

Fitting mixed random regret minimization models using `mixrandregret`.

UK Stata Meeting - London, 2022.

Presenter: Álvaro A. Gutiérrez-Vargas

- Ziyue Zhu ([GitHub](#), [in](#))'
- Álvaro A. Gutiérrez-Vargas ([GitHub](#), [in](#))'
- Martina Vandebroek

📍 Research Centre for Operations Research and Statistics ([ORSTAT](#))

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions
- ⑦ Bibliography

1 Outline

- ① Random Regret Minimization Models
Random Utility vs Random Regret
Classical Regret Function
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions

1 *What is Regret and how to use it for Choice Modeling?*

- ▶ From Utility to Regret.
- ▶ Regret: Situation where a non-chosen alternative ends up being more attractive than the chosen one for some of the attributes.
- ▶ Individuals are assumed to minimize regret.
- ▶ Consider this hypothetical situation:

altern	total_time	total_cost
First	23	6
Second	27	4
Third	35	3

If we chose alternative 2:

- ▶ Alternative 1 is faster...
- ▶ Alternative 3 is cheaper...

⇒ Regret models will (formalize and) minimize this notion of regret!

1 Formalization of the previous example.

Some (unavoidable) notation for the rest of the presentation:

id	cs	altern	total_time	total_cost	choice
1	1	First	23	6	0
1	1	Second	27	4	0
1	1	Third	35	3	1
1	2	First	27	5	0
1	2	Second	35	4	1
1	2	Third	23	6	0

- ▶ Individuals (**id**) are referred by n ; they answer s choice situations (**cs**).
- ▶ They decide among J alternatives (**altern**) (referred by i or j).
- ▶ Alternatives are described by M attributes referred by m (**total_time** and **total_cost**).
- ▶ $x_{ins,m}$: value of attribute m of alternative i for individual n in choice situation s . (yes, 4 sub-indexes; I am sorry...)
- ▶ y_{ins} : response variable (**choice**). It takes the value of 1 when alternative i is chosen by individual n in choice situation s ; 0 otherwise.

1 RUM vs RRM

► Random Utility Maximization (RUM)

Utility ← $U_{ins} = V_{ins} + \varepsilon_{ins}$ ← Systematic Utility

$$= \beta_{n,T} \times x_{ins,T} + \beta_{n,C} \times x_{ins,C} + \varepsilon_{ins}$$

► Random Regret Minimization (RRM)

Regret ← $RR_{ins} = R_{ins} + \varepsilon_{ins}$ ← Systematic Regret

$$= \sum_{j \neq i}^J R_{i \leftrightarrow jns,T} + \sum_{j \neq i}^J R_{i \leftrightarrow jns,C} + \varepsilon_{ins}$$

- The **notion of regret** is characterized by the systematic regret R_{ins} .
- R_{ins} is described in terms of **attribute level regret** ($R_{i \leftrightarrow jns,m}$).

1 The Attribute level regret $R_{i \leftrightarrow jns,m}$

- Using the same example as before:

altern	total_time	total_cost
First	23	6
Second	27	4
Third	35	3

- $R_{i \leftrightarrow jns,m}$ corresponds to pairwise combination of regret for alternatives i and j for individual n on attribute m in choice situation s .

$(x_{jns,m} - x_{ins,m})$	Attribute \ Route	$j = 1$	$j = 2$	$j = 3$
$(x_{jns,m} - x_{1ns,T})$	Travel Time	0	4	12
$(x_{jns,m} - x_{1ns,C})$	Travel Cost	0	-2	-3
$(x_{jns,m} - x_{2ns,T})$	Travel Time	-4	0	8
$(x_{jns,m} - x_{2ns,C})$	Travel Cost	2	0	-1
$(x_{jns,m} - x_{3ns,T})$	Travel Time	-12	-8	0
$(x_{jns,m} - x_{3ns,C})$	Travel Cost	3	1	0

- **Takeaway:** We will define $R_{i \leftrightarrow jns,m}$ in terms of the attribute differences.

