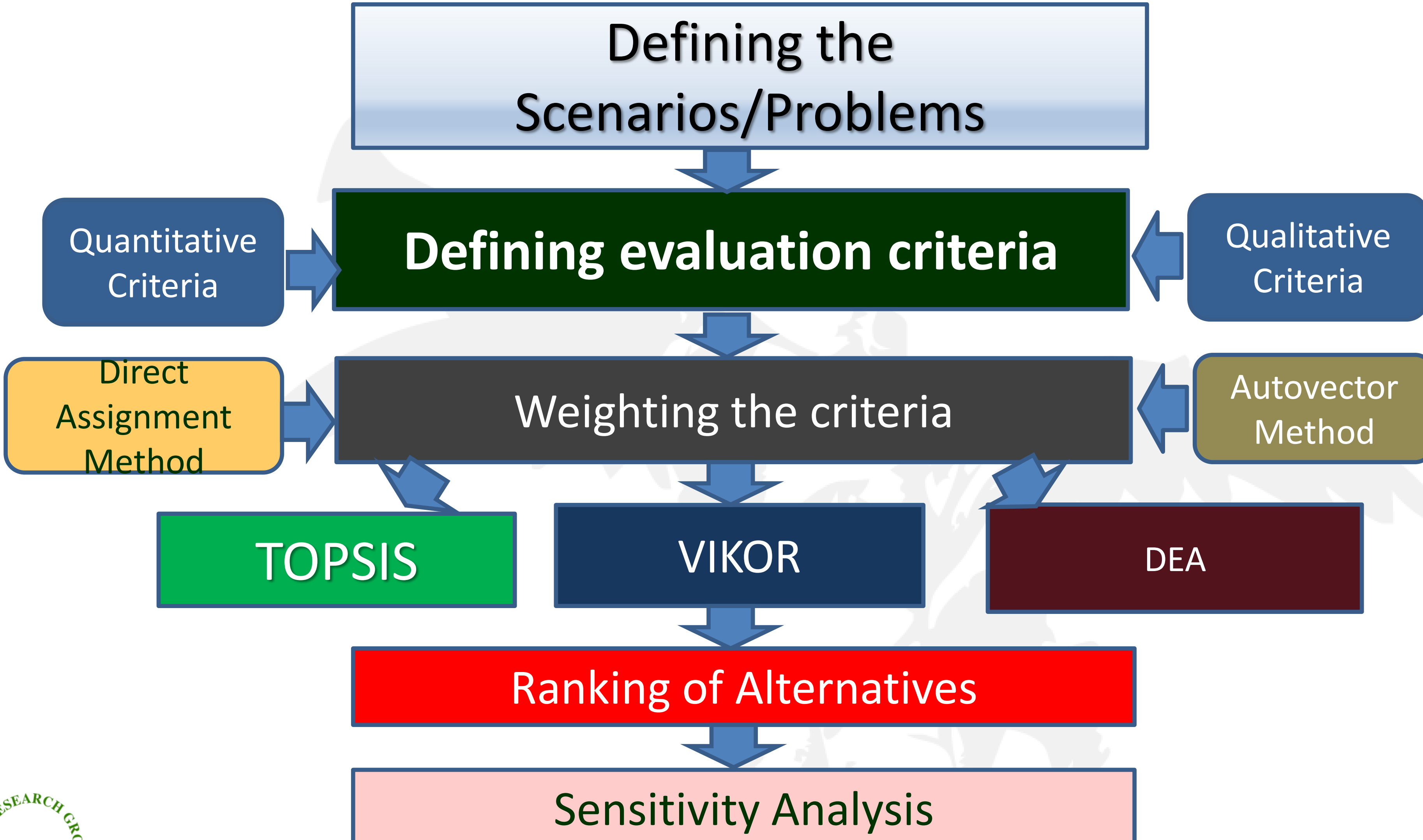
Técnicas de Toma de Decisiones

Dr. Rafael Martínez Gasca
Grupo de Investigación **IDEA**,
Tecnologías Inteligentes y de Seguridad de los Sistemas de Información
Departamento de Lenguajes y Sistemas Informáticos
Universidad de Sevilla

