1. Notation

In the rest of the document, we shall analyze a power distribution network represented as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of power lines connecting these nodes. An edge (k, t) indicates a power line that connects node k to node t. We define $\mathcal{N}_k$ as the set of nodes directly connected to node k, and $\mathcal{A}_k$ as the set of edges that either originate from or terminate at node k.

2. Second Order Conic Programming Power Flow Formulation

This work considers the power flow formulation for distribution networks introduced in [**?**]. In that research, the following variables are adopted: $c_{k,k} = e_k^2 + f_k^2$, $c_{k,t} = e_k e_t + f_k f_t$ and $s_{k,t} = e_k f_t - f_k e_t$ where e_k and f_k represents the real and imaginary components of voltage at node k, v_k, respectively. Note that $c_{k,t} = c_{t,k}$ and $s_{k,t} = -s_{t,k}$ for every pair of connected nodes.

Using the notation described above and considering the single-line equivalent circuit depicted in Figure 1, the power flowing from node k to node t can be expressed in a linear form as follows:

$$\begin{aligned} P_{k,t} &= G_{k,t} c_{k,k} - G_{k,t} c_{k,t} - B_{k,t} s_{k,t}, \\ Q_{k,t} &= B_{k,t} c_{k,k} - B_{k,t} c_{k,t} + G_{k,t} s_{k,t}, \end{aligned}$$

where $c_{k,t}^2 + s_{k,t}^2 = c_{k,k} c_{t,t}$ constraints the new problem variables.

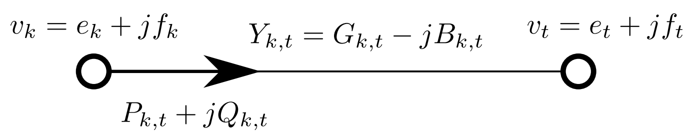

Figure 1: Distribution line model.

Thus, the set of equations that describe the power flow problem are given by:

$$\begin{aligned} P_{g,k} - P_{l,k} &= \sum_{\forall t \in \mathcal{N}_k} P_{k,t}, \quad \forall k \in \mathcal{V}, \\ Q_{g,k} - Q_{l,k} &= \sum_{\forall t \in \mathcal{N}_k} Q_{k,t}, \quad \forall k \in \mathcal{V}, \\ c_{k,t}^2 + s_{k,t}^2 &\leq c_{k,k} c_{t,t}, \quad \forall (k,t) \in \mathcal{E}, \end{aligned} \tag{1}$$

$\max c_{k,t}$ where $P_{g,k}$ and $Q_{g,k}$ are the active and reactive power injections at node k; and $P_{l,k}$ and $Q_{l,k}$ are, respectively, the active and reactive power demanded at that node. As evident from the analysis, there are two linear equations corresponding to each node, excluding the slack node, and one nonlinear equation for each line in the network that corresponds to a second order rotated conic equality constraint.

In sight of problem formulation (1), the set of searched variables can be stacked in column vector $x = [\mathrm{col}(c_{k,k})_{k \in \mathcal{V}}^\top, \mathrm{col}(c_{k,t})_{(k,t) \in \mathcal{E}}^\top, \mathrm{col}(s_{k,t})_{(k,t) \in \mathcal{E}}^\top]^\top \in \mathbb{R}^n$ in such a way that the problem can be rewritten in a compact form as:

$$\begin{aligned} Ax &= b, \\ x^\top Q^{(r,t)} x &= 0, \quad \forall (r,t) \in \mathcal{E}, \end{aligned} \tag{2}$$

where linear power flow equations are embedded in $Ax = b$; and $Q^{(r,t)}$ are a set of matrices that constitutes the rotated quadratic cone constraints [**?**].