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4AL18EC005

- Fourier Analysis:
  Fourier Transform (& wavelets)

Co-ordinate transform:

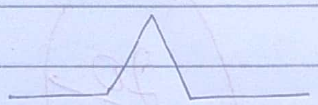
$\boxed{u(x,y,t)}$ → This is governed by heat eqn.

solve, P.D.E in mat lab & python

$u_t = \alpha \nabla^2 u$, It has Eigen values and functions.

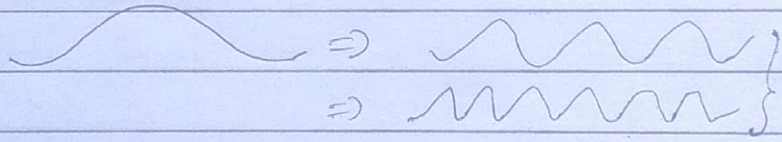
SVD = Data driven FFT.

- Orbitrary function:



we can approximate by sum of science & co-science of increasing frequency.

function space



increasing frequency.

→ Vector space x, y co-ordinates for 2-D.
this would help us in understanding Hilbert.

- Hilbert: function.

- Fast Fourier transform: (FFT).
  We compute officially in computer modern digital communication are built on FFT.

$$f(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos 2\pi kt + b_k \sin 2\pi kt)$$

Periodic function of time decomposes is sum of constants & to cosine & sine and to there frequency in terms of time.

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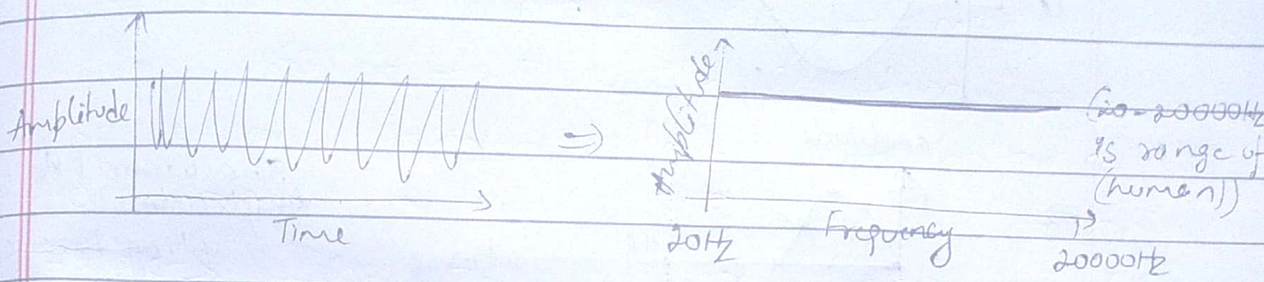
- Noise signal: $x \to$ amplitude, $y \to$ time



How to calculate co-efficients.

$$X(F) = \int_{-\infty}^{\infty} \underbrace{x(t)}_{\text{function}} \underbrace{e^{-j2\pi Ft}\,dt}_{\substack{\text{analyzing function} \\ \text{sinusoids}}}$$ For continuous.

Result: one complex co-efficient per frequency

$$X_a(F) = \int_{-\infty}^{\infty} x(t)\cos 2\pi Ft\,dt \quad , \quad X_b(F) = \int_{-\infty}^{\infty} x(t)\sin 2\pi Ft\,dt$$

Result: Two real co-efficients per frequency.

For discrete $X_k = \sum_{n=0}^{N-1} x_n \cdot e^{\frac{-j2\pi kn}{N}}$

k^{th} frequency $\quad k/N \mathrel{\hat{=}} F \quad n \mathrel{\hat{=}} t$

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-j\frac{2\pi kn}{N}} \to b_n$$

"k^{th}" frequency bin

$$X_k = x_0 e^{-b_0 j} + x_1 e^{-b_1 j} + x_2 e^{-b_2 j} + \ldots + x_n e^{-b_{N-1} j}$$

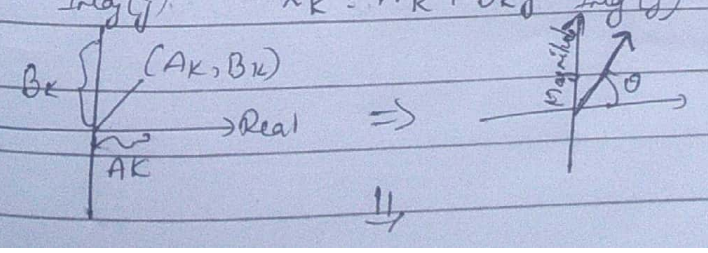
"n^{th}" sample value

Euler's formula:

$$e^{jx} = \cos x + j\sin x$$

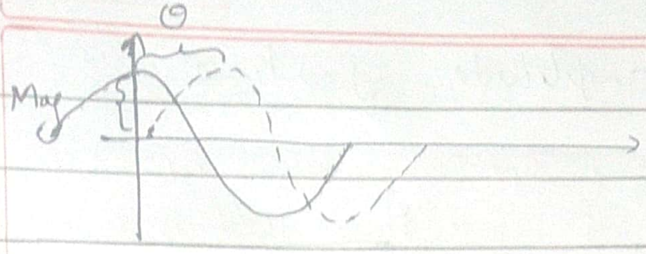
$$X_k = x_0[\cos(-b_0) + j\sin(-b_0)] + \ldots$$