- **Multi Criteria Decision Making (MCDM o MADM)**

TOPSIS Y VIKOR

MCDM.
The Process



Types of MCDM

- Theoretical base for MCDM
  - **Based on MAUT–Multi-attribute Utility Theory**, propios de la American School (AHP, ANP,…)
  - **Based on Outranking methods** (Superación o Sobreclasificación) European School. ELECTRE, PROMETHEE, VIKOR, TOPSIS,…
    - This concept was proposed by Roy (1968).
    - The basic idea is an alernative i supera a la alternativa j si para la mayor parte de los criterios Ai es al menos igual de buena que Aj (condición de concordancia), mientras que no hay ningún criterio para el cual sea notoriamente inferior (condición de discordancia).

- TOPSIS (MCDM)
  - SAW
  - WP
  - TOPSIS
- VIKOR

- **Design of Experiment (DoE)** helps researchers to determine which subset of parameters has the largest influence on the performance of business process
  - Tools: Table Anova-Minitab for two-factor interaction

Decision Matrix

Criteria	Cost	Usability	Perform.	Design
Norm. w	.3333	.2667	.2	.2
a_1	300	3	3	3
a_2	250	3	2	3
a_3	250	2	3	3
a_4	200	2	3	2
a_5	200	2	2	3
a_6	200	1	3	3
a_7	100	1	2	2

- **Simple Additive Weighting – Weighted Average – Weighted Sum** (Yoon & Hwang, 1995; Vincke, 1992...)
- A global (total) score in the SAW is obtained by adding contributions from each attribute.
- A common numerical scaling system **such as normalization** (instead of single dimensional value functions) is required to permit addition among attribute values.
- **Value (global score) of an alternative** can be expressed as: $V(a_i) = V_i = \Sigma\, w_j\, r_{ij}$