1 Classical RRM (Chorus, 2010)

- ▶ (Chorus, 2010) proposed the following attribute level regret:

$$R_{i \leftrightarrow jns,m} = \ln [1 + \exp \{ \beta_{n,m} \cdot (x_{jns,m} - x_{ins,m}) \}]$$

- ▶ As we saw, $R_{i \leftrightarrow jns,m}$ compares alternative i with alternative j in attribute m .
- ▶ $\sum_{j \neq i} R_{i \leftrightarrow jns,m}$ is the equivalent to $x_{ins,m} \times \beta_{n,m}$ in an utility model.
- ▶ In both cases, $\beta_{n,m}$ is the taste parameter of attribute m of individual n .
- ▶ However, they have drastically different interpretation (more on that later).

1 Classical RRM (Chorus, 2010)

- (Chorus, 2010) proposed the following systematic regret:

$$R_{ins} = \sum_{j \neq i} \sum_{m=1}^M R_{i \leftrightarrow jns,m} = \sum_{j \neq i} \sum_{m=1}^M \ln [1 + \exp \{ \beta_{n,m} \cdot (x_{jns,m} - x_{ins,m}) \}] \quad (1)$$

Attribute level regret.

Sum over attributes.

- In our example: $M = 2$ (Time and Cost) and $J = 3$.
- Regret of alternative 1 (R_{1ns}) will be given by:

$$\begin{aligned} R_{1ns} &= \sum_{j \neq i} \sum_{m=1}^M \ln [1 + \exp \{ \beta_{n,m} (x_{jns,m} - x_{ins,m}) \}] \\ &= \ln [1 + \exp \{ \beta_{n,T} (x_{2ns,T} - x_{1ns,T}) \}] + \ln [1 + \exp \{ \beta_{n,C} (x_{2ns,C} - x_{1ns,C}) \}] \\ &\quad + \ln [1 + \exp \{ \beta_{n,T} (x_{3ns,T} - x_{1ns,T}) \}] + \ln [1 + \exp \{ \beta_{n,C} (x_{3ns,C} - x_{1ns,C}) \}] \end{aligned}$$

1 Classical RRM (Chorus, 2010): Towards the log-likelihood.

- 1 Defining $RR_{ins} = R_{ins} + \varepsilon_{ins}$, where ε_{ins} is a type I Extreme Value i.i.d. error.
- 2 Acknowledging that the minimization of the random regret is mathematically equivalent to maximizing the negative of the regret.
- 3 Hence, the probabilities can be derived using the Multinomial Logit:

$$P_{ins} = \frac{\exp(-R_{ins})}{\sum_{j=1}^J \exp(-R_{jns})} \quad \text{for } i = 1, \dots, J \quad (2)$$

- 4 Consequently, the log-likelihood will be given by:

$$\ln L = \sum_{n=1}^N \sum_{s=1}^S \sum_{i=1}^J y_{in} \times \ln(P_{ins})$$

2 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
Taste Parameter Interpretation in RRM models
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions

2 Taste Parameter Interpretation in RRM models

- ▶ RUM: parameters are interpreted as the change in utility caused by an increase of a particular attribute level.
- ▶ RRM: parameters represent the *potential* change in regret associated with comparing a considered alternative with another alternative in terms of the attribute, caused by one unit change in a particular attribute level.
 - For instance if $\hat{\beta}_{n,m} > 0$ suggests that **regret increases** as the **level of that attribute increases in a non-chosen alternative**, in comparison to the level of the same attribute in the chosen alternative (e.g: Comfortable level).
 - For instance if $\hat{\beta}_{n,m} < 0$ suggests that **regret decreases** as the **level of that attribute increases in a non-chosen alternative**, in comparison to the level of the same attribute in the chosen alternative (e.g: Total Time).
- ▶ All in all, the parameters in RUM and RRM, are expected to have the same sign, even though their interpretation is drastically different.

3 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models**
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions
- ⑦ Bibliography

3 Mixed Random Regret Minimization Models

Additional assumptions of the mode: Individual Level Parameters.

