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Fitting mixed random regret minimization models using `mixrandregret`.

UK Stata Meeting - London, 2022.
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- Martina Vandebroek

Research Centre for Operations Research and Statistics (ORSTAT)

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
5. Implementation
6. Conclusions
7. Bibliography

1 Outline

1. Random Regret Minimization Models
   - Random Utility vs Random Regret
   - Classical Regret Function
2. Differences between RUM and RRM models.
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
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$\Rightarrow$ Regret models will (formalize and) minimize this notion of regret!

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- $x_{ins,m}$: value of attribute m of alternative i for individual n in choice situation s. (yes, 4 sub-indexes; I am sorry...)
- y_{ins}: response variable (**choice**). It takes the value of 1 when alternative i is chosen by individual n in choice situation s; 0 otherwise.

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  - The notion of *regret* is characterized by the systematic regret R_{ins}.
  - R_{ins} is described in terms of ***attribute level regret*** ($R_{i \leftrightarrow jns,m}$).

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$(x_{jns,m} - x_{ins,m})$	Attribute \ Route	$j = 1$	$j = 2$	$j = 3$
$(x_{jns,m} - x_{1ns,T})$	Travel Time	0	4	12
$(x_{jns,m} - x_{1ns,C})$	Travel Cost	0	-2	-3
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$(x_{jns,m} - x_{2ns,C})$	Travel Cost	2	0	-1
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- Takeaway: We will define $R_{i\leftrightarrow jns,m}$ in terms of the attribute differences.

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- (Chorus, 2010) proposed the following attribute level regret:

$$R_{i\leftrightarrow jns,m} = \ln\left[1 + \exp\left\{\beta_{n,m} \cdot (x_{jns,m} - x_{ins,m})\right\}\right]$$

- As we saw, $R_{i\leftrightarrow jns,m}$ compares alternative i with alternative j in attribute m.

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- However, they have drastically different interpretation(more on that later).

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$$\begin{aligned} R_{1ns} &= \sum_{j \neq i}^{3} \sum_{m=1}^{M} \ln \left[1 + \exp \left\{ \beta_{n,m} (x_{jns,m} - x_{ins,m}) \right\} \right] \\ &= \ln \left[1 + \exp \left\{ \beta_{n,T} \left(x_{2ns,T} - x_{1ns,T} \right) \right\} \right] + \ln \left[1 + \exp \left\{ \beta_{n,c} \left(x_{2ns,C} - x_{1ns,C} \right) \right\} \right] \\ &+ \ln \left[1 + \exp \left\{ \beta_{n,T} \left(x_{3ns,T} - x_{1ns,T} \right) \right\} \right] + \ln \left[1 + \exp \left\{ \beta_{n,C} \left(x_{3ns,C} - x_{1ns,C} \right) \right\} \right] \end{aligned}$$

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4. Consequently, the log-likelihood will be given by:

$$\ln L = \sum_{n=1}^{N} \sum_{s=1}^{S} \sum_{i=1}^{J} y_{in} \times \ln\left(P_{ins}\right)$$

2 Outline

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
   Taste Parameter Interpretation in RRM models
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
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    suggests that regret decreases as the level of that attribute increases **in a non-chosen alternative**, in comparison to the level of the same attribute in the chosen alternative (e.g: Total Time).
- All in all, the parameters in RUM and RRM, are expected to have the same sign, even though their interpretation is drastically different.

3 Outline

1. Random Regret Minimization Models
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$$\ln L(\boldsymbol{\beta}) = \sum_{n=1}^{N} \ln \left[\int_{\boldsymbol{\beta}} P_n(\boldsymbol{\beta}) f\left(\boldsymbol{\beta}|\varphi\right) d\boldsymbol{\beta} \right] \tag{4}$$

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- R is the number of draws and r is the r-th draw from $f(\boldsymbol{\beta}|\varphi)$.

4 Outline

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- For this estimation we will use the command `mixrbeta` after estimating the population parameters using `mixrandregret` (Zhu, 2022).

5 Outline

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
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   - Syntax
   - Outputs
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5 Syntax

`mixrandregret` (Zhu, 2022) is implemented as a Mata-based `gf-0 ml` evaluator. The command allows the inclusion of normally and log-normally distributed random parameters.

`mixrandregret` *depvar* [*indepvars*] [*if*] [*in*] *group(varname)*
alternative(varname) *rand(varlist)* [, `id`(*varname*)
`basealternative`(*string*) `noconstant` `ln`(*string*) `nrep`(*string*)
`burn`(*string*) `robust` `cluster`(*varname*) `level`(#) *maximize_options*]

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The command `mixrbeta` can be used after `mixrandregret` to calculate individual-level parameters corresponding to the variables in the specified *varlist* using equation (7).