Decision Matrix and Global Scores

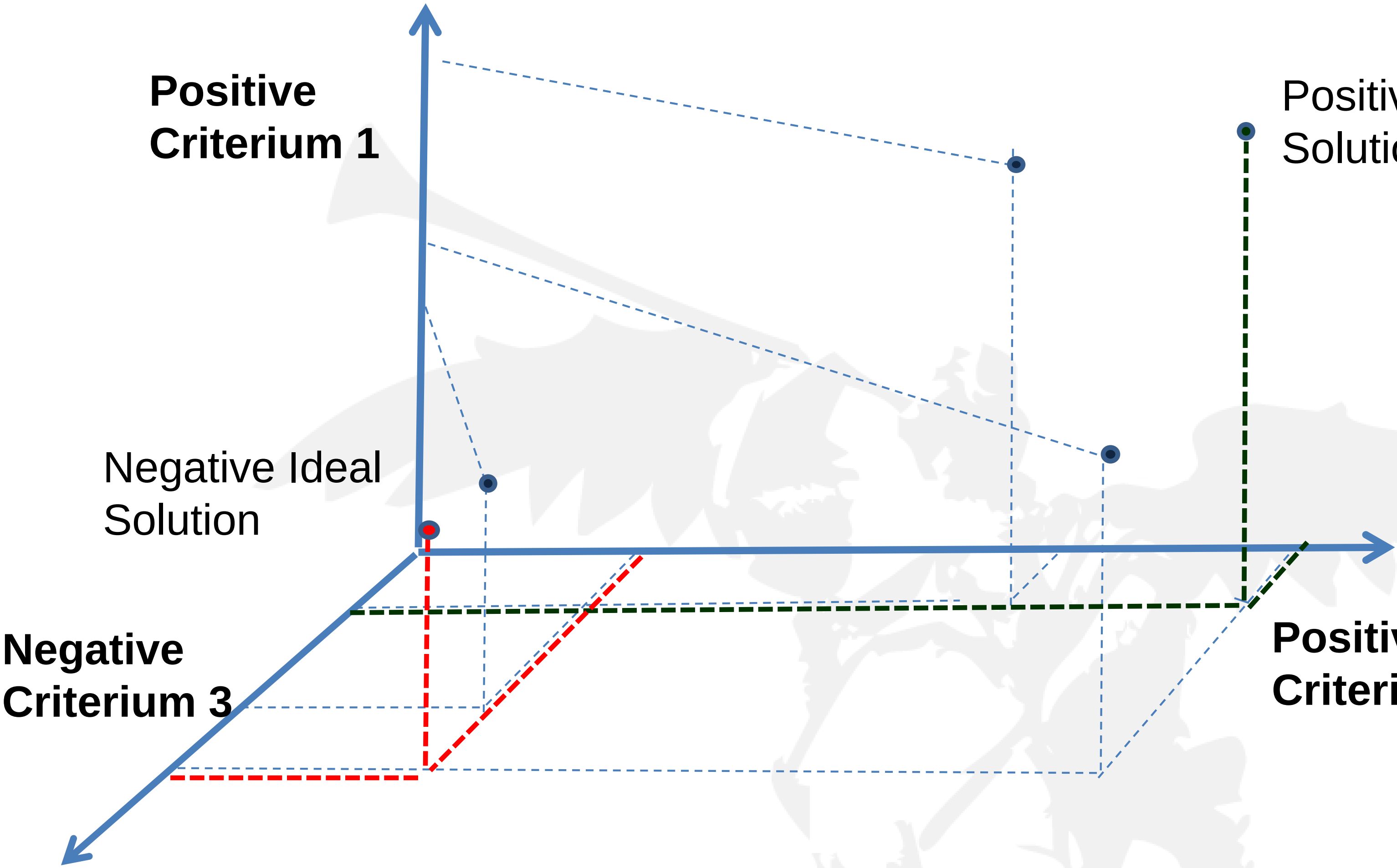
Criteria	Cost	Usability	Perform.	Design	V_i
Norm. w	**0.3333**	**0.2667**	**0.2**	**0.2**	
a_1	**0.3333**	**1**	**1**	**1**	**.7778**
a_2	**0.4**	**1**	**0.6667**	**1**	.7334
a_3	**0.4**	**0.6667**	**1**	**1**	.7111
a_4	**0.5**	**0.6667**	**1**	**0.6667**	.6778
a_5	**0.5**	**0.6667**	**0.6667**	**1**	.6778
a_6	**0.5**	**0.3333**	**1**	**1**	.6555
a_7	**1**	**0.3333**	**0.6667**	**0.6667**	.6889

- **Weighted Product** (Yoon & Hwang, 1995)
- **Normalization is not necessary!**
- When WP is used weights become exponents associated with each attribute value;
  - a positive power for benefit attributes
  - a negative power for cost attributes
- Because of the exponent property, this method requires that **all ratings be greater than 1**. When an attribute has fractional ratings, all ratings in that attribute are multiplied by 10^m to meet this requirement

$$V_i = \prod_j (x_{ij})^{w_j}$$

Decision Matrix and Global Scores

Criteria	Cost	Usability	Perform.	Design	V_i
Norm. w	.3333	.2667	.2	.2	
a_1	300	3	3	3	.3108
a_2	250	3	2	3	.3045
a_3	250	2	3	3	.2964
a_4	200	2	3	2	.2944
a_5	200	2	2	3	.2944
a_6	200	1	3	3	.2654
a_7	100	1	2	2	.2843



Positive Criterium 1
Positive Ideal Solution
Negative Ideal Solution
Negative Criterium 3
Positive Criterium 2

- **Technique for Order Preference by Similarity to Ideal Solution** (Yoon & Hwang, 1995; Hwang & Lin, 1987)
- Concept: Chosen alternative should **have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution**
- Steps:
  1. **Calculate normalized ratings**
  2. **Calculate weighted normalized ratings**
  3. **Identify positive-ideal and negative-ideal solutions**
  4. **Calculate separation measures**
  5. **Calculate similarities to positive-ideal solution**
  6. **Rank preference order**

