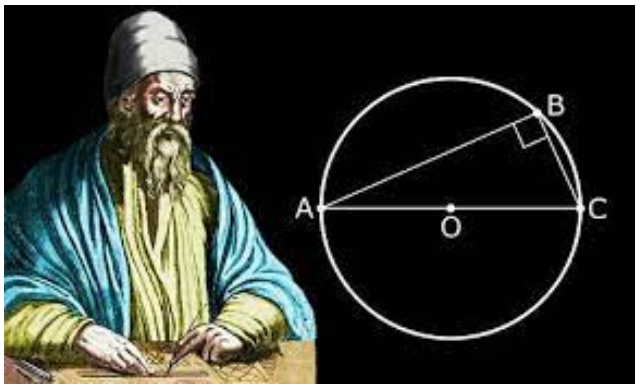
MATH CIRCLE TTU

Geometry

Geodesics



Euclid's First Postulate

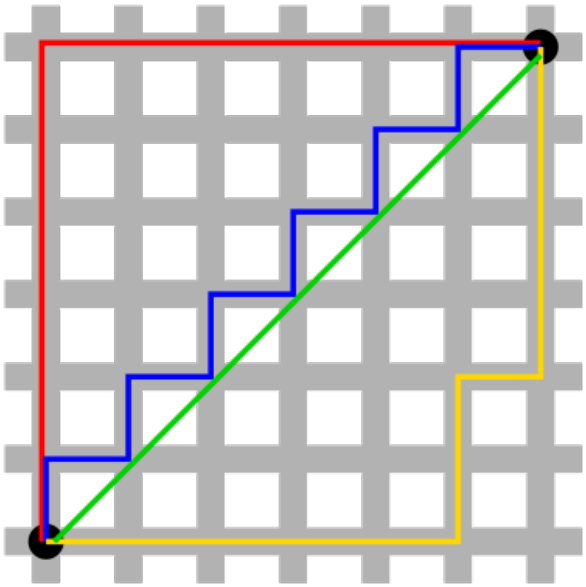
Every two points can be joined by a unique straight line segment.

Question. Draw two points and find the line that joins them. Is this the shortest path between the two points?

Geodesics

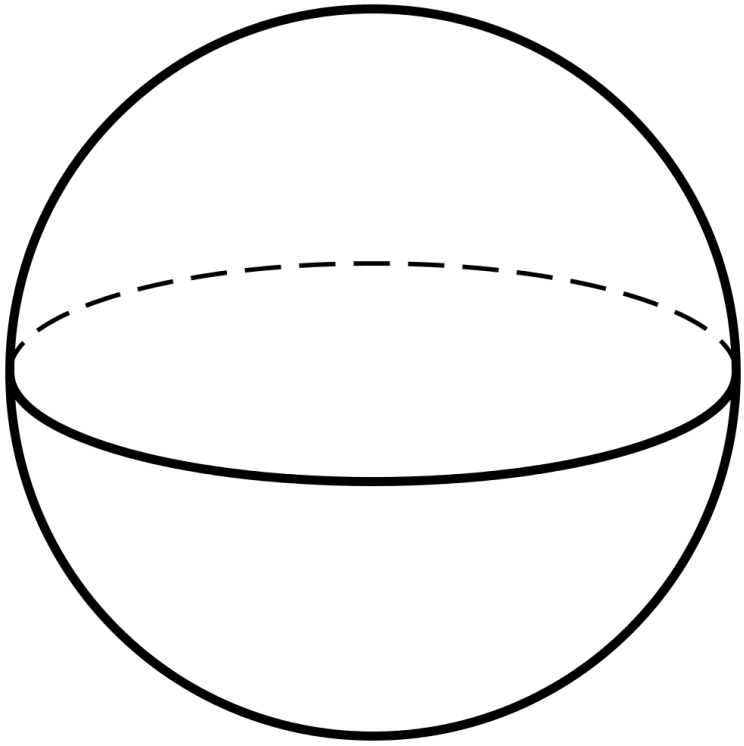
The shortest path between two points is called a **geodesic**.

Question. In the next figure, you can see two points and several paths highlighted. The empty squares are buildings (it is not Halloween anymore so we are not ghosts and we cannot cross the buildings). If you are a taxi driver and need to drive between the two points, what path would you pick up? Why? (There are no traffic lights and your car's speed is always 35mph.)



