Problem 3

Ⓐ (As seen in Lecture 4 - Slide 15).

Moran Process with:

- $N = 1000$
- Mutants arise with probability μ and have a relative fitness r. ($r < 1$ is deleterious).
- Mutants decrease in fitness by 1%.

$\Rightarrow r = 0.99$.

The fixation probability (Slide 15):

$$\rho = x_1 = \frac{(1 - 1/r)}{(1 - 1/r^N)} = \frac{1 - 1/0.99}{1 - 1/0.99^{1000}} = \boxed{4.36 \times 10^{-7}}$$

Extra:

The probability that a mutant has been fixed by time t is:

$$\boxed{P(t) = 1 - e^{-N\mu\rho t}}$$

1000

"Very low" according to text.

4.36 e^{-7}

$P(t)$ is decreasing, because $r < 1$ (see slide 15)

(B) • If we have that the probability for mutation at one site is 10^{-5}.

• Probability of acquiring AT LEAST one mutation:

$$P(X \geq 1) = 1 - P(X < 1).$$

$P(X<1) \Rightarrow$ Prob of acquiring NO mutation.

At each site:

$$P(X<1) = 1 - 10^{-5}.$$

Therefore for all possible sites.

$$P(X<1) = (1-10^{-5})^{10^6} \simeq 0.$$

Thus for the whole genome

$$\boxed{P(X \geq 1) = 1 - P(X<1) \simeq 1}$$

It is almost certain that at least one mutation will occur in the genome.

(c) For this population, given that $P(X \geq 1) \simeq 1$, all individuals will end up with the deleterious mutation.
Depending on the fitness reduction, this could mean that for the long term the whole population can become extinct.

(d) In natural populations we don't have the finite effects of the Moran process. That is, we have populations that are dynamic (growing or shrinking) and not fixed as in the Moran.
Also, we dont have a one-dies, one-reproduces scenario, but instead a more complex reality.
This means that such deleterious mutations can quickly disappear, leaving the fitter population as dominant.
Also depending on the type of populations (for example viruses), even if fixation probability is 1, the time it takes to happen, $P(t)$ can be so large that in reality the genome undergoes many more mutations and fixation never happens.