1. **Calculate normalized ratings**
   - Vector normalization (Euclidean) is used
   - Do not take the inverse rating for cost attributes!
2. **Calculate weighted normalized ratings**
   - $v_{ij} = w_j * r_{ij}$
3. **Identify positive-ideal and negative-ideal solutions**

$$a^* = \{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\} = \left\{ \left(\max_i v_{ij} \middle| j \in J_1 \right), \left(\min_i v_{ij} \middle| j \in J_2 \right) \middle| i = 1, \ldots, m \right\}$$

$$a^- = \{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\} = \left\{ \left(\min_i v_{ij} \middle| j \in J_1 \right), \left(\max_i v_{ij} \middle| j \in J_2 \right) \middle| i = 1, \ldots, m \right\}$$

where J_1 is a **set of benefit attributes** and J_2 is a **set of cost attributes**

4. Calculate separation measures

- Euclidean distance (separation) of each alternative from the ideal solutions are measured:

$$S_i^* = \sqrt{\sum_j (v_{ij} - v_j^*)^2} \qquad S_i^- = \sqrt{\sum_j (v_{ij} - v_j^-)^2}$$

5. Calculate similarities to positive-ideal solution

$$C_i^* = S_i^- / (S_i^* + S_i^-)$$

6. Rank preference order

– Rank the alternatives according to similarities in descending order.

– Recommend the alternative with the maximum similarity

- Normalized (Vector) Decision Matrix

	Cost	**Usability**	**Perf.**	**Design**
Norm. w	0.3333	0.2667	0.2	0.2
a_1	0.5108	0.5303	0.433	0.4121
a_2	0.4256	0.5303	0.2887	0.4121
a_3	0.4256	0.3536	0.433	0.4121
a_4	0.3405	0.3536	0.433	0.2747
a_5	0.3405	0.3536	0.2887	0.4121
a_6	0.3405	0.1768	0.433	0.4121
a_7	0.1703	0.1768	0.2887	0.2747

Weighted Normalized Ratings & Positive–Negative Ideal

	Cost	Usability	Perf.	Design
a_1	0.1703	0.1414	0.0866	0.0824
a_2	0.1419	0.1414	0.0577	0.0824
a_3	0.1419	0.0943	0.0866	0.0824
a_4	0.1135	0.0943	0.0866	0.0549
a_5	0.1135	0.0943	0.0577	0.0824
a_6	0.1135	0.0471	0.0866	0.0824
a_7	0.0568	0.0471	0.0577	0.0549
a^*	.0568	.1414	.0866	.0824
a^-	.1703	.0471	.0577	.0549

Separation Measures & Similarities to Positive Ideal Solution

	S^*	S^-	C^*	Rank
a_1	0,1135	0,1024	0,4742	5
$\boldsymbol{a_2}$	0,0899	0,1022	**0,5321**	1
a_3	0,0973	0,0679	0,4111	6
a_4	0,0787	0,0792	0,5016	3
a_5	0,0792	0,0787	0,4984	4
a_6	0,11	0,0693	0,3866	7
a_7	0,1024	0,1135	0,5258	2

- If an alternative is removed or added, the total process for **TOPSIS should be redone**, which is laborious and time-consuming
- TOPSIS is not able to handle the stochastic alternatives (Hellinger TOPSIS uses the Hellinger distance)
- Two reference points are often insufficient, especially for nonlinear problems. As a consequence of this situation, the final result ranking is prone to errors, including **the rank reversals phenomenon.**
  - To avoid this phenomenon: The characteristic objects method (COMET). This method takes into account the existence of a correlation between components of an MCDM function. Additionally, comparisons between COs are easier than comparisons between alternatives

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– TOPSIS (MCDM)
  - SAW
  - WP
  - TOPSIS
– VIKOR

- The VIKOR method of compromise ranking determines a compromise solution, providing
  - A maximum “group utility” for the “majority” and
  - A minimum of an individual regret for the “opponent”.
- Online Tools for TOPSIS and VIKOR
  - **TOPSIS https://rdrr.io/cran/MCDM/man/TOPSISLinear.html**
  - **VIKOR https://rdrr.io/cran/MCDM/man/VIKOR.html**