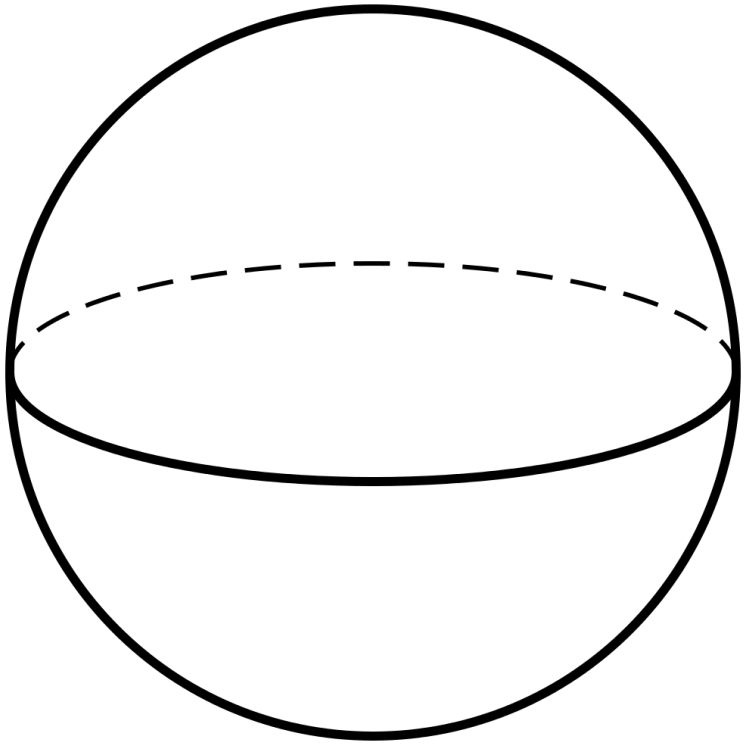
Geodesics in the Sphere

Question. Given two points in a sphere, how many geodesics (shortest paths) can you draw joining them?



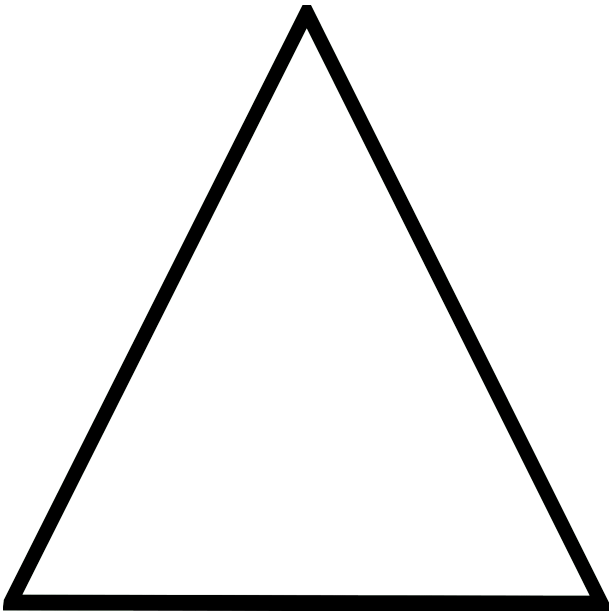
Non-Euclidean Geometries

Question. What happens if those points are the north and south poles (or any other antipodal points)? How many shortest paths can there be?

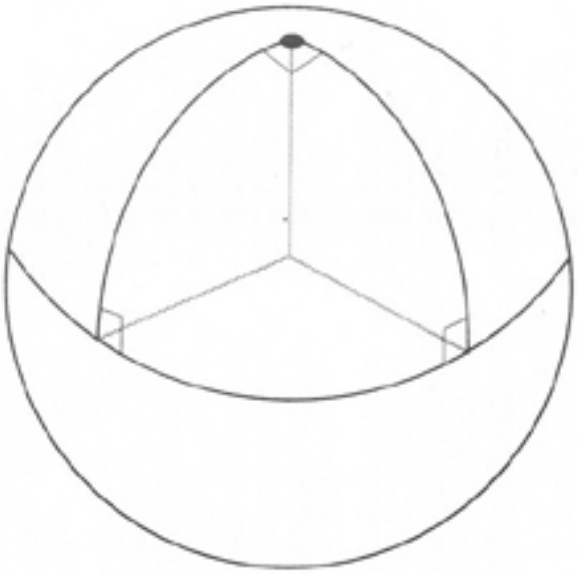


Another Difference Between Geometries

Question. What is the sum of the interior angles of a triangle?



Question. And, of a triangle in a sphere?



Hyperbolic Geometry

Question. Think what happens on a surface like the following one: