

Akshatha. Y.E

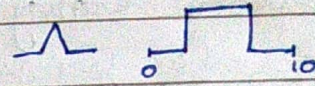
4AL18EC005

DAY 2: Fourier Series and Gibbs Phenomena:

$$f(x) \approx \sum_{k=0}^{\infty} a_k \cos\left(k \frac{2\pi x}{L}\right) + b_k \sin\left(k \frac{2\pi x}{L}\right)$$

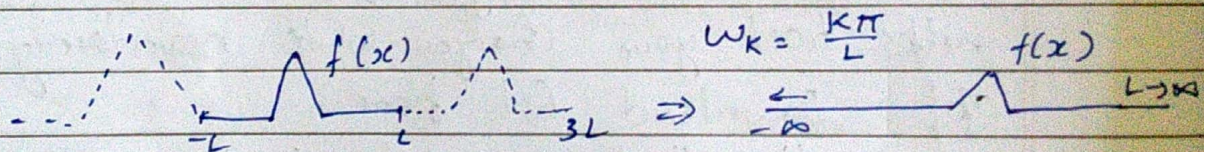
$$a_k = \langle f(x), \cos\left(k \frac{2\pi x}{L}\right) \rangle$$

$$b_k = \langle f(x), \sin\left(k \frac{2\pi x}{L}\right) \rangle$$

• Fourier Transform:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ik\pi x/L}$$

$$c_k = \frac{1}{2\pi} \langle f(x), \psi_k \rangle = \frac{1}{2\pi} \int_{-L}^L f(x) \underbrace{e^{-ik\pi x/L}}_{\psi_k} dx$$



$$W_L = \frac{K\pi}{L} = K \Delta W \Rightarrow \Delta W = \pi/L$$

$$f(x) = \lim_{\substack{\Delta W \rightarrow 0 \\ (L \rightarrow \infty)}} \sum_{k=-\infty}^{\infty} \frac{\Delta W}{2\pi} \int_{-\pi/\Delta W}^{\pi/\Delta W} f(\xi) e^{-iK\Delta W \xi} d\xi e^{iK\Delta W x}$$

[ξ - dummy variable]

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi} \underbrace{\int_{-\infty}^{\infty} f(\xi) e^{-i\omega \xi} d\xi}_{f(\omega)} e^{i\omega x} d\omega$$

$$\boxed{\hat{f}(\omega) = F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx}$$

$$\boxed{f(x) = F^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x} d\omega}$$

Fourier transform.

Fourier transform & Derivative:

SURYA Gold

Date

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$$\begin{aligned}
 F\left(\frac{d}{dx}f(x)\right) &= \int_{-\infty}^{\infty} \underbrace{\frac{df}{dx}}_{dv} \underbrace{e^{-i\omega x}}_u dx \\
 &= \underbrace{\left[f(x)e^{-i\omega x}\right]}_{uv=0} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \underbrace{f(x)}_v \underbrace{(-i\omega e^{-i\omega x})}_{du} dx \\
 &= i\omega \underbrace{\int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx}_{F(f(x))} = i\omega \underbrace{F(f(x))}_{F\left(\frac{d}{dx}f(x)\right)}
 \end{aligned}$$

$$\begin{aligned}
 U_{tt} &= CU_{xx} \quad (\text{PDE}) \xrightarrow{F} \boxed{\hat{U}_{tt} = -\omega^2 \hat{U} \quad (\text{ODE})} \\
 U(x, t) &\xrightarrow{F} \hat{U}(\omega, t)
 \end{aligned}$$

Fourier transform and Convolution Integrals:

$$(f * g) = \int_{-\infty}^{\infty} f(x - \xi) g(\xi) d\xi \rightarrow (1)$$

$$F(f * g) = F(f)F(g) = \hat{f}\hat{g}$$

$$\begin{aligned}
 F^{-1}(\hat{f}\hat{g})(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \hat{g}(\omega) e^{i\omega x} d\omega \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \left(\int_{-\infty}^{\infty} g(y) e^{-i\omega y} dy \right) e^{i\omega x} d\omega \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(\omega) g(y) e^{i\omega(x-y)} d\omega dy \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} g(y) \underbrace{\int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega(x-y)} d\omega}_{f(x-y)} dy
 \end{aligned}$$

$$(f * g) = \int_{-\infty}^{\infty} g(y) f(x-y) dy \rightarrow (2)$$

$$F^{-1}(\hat{f}\hat{g})(x) = f * g$$

• Application of Z-transform:

Difference eqⁿ: $\Delta y_{n+1} + y_n = 2$
 $\Delta y_{n+1} + \Delta^2 y_n = 1$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta y_{n+1} = y_{n+2} - y_{n+1} \text{ \& hence}$$

$$\Delta y_{n+1} + y_n = 2 \Rightarrow y_{n+2} - y_{n+1} + y_n = 1$$

$$\Delta y_{n+1} + \Delta^2 y_n = 1$$

$$y_{n+2} - y_{n+1} + \Delta(\Delta y_n) = 1 \Rightarrow y_{n+2} - y_{n+1} + 2y_{n+1} - 2y_n = 1$$

Now, $\Delta y_{n-1} = y_n - y_{n-1}$

$$y_{n+2} - 2y_n + y_{n+1} = 1$$

$$\Delta(\Delta y_{n-1}) = \Delta(y_n - y_{n-1})$$

$$= \Delta y_n - \Delta y_{n-1}$$

$$= y_{n+1} - y_n - (y_n - y_{n-1})$$

$$= y_{n+1} - 2y_n + y_{n-1}$$

$$\textcircled{2} \quad \frac{(n+2)-n}{1} = 2 \quad \frac{(n+2)-(n-1)}{1} = 3$$

Properties $y_n = u_n + v_n$

u_n = complementary function.

v_n = Particular function.

• Python:

Graphical User Interfaces with Tkinter

→ Bookstore - it allows to store book information in database.

How to add entry in bookstore.

① Import Tkinter or

1. from tkinter import *

2.

3. window = tk()

4.

5. window.mainloop()

In o/t tk is not defined so Tk.

$b_1 = \text{Button}(\rightarrow \text{window}, \text{text} = \text{"Execute"})$

$b_1.grid()$ we have control on button

where we have to put column or row number

- Introduction to Tkinter.
- Setting up a GUI with widgets.
- Connecting GUI widgets with Callback Functions.
- Interacting with Databases:
 - Introduction to "Python with databases".
 - Python is able to interact with many databases
MySQL, PostgreSQL, Oracle, SQLITE
 - Connecting and Inserting Data to SQLITE via Python
① import sqlite3.
 - ① Connect to a Database.
 - ② Create a cursor object.
 - ③ Write an SQL query
 - ④ Commit changes.
 - Selecting, Inserting, deleting & uploading
SQLITE Records.
 - Introduction to PostgreSQL psycopg2.
 - Selecting, Inserting, deleting & uploading
PostgreSQL Records.
import psycopg2