

Fitting mixed random regret minimization models using mixrandregret.

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- 1 Random Regret Minimization Models
- 2 Differences between RUM and RRM models.
- 3 Mixed Random Regret Minimization Models
- 4 Individual Level Parameters
- 5 Implementation
- 6 Conclusions
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1 Outline

① Random Regret Minimization Models

Random Utility vs Random Regret
Classical Regret Function

② Differences between RUM and RRM models.

③ Mixed Random Regret Minimization Models

④ Individual Level Parameters

⑤ Implementation

⑥ Conclusions

1 *What is Regret and how to use it for Choice Modeling?*

- ▶ From Utility to Regret.
- ▶ Regret: Situation where a non-chosen alternative ends up being more attractive than the chosen one
- ▶ Individuals are assumed to minimize regret.

Table: Hypothetical Choice Situation

Attribute \ Route	1	2	3
Travel Time	23 min.	27 min.	35 min.
Travel Cost	6 euros	4 euros	3 euros

If we chose alternative 2:

- ▶ Alternative 1 is faster...
- ▶ Alternative 3 is cheaper...

⇒ RRM models will (formalize and) minimize this notion of regret!

1 Formalization of the later example.

Attribute \ Route	1	2	3
Travel Time	23 min.	27 min.	35 min.
Travel Cost	6 euros	4 euros	3 euros

Table: Hypothetical Choice Situation

We will denote in the following:

- ▶ Decision-makers (referred to n) answer s choice situations
- ▶ They decide among J alternatives (referred to i or j indistinctly).
- ▶ Each alternative is described by M attributes (referred to m).
- ▶ $x_{ins,m}$ The value of attribute m of alternative i for individual n in choice situation s is denoted by .
- ▶ y_{ins} is the response variable that takes the value of 1 when alternative i is chosen by individual n in choice situation s and 0 otherwise.

1 RUM vs RRM

► Random Utility Maximization (RUM)

Utility ← $U_{ins} = V_{ins} + \varepsilon_{ins}$ ← Systematic Utility

$$= \beta_{n,T} \times x_{ins,T} + \beta_{n,C} \times x_{ins,C} + \varepsilon_{ins}$$

► Random Regret Minimization (RRM)

Regret ← $RR_{ins} = R_{ins} + \varepsilon_{ins}$ ← Systematic Regret

$$= \sum_{j \neq i}^J R_{i \leftrightarrow jns,T} + \sum_{j \neq i}^J R_{i \leftrightarrow jns,T} + \varepsilon_{ins}$$

- The **notion of regret** is characterized by the systematic regret R_{ins} .
- R_{in} is described in terms of **attribute level regret** ($R_{i \leftrightarrow jns,m}$).

1 The Attribute level regret $R_{i \leftrightarrow jns,m}$

Attribute \ Route	1	2	3
Travel Time	23 min.	27 min.	35 min.
Travel Cost	6 euros	4 euros	3 euros

- $R_{i \leftrightarrow jns,m}$ describes the pairwise combinations of regret derived from alternatives.

$(x_{jns,m} - x_{ins,m})$	Attribute \ Route	$j = 1$	$j = 2$	$j = 3$
$(x_{jns,m} - x_{1ns,T})$	Travel Time	0	4	12
$(x_{jns,m} - x_{1ns,C})$	Travel Cost	0	-2	-3
$(x_{jns,m} - x_{2ns,T})$	Travel Time	-4	0	8
$(x_{jns,m} - x_{2ns,C})$	Travel Cost	2	0	-1
$(x_{jns,m} - x_{3ns,T})$	Travel Time	-12	-8	0
$(x_{jns,m} - x_{3ns,C})$	Travel Cost	3	1	0

- **Takeaway:** We will define $R_{i \leftrightarrow jns,m}$ in terms of the attribute differences.