- ▶ $\beta_n = (\beta_{n,1}, \dots, \beta_{n,m})$ follows a parametric distribution: $f(\beta|\varphi)$.
- ▶ φ are the parameters that describe the distribution (e.g., mean and variance of a Normal distribution).
- ▶ We define the conditional probability (CP) of the observed sequence of choices of individual n (conditional on knowing β_n) as:

$$P_n(\beta_n) = \prod_{s=1}^S \prod_{i=1}^J \{P_{ins}\}^{y_{ins}} \quad (3)$$

- ▶ The unconditional probability of the observed sequence of choices is the CP integrated over the entire domain of the distribution of β

$$\ln L(\beta) = \sum_{n=1}^N \ln \left[\int_{\beta} P_n(\beta) f(\beta|\varphi) d\beta \right] \quad (4)$$

3 Mixed Random Regret Minimization Models

- ▶ Given that equation (4) has no closed form we will approximate it using simulations (Train, 2009).
- ▶ Hence, we will estimate the model using Maximum Simulated Likelihood where we will maximize the following simulated log-likelihood function:

$$SLL(\beta) = \sum_{n=1}^N \ln \left\{ \frac{1}{R} \sum_{r=1}^R P_n(\beta^r) \right\} \quad (5)$$

- ▶ R is the number of draws and r is the r -th draw from $f(\beta|\varphi)$.

4 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters**
- ⑤ Implementation
- ⑥ Conclusions
- ⑦ Bibliography

4 Individual Level Parameters

- We can obtain estimates of the individual level parameter (β_n) by:

$$\bar{\beta}_n = \frac{\int_{\beta} \beta \times P_n(\beta) f(\beta|\varphi) d\beta}{\int_{\beta} P_n(\beta) f(\beta|\varphi) d\beta} \quad (6)$$

- We will approximate this individual level parameter by simulation using:

$$\check{\beta}_n = \sum_{r=1}^R \left(\frac{\beta^r \times P_n(\beta^r)}{\sum_{r=1}^R P_n(\beta^r)} \right) \quad (7)$$

- For this estimation we will use the command `mixrbeta` after estimating the population parameters using `mixrandregret` (Zhu, 2022).

5 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
 - Syntax
 - Outputs
- ⑥ Conclusions

5 Syntax

`mixrandregret` (Zhu, 2022) is implemented as a Mata-based `gf-0 ml` evaluator. The command allows the inclusion of normally and log-normally distributed random parameters.

```
mixrandregret depvar [indepvars] [if] [in] group(varname)
alternative(varname) rand(varlist) [, id(varname)
basealternative(string) noconstant ln(string) nrep(string)
burn(string) robust cluster(varname) level(#) maximize_options]
```

The command `mixrbeta` can be used after `mixrandregret` to calculate individual-level parameters corresponding to the variables in the specified *varlist* using equation (7).

```
mixrbeta varlist saving(filename) [, plot nrep(#) burn(#)]
```

5 The Data

- Data from [van Cranenburgh \(2018\)](#): Stated Choice (SC) experiment.

```
. list id cs altern total_time total_cost choice in 1/6, sepby(cs) ab(10) noo
```

id	cs	altern	total_time	total_cost	choice
1	1	First	23	6	0
1	1	Second	27	4	0
1	1	Third	35	3	1
1	2	First	27	5	0
1	2	Second	35	4	1
1	2	Third	23	6	0

- Three unlabeled route alternatives ($J = 3$).
 - Described by `total_time` and `total_cost` ($M = 2$).
- Each respondent (`id`) answered a total of 10 choice situations.
 - Variables `choice` and `altern` allows us to identify each choice.

5 Fixed Parameter RRM model

- First we estimate a fixed parameters RRM model.

```
. randregret choice total_time total_cost , gr(cs) alt(altern) rrmfn(classic) ///  
> nocons cluster(id) nolog
```

Fitting Classic RRM Model

RRM: Classic Random Regret Minimization Model

Case ID variable: cs	Number of cases	=	1060
Alternative variable: altern	Number of obs	=	3180
	Wald chi2(2)	=	40.41
Log likelihood = -1118.4784	Prob > chi2	=	0.0000
	(Std. Err. adjusted for	106 clusters in id)	

choice	Robust		z	P> z	[95% Conf. Interval]	
	Coef.	Std. Err.				
RRM						
total_time	-.102813	.0182526	-5.63	0.000	-.1385874	-.0670386
total_cost	-.417101	.068059	-6.13	0.000	-.5504943	-.2837078

```
. matrix b_rrm = e(b)
```

- As expected, both parameter estimates are negative.