`mixrbeta` *varlist* `saving`(filename) [, `plot` `nrep`(#) `burn`(#)]

5 The Data

- Data from van Cranenburgh (2018): Stated Choice (SC) experiment.

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```
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```

id	cs	altern	total_time	total_cost	choice
1	1	First	23	6	0
1	1	Second	27	4	0
1	1	Third	35	3	1
1	2	First	27	5	0
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  - Three unlabeled route alternatives ($J = 3$).

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- Variables **choice** and **altern** allows us to identify each choice.

5 Fixed Parameter RRM model

- First we estimate a fixed parameters RRM model.

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```
. randregret choice total_time total_cost , gr(cs) alt(altern) rrmfn(classic) ///
> nocons cluster(id)  nolog
------------------------------------------------------------------------------
Fitting Classic RRM Model
------------------------------------------------------------------------------

RRM: Classic Random Regret Minimization Model
Case ID variable: cs                            Number of cases     =      1060
Alternative variable: altern                    Number of obs       =      3180
                                                Wald chi2(2)        =     40.41
Log likelihood = -1118.4784                     Prob > chi2         =    0.0000
                                 (Std. Err. adjusted for  106 clusters in id)
------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
RRM          |
  total_time |   -.102813   .0182526    -5.63   0.000    -.1385874   -.0670386
  total_cost |   -.417101    .068059    -6.13   0.000    -.5504943   -.2837078
------------------------------------------------------------------------------

. matrix b_rrm = e(b)
```

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- As expected, both parameter estimates are negative.

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```
. mixrandregret choice  total_cost , gr(cs) alt(altern) rand(total_time) id(id) ///
> nocons cluster(id)  nrep(500) from(init_mix_rrm) tech(bhhh)  nolog

Case ID variable: cs                                Number of cases    =       1060
Alternative variable: altern
Random variable(s): total_time
                                (Std. Err. adjusted for  106 clusters in id)

Mixed random regret model                            Number of obs      =      3,180
                                                     Wald chi2(2)       =     606.11
Log likelihood = -771.05731                          Prob > chi2        =     0.0000

------------------------------------------------------------------------------
             |                OPG
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Mean         |
  total_cost |  -1.102136   .0449727   -24.51   0.000    -1.190281   -1.013991
  total_time |  -.3580736   .0581449    -6.16   0.000    -.4720355   -.2441117
-------------+----------------------------------------------------------------
SD           |
  total_time |   .5068268    .041366    12.25   0.000      .425751    .5879027
------------------------------------------------------------------------------

The sign of the estimated standard deviations is irrelevant: interpret them as
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- The mean of `total_time` is negative, as expected.

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- `mixrbeta` creates a new data set with one observation per individual (`id`) and its corresponding parameters estimates.

```
. preserve

. /* Computing Individual Level Parameters */
. qui mixrbeta total_time , nrep(500)  replace saving("${graphs_route}\mixRRM_normal_idl")

. use "${graphs_route}\mixRRM_normal_idl" , replace

. list id  total_time  in 1/5
```

	id	total_time
1.	1	.37640482
2.	2	-.05517462
3.	3	.37672848
4.	4	.38495822
5.	5	.37607978

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  4. |  4     .38495822 |
  5. |  5     .37607978 |
     +-----------------+
```

- We observe that some of the individuals has a positive coefficient for Total Time (**`total_time`**).

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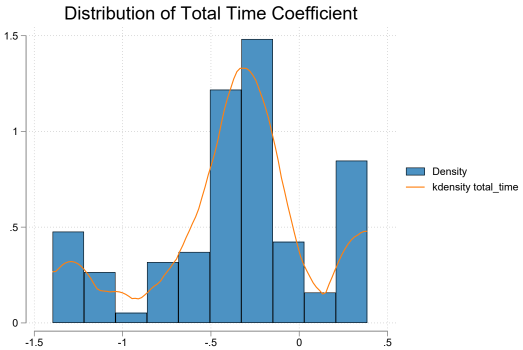

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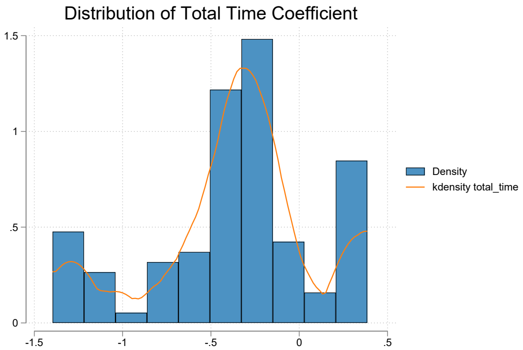

- We see some individuals with positive estimates.