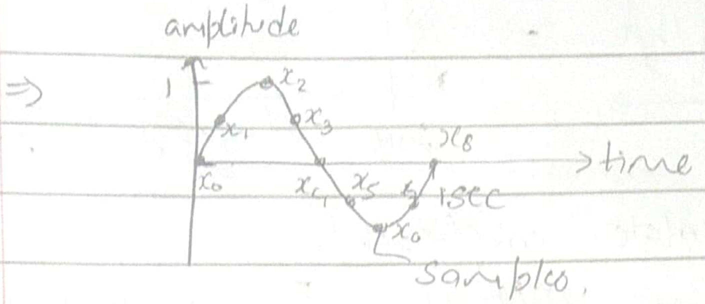
$$X_k = A_k + B_k j$$



$Mg = \sqrt{A_k^2 + B_k^2}$

$\theta = \tan^{-1}\frac{B_k}{A_k}$





Sine wave 1 Hz,
Amplitude = 1
Sampling Frequency = 8 Hz
samples (N) : 8

$x_0 = 0$
$x_1 = 0.707$
$x_2 = 1$
$x_3 = 0.707$
$x_4 = 0$
$x_5 = -0.707$
$x_6 = 1$
$x_7 = -0.707$

$$X_K = \sum_{n=0}^{N-1} x_n \cdot e^{-j\frac{2\pi k n}{N}}$$

$$X_1 = 0 \cdot e^{-j\frac{2\pi(1)(0)}{8}} + 0.707 \cdot e^{-j\frac{(2\pi)(1)(1)}{8}} + 1 \cdot e^{-j\frac{2\pi(1)(2)}{8}} + \dots$$

$$X_1 = 0 + 0.707\left[\cos(-\pi/4) + j\sin(-\pi/4)\right] + 1\left[\cos(-\pi/2) + j\sin(-\pi/2)\right] + \dots$$

$$X_1 = 0 + (0.5 - 0.5j) + (-j) + (-0.5 - 0.5j) + (0.5 - 0.5j) + (-j) + (-0.5 - 0.5j)$$

$$X_1 = -4j$$

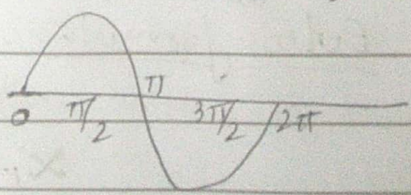
Same can be done for $X_2, X_3 \dots$

$$\text{Mag} = \sqrt{A_k^2 + B_k^2} = \sqrt{(-4)^2} = 4$$

$X_0 = 0$
$X_1 = 0 - 4j$
$X_2 = 0 - 0j$, $X_3 = 0$, $X_4 = 0$, $X_5 = 0$, $X_6 = 0$, $X_7 = 0 + 4j$

Mag of all the frequency is equal to the sampling frequency divided by samples.
Nyquist limit = sampling frequency /2.

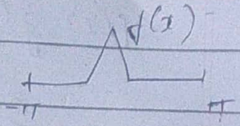
Amplitude = 1, $\theta = 3\pi/2$



sine wave has amplitude 1

- Fourier series:

$$\langle f(x), g(x)\rangle = \int_a^b f(x)\bar{g}(x)\,dx$$

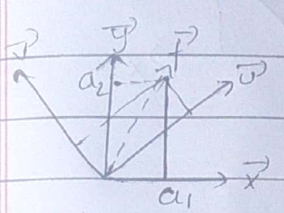


$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty}\left(A_k\cos(kx) + B_k\sin(kx)\right)$$ of increasing high frequency.

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx \quad , \quad B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx$$

normalize

$$\frac{1}{\|\cos(kx)\|^2}\langle f(x), \cos(kx)\rangle \quad \Big| \quad = \frac{1}{\|\sin k(x)\|^2}\langle f(x), \sin(kx)\rangle$$



$$\vec{f} = \frac{\langle \vec{f}, \vec{x}\rangle}{\|x\|^2}\vec{x} + \frac{\langle \vec{f}, \vec{y}\rangle}{\|y\|^2}\vec{y}$$

$$= \frac{\langle \vec{F}, \vec{u}\rangle}{\|\vec{u}\|^2}\vec{u} + \frac{\langle \vec{f}, \vec{v}\rangle}{\|\vec{v}\|^2}\vec{v}$$

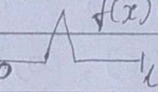
Part -2

$$\langle f(x), g(x)\rangle = \int_a^b f(x)\bar{g}(x)\,dx$$

for gono function

$f(x) \in L(0, L)$

$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty}\left(A_k\cos\left(\frac{2\pi kx}{L}\right) + B_k\sin\left(\frac{2\pi kx}{L}\right)\right)$$