5 Mixed RRM model: Normal Distribution

- ▶ `total_time` assumed to be normally distributed: $\beta_T \sim \mathcal{N}(\mu_T, \sigma_T)$
- ▶ We estimate the two parameters of a normal distribution: μ_T and σ_T

```
. mixrandregret choice total_cost , gr(cs) alt(altern) rand(total_time) id(id) ///
> nocons cluster(id) nrep(500) from(init_mix_rrm) tech(bhhh) nolog

Case ID variable: cs                      Number of cases   =       1060
Alternative variable: altern
Random variable(s): total_time

                               (Std. Err. adjusted for 106 clusters in id)

Mixed random regret model                Number of obs     =       3,180
                                         Wald chi2(2)       =       606.11
Log likelihood = -771.05731              Prob > chi2        =       0.0000
```

choice	OPG					[95% Conf. Interval]
	Coef.	Std. Err.	z	P> z		
Mean						
total_cost	-1.102136	.0449727	-24.51	0.000	-1.190281	-1.013991
total_time	-.3580736	.0581449	-6.16	0.000	-.4720355	-.2441117
SD						
total_time	.5068268	.041366	12.25	0.000	.425751	.5879027

The sign of the estimated standard deviations is irrelevant: interpret them as being positive

```
. matrix b_mixrrm = e(b)
```

- ▶ The mean of `total_time` is negative, as expected.

5 Mixed RRM model: Normal Distribution

- ▶ We can compute the individual level parameters of Equation (7) using `mixrbeta`.
- ▶ `mixrbeta` creates a new data set with one observation per individual (`id`) and its corresponding parameters estimates.

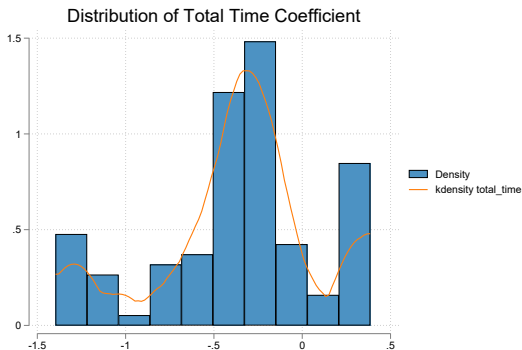
```
. preserve
. /* Computing Individual Level Parameters */
. qui mixrbeta total_time , nrep(500) replace saving("${graphs_route}\mixRRM_normal_id1")
. use "${graphs_route}\mixRRM_normal_id1" , replace
. list id total_time in 1/5
```

	id	total_time
1.	1	.37640482
2.	2	-.05517462
3.	3	.37672848
4.	4	.38495822
5.	5	.37607978

- ▶ We observe that some of the individuals has a positive coefficient for Total Time (`total_time`).

5 Mixed RRM model: Individual Level Parameters

- We can plot the individual level parameters for `total_time` when we assume it as Normally distributed.



- We see some individuals with positive estimates.
- To prevent this from happening we can use a bounded distribution...

5 Mixed RRM model: Log-normal Distribution

- ▶ `total_time` assumed Log-normal: $\beta_T \sim -1 \times \exp(\mathcal{N}(\mu_T, \sigma_T))$
- ▶ Given that `total_time` is expected to be negative, we created (`ntt=-total_time`), since the log-normal distribution implies that the coefficient is positive.

```
. gen ntt = -1 * total_time
. mixrandregret choice total_cost , gr(cs) alt(altern) rand(ntt) ln(1) id(id) ///
> nocons cluster(id) nrep(500) tech(bhhh) from(b_mixrrm) nolog
Case ID variable: cs                      Number of cases      =      1060
Alternative variable: altern
Random variable(s): ntt

                               (Std. Err. adjusted for 106 clusters in id)

Mixed random regret model              Number of obs      =       3,180
                                       Wald chi2(2)         =      1230.55
Log likelihood = -785.27671             Prob > chi2         =       0.0000
```

choice	OPG					
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Mean						
total_cost	-1.217682	.0442047	-27.55	0.000	-1.304321	-1.131042
ntt	-1.312285	.1562202	-8.40	0.000	-1.618471	-1.006099
SD						
ntt	1.363632	.1185994	11.50	0.000	1.131181	1.596082

The sign of the estimated standard deviations is irrelevant: interpret them as being positive