1 Classical RRM (Chorus, 2010)

- ▶ (Chorus, 2010) proposed the following attribute level regret:

$$R_{i \leftrightarrow jns,m} = \ln [1 + \exp \{ \beta_{n,m} \cdot (x_{jns,m} - x_{ins,m}) \}]$$

- ▶ $R_{i \leftrightarrow jns,m}$ compares alternative i with alternative j in attribute m .
- ▶ $\sum_{j \neq i} R_{i \leftrightarrow jns,m}$ is the equivalent to $x_{ins,m} \times \beta_{n,m}$ in an utilitarian model.
- ▶ $\beta_{n,m}$ is the taste parameter of attribute m of individual n .

1 Classical RRM (Chorus, 2010)

- (Chorus, 2010) proposed the following systematic regret:

$$R_{ins} = \sum_{j \neq i} \sum_{m=1}^M R_{i \leftrightarrow jns,m} = \sum_{j \neq i} \sum_{m=1}^M \ln [1 + \exp \{\beta_{n,m} \cdot (x_{jns,m} - x_{ins,m})\}]$$

Attribute level regret.

Linear sum of all attribute level regret.

(1)

- From our example: $\mathcal{M} = \{T, C\}$, $J = 3$.
- Regret of alternative 1 (R_{1ns}) will be described by:

$$\begin{aligned} R_{1ns} &= \sum_{j \neq i} \sum_{m \in \mathcal{M}} \ln [1 + \exp \{\beta_{n,m} (x_{jns,m} - x_{ins,m})\}] \\ &= \ln [1 + \exp \{\beta_{n,T} (x_{2ns,T} - x_{1ns,T})\}] + \ln [1 + \exp \{\beta_{n,C} (x_{2ns,C} - x_{1ns,C})\}] \\ &\quad + \ln [1 + \exp \{\beta_{n,T} (x_{3ns,T} - x_{1ns,T})\}] + \ln [1 + \exp \{\beta_{n,C} (x_{3ns,C} - x_{1ns,C})\}] \end{aligned}$$

1 Classical RRM (Chorus, 2010): Towards the log-likelihood.

- 1 Defining $RR_{ins} = R_{ins} + \varepsilon_{ins}$, where ε_{ins} is a type I Extreme Value i.i.d. error.
- 2 Acknowledging that the minimization of the random regret is mathematically equivalent to maximizing the negative of the regret.
- 3 Hence, the probabilities may be derived using the Multinomial Logit:

$$P_{ins} = \frac{\exp(-R_{ins})}{\sum_{j=1}^J \exp(-R_{jns})} \quad \text{for } i = 1, \dots, J \quad (2)$$

- 4 Consequently, the log-likelihood will be described by:

$$\ln L = \sum_{n=1}^N \sum_{s=1}^S \sum_{i=1}^J y_{in} \times \ln(P_{ins})$$

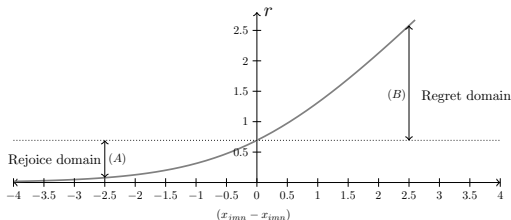
2 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
Taste Parameter Interpretation in RRM models
Semi-compensatory Behavior and the Compromise Effect
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions

2 Taste Parameter Interpretation in RRM models

- ▶ RUM: parameters are interpreted as the change in utility caused by an increase of a particular attribute level.
- ▶ RRM: parameters represent the *potential* change in regret associated with comparing a considered alternative with another alternative in terms of the attribute, caused by one unit change in a particular attribute level.
 - For instance: $\hat{\beta}_m > 0$
suggests that **regret increases** as the **level of that attribute increases in a non-chosen alternative**, in comparison to the level of the same attribute in the chosen alternative (e.g: Comfortable level).
 - For instance: $\hat{\beta}_m < 0$
suggests that **regret decreases** as the **level of that attribute increases in a non-chosen alternative**, in comparison to the level of the same attribute in the chosen alternative (e.g: Total Time).
- ▶ All in all. the parameters in RUM and RRM, are expected to have the same sign, even though their interpretation is dramatically different.