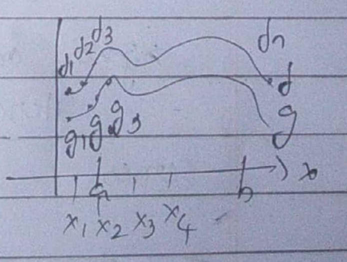


$$A_k = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2\pi k}{L}x\right)dx$$

$$B_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2\pi k}{L}x\right)dx$$

- Inner products of Hilbert space

$$\langle f(x), g(x)\rangle = \int_a^b f(x)\bar{g}(x)\,dx$$



$$\underline{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} \quad \underline{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix}$$

$$\Delta x = \frac{b-a}{n-1}$$

row vec, column vector

$$\langle \underline{f}, \underline{g}\rangle = \underline{g}^* \underline{f} = \boxed{\underline{g}}\,\boxed{f}$$

$$\boxed{\langle \underline{f}, \underline{g}\rangle \Delta x = \sum_{k=1}^{n} f(x_k)\bar{g}(x_k)\Delta x}$$

$$= \sum_{k=1}^{n} f_k \bar{g}_k$$

• Complex fourier series

$\langle f(x), g(x)\rangle = \int_{-\pi}^{\pi} f(x)\,\bar{g}(x)\,dx$

$e^{ikx} = \cos(kx) + i\sin(kx) = \psi_k$

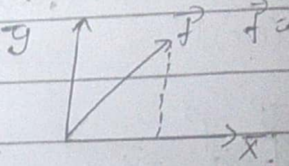
$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx} = \sum_{k=-\infty}^{\infty} (\alpha_k + i\beta_k)(\cos(kx) + i\sin(kx))$

complex valued functions

$(c_k = \bar{c}_{-k}$ if $f(x)$ real$)$

$\langle \psi_j, \psi_k\rangle = \int_{-\pi}^{\pi} e^{ijx} e^{-ikx}\,dx = \int_{-\pi}^{\pi} e^{ix(j-k)}\,dx = \frac{1}{i(j-k)}\left[e^{i(j-k)x}\right]$

$= \begin{cases} 0 & \text{if } j \neq k \\ 2\pi & \text{if } j = k \end{cases}$



$\vec{f} = \langle \vec{f}, \vec{x}\rangle \vec{x} + \langle \vec{f}, \vec{y}\rangle$

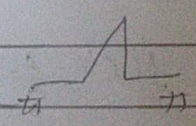
$f(x) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \underbrace{\langle f(x), \psi_k\rangle}_{c_k} \underbrace{\psi_k}_{e^{ikx}}$

• Fourier series (Matlab)

$f(x) \cong \sum_{k=0}^{\infty} a_k \cos\left(k\frac{2\pi x}{L}\right) + b\sin\left(k\frac{2\pi x}{L}\right)$

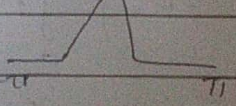
$a_k = \langle f(x), \cos\left(k\frac{2\pi x}{L}\right)\rangle$

$b_k = \langle f(x), \sin\left(k\frac{2\pi x}{L}\right)\rangle$



• Fourier ~~sins~~ series (Python)

$f(x) \cong \sum_{k=0}^{\infty} a_k \cos\left(k\frac{2\pi x}{L}\right) + b\sin\left(k\frac{2\pi x}{L}\right)$

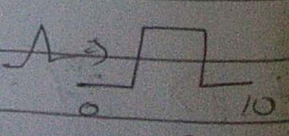


Higher order better approximation.

Here domain, hat function, compute fourier series.

• Fourier series and Gibbs Phenomena.

Discontinuous function.



$f(x) \cong \sum_{k=0}^{100} a_k \cos\left(k\frac{2\pi x}{L}\right) + b_k \sin\left(k\frac{2\pi x}{L}\right)$

Gibbs Phenomena - Winging Phenomenon.

It is obtained by adding all the sine & cosine values.

Scanned with CamScanner

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Phython: Application 4: Build a personal website with Python & Flask.

→ Personal website - How the o/p will look like

→ Your First Website, HTML templates, Navigation Menu.

~~Browser Caching~~: If you deployed your website on Heroku but when you visit the website on the browser you see an error, you probably did something wrong during the deployment. $\boxed{\text{heroku logs}}$

This command will show a series of messages. Carefully read the logs to understand what went wrong.

→ CSS styling

→ Creating a Python Virtual Environment.

→ How to install Git

Git is version control system allowing you to upload the project files to a server and helps track your changes while maintaining the web app. Download Git from

https://git-scm.com/downloads.

→ Note on Browser Caching:

Sometimes, when you make a change to the CSS file and reload the webpage, the changes are not shown because the browser uses the previous cached styling. If this happens, open the browser in private (incognito) mode and load the webpage there.

→ Deploying the website to a live server.

→ Maintaining the live website.