5 Mixed RRM model: Log-normal Distribution

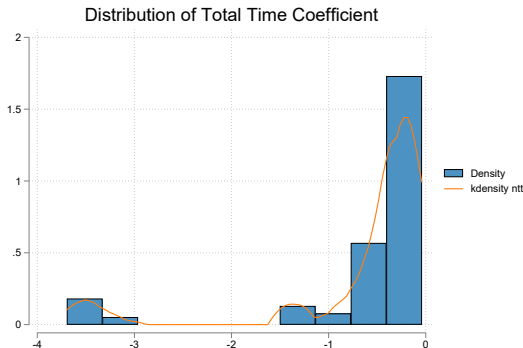
- Similarly, we can compute the individual level parameters for the log-normally distributed variable `tt` using `mixrbeta`.

```
. /* Computing Individual Level Parameters */  
. qui mixrbeta ntt , nrep(500) replace saving("${graphs_route}\mixRRM_ln_id1")  
. use "${graphs_route}\mixRRM_ln_id1" , replace  
. replace ntt = -1 * ntt /*reverse sign for graph*/  
(106 real changes made)  
. list id ntt in 1/5
```

	id	ntt
1.	1	-.04032598
2.	2	-.08142616
3.	3	-.04047817
4.	4	-.04110615
5.	5	-.04025335

5 Mixed RRM model: Log-normal Distribution

- Individual Level Parameters when total time is assumed to be Log-normally distributed.



- Now we observe that the individual level parameters are all negative.

5 Mixed RRM model: Log-normal Distribution

- ▶ The parameters we estimated are the mean (β_T) and standard deviation (σ_T) of the natural logarithm of the total time coefficient.
- ▶ Hence, the mean, median and variance of log-normal distributed parameter are equal to $\exp(\beta_T)$, $\exp(\beta_T + \sigma_T/2)$ and $\exp(\beta_T + \sigma_T/2) \times \sqrt{\exp(\sigma_T^2) - 1}$, respectively.
- ▶ Finally, we can compute them using nlcom.

```
. nlcom ///  
> (mean_time: -1*exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)) ///  
> (med_time : -1*exp([Mean]_b[ntt])) ///  
> (sd_time : exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)*sqrt(exp([SD]_b[ntt]^2)-1))  
mean_time: -1*exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)  
med_time: -1*exp([Mean]_b[ntt])  
sd_time: exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)*sqrt(exp([SD]_b[ntt]^2)-1)
```

choice	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
mean_time	-.682127	.1587961	-4.30	0.000	-.9933616	-.3708923
med_time	-.2692041	.0420551	-6.40	0.000	-.3516307	-.1867776
sd_time	1.588122	.6295756	2.52	0.012	.3541763	2.822067

6 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions
- ⑦ Bibliography

6 Conclusions

- ▶ The `mixrandregret` ([Zhu, 2022](#)) command extends its predecessor `randregret` ([Gutiérrez-Vargas et al., 2021](#)) by allowing the inclusion of random coefficients in the regret functions.
- ▶ The parameters are estimated by Maximum Simulated Likelihood.
- ▶ The random parameters can follow either a Normal or Log-normal distribution.
- ▶ Additionally, we can compute the individual level parameters using the `mixrbeta` command.
- ▶ The programs can be downloaded from [Ziyue's Github account](#).
- ▶ The example code used in this presentation is available [here](#).

7 Outline

- ① Random Regret Minimization Models
- ② Differences between RUM and RRM models.
- ③ Mixed Random Regret Minimization Models
- ④ Individual Level Parameters
- ⑤ Implementation
- ⑥ Conclusions
- ⑦ Bibliography**

8 Bibliography

- Chorus, C. G. (2010). A new model of random regret minimization. *European Journal of Transport and Infrastructure Research*, 10(2):181–196.
- Gutiérrez-Vargas, A. A., Meulders, M., and Vandebroek, M. (2021). randregret: A command for fitting random regret minimization models using stata. *The Stata Journal*, 21(3):626–658.
- Train, K. E. (2009). *Discrete Choice Methods with Simulation*. Cambridge University Press, 2 edition.
- van Cranenburgh, S. (2018). Small value-of-time experiment, netherlands. 4TU.Centre for Research Data, Dataset <https://doi.org/10.4121/uuid:1ccca375-68ca-4cb6-8fc0-926712f50404>.
- Zhu, Z. (2022). *mixrandregret: A command for fitting mixed random regret minimization models using Stata*. Master's thesis, KU Leuven.

GitHub with Slides + Example code here:



Thanks 🙌