2 Semi-compensatory Behavior and the Compromise Effect



- ▶ **Attribute level regret** $R_{i \leftrightarrow j, mn}$ with $\beta_m = 1$.
- ▶ $(A) = \text{rejoice}$ and $(B) = \text{regret}$ on an equal difference of attribute level.
- ▶ For an equal difference of the attribute levels \Rightarrow regret $\gg \gg$ rejoice
- ▶ Linear RUM models \Rightarrow fully-compensatory model.
- ▶ **Compromise Effect:** Alternatives with “*balanced*” performance in all attributes are more attractive than alternatives with a severe poor performance in one attribute.

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3 Mixed Random Regret Minimization Models

New assumptions of the model.

- ▶ β_n follows a parametric distribution, denoted by $f(\beta|\varphi)$.
- ▶ φ are the parameters that describe the distribution (e.g., mean and variance of a Normal distribution).
- ▶ We define the conditional probability (CP) of the observed sequence of choices of individual n (conditional on knowing β_n) as:

$$P_n(\beta_n) = \prod_{s=1}^S \prod_{i=1}^J \{P_{ins}\}^{y_{ins}} \quad (3)$$

- ▶ The unconditional probability of the observed sequence of choices is the CP integrated over the entire domain of the distribution of β

$$\ln L(\beta) = \sum_{n=1}^N \ln \left[\int_{\beta} P_n(\beta) f(\beta|\varphi) d\beta \right] \quad (4)$$

3 Mixed Random Regret Minimization Models

- ▶ Given that equation (4) has no closed form we will approximate it using simulations (Train, 2009).
- ▶ Hence, we will estimate the model using Maximum Simulate Likelihood where we will maximize the following simulated log-likelihood function:

$$SLL(\beta) = \sum_{n=1}^N \ln \left\{ \frac{1}{R} \sum_{r=1}^R P_n(\beta^r) \right\} \quad (5)$$

- ▶ R is the number of replications and r is the r -th drawn from $f(\beta|\varphi)$.

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4 Individual Level Parameters

- ▶ We can obtain estimates of the individual level parameters (β_n):

$$\bar{\beta}_n = \frac{\int_{\beta} \beta \times P_n(\beta) f(\beta|\varphi) d\beta}{\int_{\beta} P_n(\beta) f(\beta|\varphi) d\beta} \quad (6)$$

- ▶ We will approximate this individual level parameter by simulation using:

$$\check{\beta}_n = \sum_{r=1}^R \left(\frac{\beta^r \times P_n(\beta^r)}{\sum_{r=1}^R P_n(\beta^r)} \right) \quad (7)$$

- ▶ For this estimation we will use the command `mixrbeta` after estimating the population parameters using `mixrandregret`.

5 Outline

- ① Random Regret Minimization Models
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- ④ Individual Level Parameters
- ⑤ **Implementation**
Syntax
Outputs
- ⑥ Conclusions

5 Syntax

`mixrandregret` is implemented as a Mata-based `gf-0 ml` evaluator. The command allows the inclusion of normally and log-normally distributed random parameters.

```
mixrandregret depvar [indepvars] [if] [in] group(varname)
alternative(varname) rand(varlist) [, id(varname)
basealternative(string) noconstant ln(string) nrep(string)
burn(string) robust cluster(varname) level(#) maximize_options]
```

The command `mixrbeta` can be used after `mixrandregret` to calculate individual-level parameters corresponding to the variables in the specified varname using the method proposed by [Train \(2009\)](#).

```
mixrbeta varlist saving(filename) [, plot nrep(#) burn(#)]
```

5 The Data

- Data from [van Cranenburgh \(2018\)](#): Stated Choice (SC) experiment.

```
. rename (choice) (choice_w)
. qui reshape long tt tc , i(obs) j(altern)
. generate choice = 0
. replace choice = 1 if choice_w==altern
(1,060 real changes made)
. label define alt_label 1 "First" 2 "Second" 3 "Third"
. label values altern alt_label
. list obs altern choice id tt tc in 1/6, sepby(obs)
```

	obs	altern	choice	id	tt	tc
1.	1	First	0	1	23	6
2.	1	Second	0	1	27	4
3.	1	Third	1	1	35	3
4.	2	First	0	1	27	5
5.	2	Second	1	1	35	4
6.	2	Third	0	1	23	6

- Three unlabeled route alternatives ($J = 3$).
 - Described by Travel Cost (tc) and Travel Time (tt) ($M = 2$).
- Each respondent (id) answered a total of 10 choice situations.