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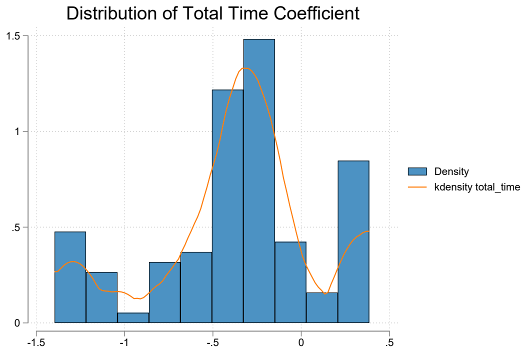

- We see some individuals with positive estimates.
- To prevent this from happening we can use a bounded distribution...

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- `total_time` assumed Log-normal: $\beta_T \sim -1 \times \exp\left(\mathcal{N}\left(\mu_T, \sigma_T\right)\right)$
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- Given that `total_time` is expected to be negative, we created (`ntt=-total_time`), since the log-normal distribution implies that the coefficient is positive.

```
. gen ntt = -1 * total_time

. mixrandregret choice  total_cost , gr(cs) alt(altern) rand(ntt ) ln(1) id(id) ///
> nocons cluster(id)  nrep(500) tech(bhhh) from(b_mixrrm) nolog

Case ID variable: cs                               Number of cases    =       1060
Alternative variable: altern
Random variable(s): ntt

                                 (Std. Err. adjusted for  106 clusters in id)

Mixed random regret model                           Number of obs     =      3,180
                                                    Wald chi2(2)      =    1230.55
Log likelihood = -785.27671                         Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                OPG
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Mean         |
  total_cost |  -1.217682   .0442047   -27.55   0.000    -1.304321   -1.131042
         ntt |  -1.312285   .1562202    -8.40   0.000    -1.618471   -1.006099
-------------+----------------------------------------------------------------
SD           |
         ntt |   1.363632   .1185994    11.50   0.000     1.131181    1.596082
------------------------------------------------------------------------------

The sign of the estimated standard deviations is irrelevant: interpret them as
being positive
```

5 Mixed RRM model: Log-normal Distribution

- Similarly, we can compute the individual level parameters for the log-normally distributed variable `tt` using mixrbeta.

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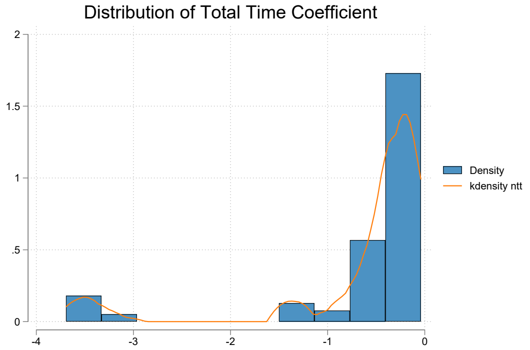
```
. /* Computing Individual Level Parameters */
. qui mixrbeta ntt , nrep(500)  replace saving("${graphs_route}\mixRRM_ln_idl")
. use "${graphs_route}\mixRRM_ln_idl" , replace
. replace ntt = -1 * ntt /*reverse sign for graph*/
(106 real changes made)
. list id  ntt  in 1/5
```

	id	ntt
1.	1	-.04032598
2.	2	-.08142616
3.	3	-.04047817
4.	4	-.04110615
5.	5	-.04025335

5 Mixed RRM model: Log-normal Distribution

- Individual Level Parameters when total time is assumed to be Log-normally distributed.



- Now we observe that the individual level parameters are all negative.

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- Hence, the mean, median and variance of log-normal distributed parameter are equal to $\exp(\beta_T)$, $\exp(\beta_T + \sigma_T/2)$ and $\exp(\beta_T + \sigma_T/2) \times \sqrt{\exp(\sigma_T^2) - 1}$, respectively.

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- Finally, we can compute them using nlcom.

```
. nlcom ///
> (mean_time: -1*exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)) ///
> (med_time : -1*exp([Mean]_b[ntt])) ///
> (sd_time  :    exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)*sqrt(exp([SD]_b[ntt]^2)-1))

   mean_time:  -1*exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)
    med_time:  -1*exp([Mean]_b[ntt])
     sd_time:  exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)*sqrt(exp([SD]_b[ntt]^2)-1)
```

choice	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
mean_time	-.682127	.1587961	-4.30	0.000	-.9933616	-.3708923
med_time	-.2692041	.0420551	-6.40	0.000	-.3516307	-.1867776
sd_time	1.588122	.6295756	2.52	0.012	.3541763	2.822067

6 Outline

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
5. Implementation
6. Conclusions
7. Bibliography

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8 Bibliography

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GitHub with Slides + Example code here:

Thanks ✌