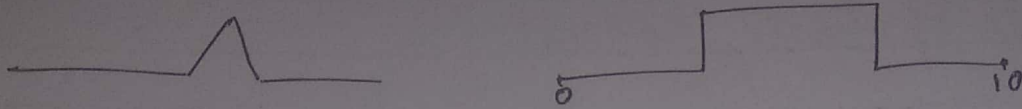


DSP

Fourier series Gibbs phenomena using python.



$$f(x) \cong \sum_{k=0}^{100} a_k \cos\left(\frac{2\pi k x}{L}\right) + b_k \sin\left(\frac{k 2\pi x}{L}\right)$$

$$a_k = \langle f(x) \cos\left(\frac{k 2\pi x}{L}\right) \rangle, \quad b_k = \langle f(x) \sin\left(\frac{k 2\pi x}{L}\right) \rangle$$

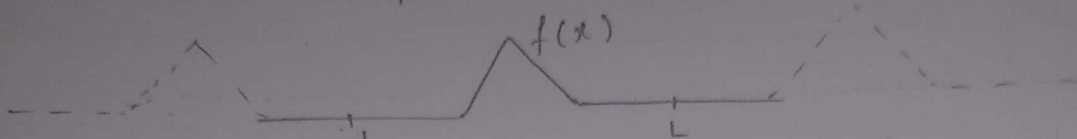
```
import numpy as np
import matplotlib.pyplot as plt
plt.rcParams['figure.figsize'] = (0.8)
plt.rcParams.update({'font.size': 10})
dx = 0.0;
L = 2*np.pi
x = np.arange(0, L+dx, dx)
n = len(x)
nquart = int(np.floor(n/4))
f = np.zeros_like(x)
f[nquart : 3*nquart] = 1
A0 = np.sum(f * np.ones_like(x)) * dx * 2/L
fFS = A0/2 * np.ones_like(f)
for k in range(1, 101):
    Ak = np.sum(f * np.cos(2*np.pi*k*x/L)) * dx * 2/L
    Bk = np.sum(f * np.sin(2*np.pi*k*x/L)) * dx * 2/L
    fFS = fFS + Ak * np.cos(2*k*np.pi*x/L) + Bk * np.sin(2*k*np.pi*x/L)
plt.plot(x, f, color='k', linewidth=2)
plt.plot(x, fFS, '=', color='r', linewidth=1.5)
plt.show()
```

Fourier transform:

$$f(x) \quad f(x) = \sum_{k=-\infty}^{\infty} C_k e^{i\pi x/L}$$

$$C_k = \frac{1}{2L} \int_{-L}^L f(x) \underbrace{e^{-j k \pi x/L}}_{\Psi_k} dx$$

Fourier transform Derivatives.



$$\hat{f}(\omega) = F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx.$$

$$f(x) = F^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{j\omega x} d\omega.$$

$$F\left(\frac{d}{dx} f(x)\right) = \int_{-\infty}^{\infty} \underbrace{\frac{df}{dx}}_{u} \underbrace{e^{-j\omega x}}_u dx$$

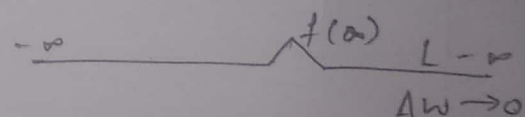
$$u_{tt} = c u_{xx} \xrightarrow{F} \hat{u}_{tt} = -\omega^2 \hat{u}$$

(PDE)

$$u(x,t) \xrightarrow{F} \hat{u}(\omega, t)$$

Fourier Transform & convolution.

$$(f * g) = \int_{-\infty}^{\infty} f(x-\xi) g(\xi) d\xi.$$



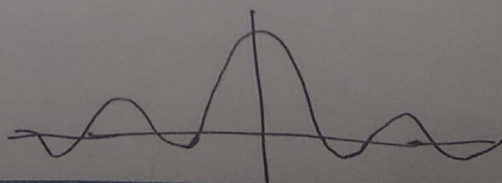
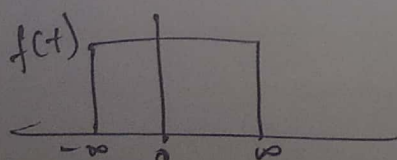
$$\hat{f}(\omega) = F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx.$$

$$f(x) = F^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) d\omega.$$

$$F(f * g) = F(f)F(g) = \hat{f} \hat{g}$$

$$= f * g.$$

Intuition of Fourier.



$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt.$$

$$e^{-i\omega t} = \cos(-\omega t) + i\sin(-\omega t)$$

Application of Z-transform.

- Uses to analysis of digital filters
- Used to simulate the continuous systems.
- + Helps in system design and analysis & also checks the systems stability.

Python:

HTML templates:

```
from flask import flask, render_template.
```

→ return render_template("home.html")

→ return render_template("about.html").