5 Fixed Parameter RRM model

- ▶ First we estimate a fixed parameters regret model.
- ▶ As expected, both parameter estimates are negative.

```
. randregret choice tc tt, gr(obs) alt(altern) rrmfn(classic) ///  
> nocons cluster(id) nolog
```

Fitting Classic RRM Model

RRM: Classic Random Regret Minimization Model

Case ID variable: obs	Number of cases	=	1060
Alternative variable: altern	Number of obs	=	3180
	Wald chi2(2)	=	40.41
Log likelihood = -1118.4784	Prob > chi2	=	0.0000
	(Std. Err. adjusted for	106 clusters in id)	

choice		Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
RRM	tc	-.417101	.068059	-6.13	0.000	-.5504943	-.2837078
	tt	-.102813	.0182526	-5.63	0.000	-.1385874	-.0670386

```
. matrix b_rrm = e(b)
```

5 Mixed RRM model: Normal Distribution

- ▶ Mixed model with total time (tt) being Normally distributed.
- ▶ The mean of tt is negative, as expected, however...

```
. mixrandregret choice tc , gr(obs) alt(altern) rand(tt) id(id) ///  
> nocons cluster(id) nrep(500) from(init_mix_rrm) tech(bhhh) nolog  
Case ID variable: obs                      Number of cases      =      1060  
Alternative variable: altern  
Random variable(s): tt  
  
                                (Std. Err. adjusted for 106 clusters in id)  
Mixed random regret model                Number of obs      =      3,180  
                                           Wald chi2(2)        =      606.11  
Log likelihood = -771.05731               Prob > chi2         =      0.0000
```

choice		Coef.	OPG Std. Err.	z	P> z	[95% Conf. Interval]	
Mean							
	tc	-1.102133	.0449728	-24.51	0.000	-1.190278	-1.013988
	tt	-.3579483	.0581553	-6.16	0.000	-.4719307	-.243966
SD							
	tt	.5068777	.0413721	12.25	0.000	.4257899	.5879656

The sign of the estimated standard deviations is irrelevant: interpret them as being positive

5 Mixed RRM model: Normal Distribution

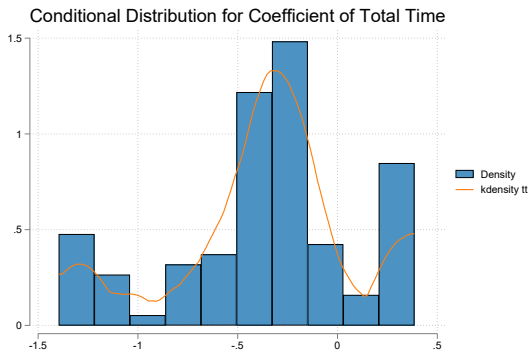
- ▶ `mixrbeta` allows us to compute the individual level parameters of Equation (7).
- ▶ It creates a new data set with one observation per individual (`id`) and its corresponding parameters estimates.

```
. preserve
. /* Computing Individual Level Parameters */
. qui mixrbeta tt , nrep(500) replace saving("${graphs_route}\mixRRM_normal_id1")
. use "${graphs_route}\mixRRM_normal_id1" , replace
. list id tt in 1/5
```

	id	tt
1.	1	.37648493
2.	2	-.05517235
3.	3	.37680846
4.	4	.38503775
5.	5	.37615914

5 Mixed RRM model: Individual Level Parameters

- Individual Level Parameters for total time when we assume it as Normally distributed.



- We see some individuals with positive estimates.
- To prevent this from happening we can use a bounded distribution...

5 Mixed RRM model: Log-normal Distribution

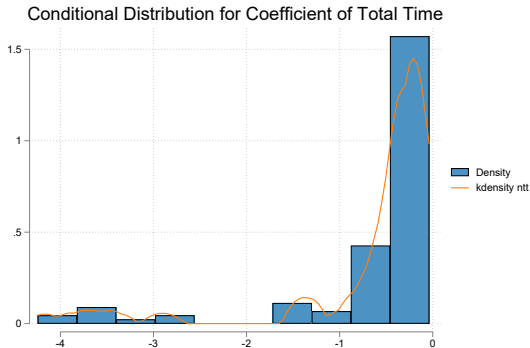
- ▶ Similarly, we can compute the individual level parameters for the log-normally distributed variable tt .

```
. /* Computing Individual Level Parameters */  
. qui mixrbeta ntt , nrep(100) replace saving("${graphs_route}\mixRRM_ln_id1")  
. use "${graphs_route}\mixRRM_ln_id1" , replace  
. replace ntt = -1 * ntt /*reverse sign for graph*/  
(106 real changes made)  
. list id ntt in 1/3
```

	id	ntt
1.	1	-.04005948
2.	2	-.08146095
3.	3	-.04124789

5 Mixed RRM model: Log-normal Distribution

- Individual Level Parameters when total time is assumed to be Log-normally distributed.



- Now we observe that the individual level parameters are all negative.

5 Mixed RRM model: Log-normal Distribution

- ▶ The parameters we estimated are the mean (β_{tt}) and standard deviation (s_{tt}) of the natural logarithm of the total time coefficient.
- ▶ Hence, the mean, median and variance of log-normal distributed parameter are equal to $\exp(\beta_{tt})$, $\exp(\beta_{tt} + s_{tt}/2)$ and $\exp(\beta_{tt} + s_{tt}/2) \times \sqrt{\exp(s_{tt}^2) - 1}$, respectively.
- ▶ Finally, we can compute them using nlcom.

```
. nlcom (mean_time: -1*exp([Mean]_b[ntt ]+0.5*[SD]_b[ntt ]^2)) ///  
> (med_time: -1*exp([Mean]_b[ntt ])) ///  
> (sd_time: exp([Mean]_b[ntt ]+0.5*[SD]_b[ntt ]^2) * sqrt(exp([SD]_b[ntt ]^2)-1))  
    mean_time:  -1*exp([Mean]_b[ntt ]+0.5*[SD]_b[ntt ]^2)  
    med_time:   -1*exp([Mean]_b[ntt ])  
    sd_time:    exp([Mean]_b[ntt ]+0.5*[SD]_b[ntt ]^2) * sqrt(exp([SD]_b[ntt ]^2)-1)
```

choice	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
mean_time	-.6945106	.1603422	-4.33	0.000	-1.008776	-.3802456
med_time	-.2745346	.0425375	-6.45	0.000	-.3579065	-.1911627
sd_time	1.613861	.6351531	2.54	0.011	.3689834	2.858738

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6 Conclusions

- ▶ The `mixrandregret` command extends its predecessor `randregret` ([Gutiérrez-Vargas et al., 2021](#)) by allowing the inclusion of random coefficients in the regret functions.
- ▶ The parameters are estimated by Simulated Maximum Likelihood.
- ▶ The random parameters can follow either a Normal or Log-normal distribution.
- ▶ Additionally, we can compute the individual level parameters using the `mixrbeta` command.
- ▶ The programs can be downloaded from [Ziyue's Github account](#).
- ▶ The example code used in this presentation is available [here](#).

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8 Bibliography

- Chorus, C. G. (2010). A new model of random regret minimization. *European Journal of Transport and Infrastructure Research*, 10(2):181–196.
- Gutiérrez-Vargas, A. A., Meulders, M., and Vandebroek, M. (2021). randregret: A command for fitting random regret minimization models using stata. *The Stata Journal*, 21(3):626–658.
- Train, K. E. (2009). *Discrete Choice Methods with Simulation*. Cambridge University Press, 2 edition.
- van Cranenburgh, S. (2018). Small value-of-time experiment, netherlands. 4TU.Centre for Research Data, Dataset <https://doi.org/10.4121/uuid:1ccca375-68ca-4cb6-8fc0-926712f50404>.