Fourier transform

It is a co-ordinate transform.

Heat equation $u(x,y,t)$

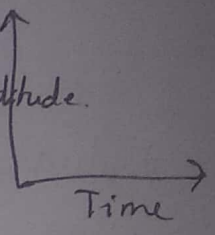
$$\boxed{u_t = \alpha \nabla^2 u}$$

Fast fourier transform (FFT)

To compute fourier series efficiently in a compute

Fourier series.

$$f(t) = \frac{1}{2}a_0 + \sum_{K=1}^{\infty}(a_K \cos 2\pi Kt + b_K \sin 2\pi Kt)$$



Here a_K & b_K are co-efficient

Fourier transform.

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

analysing function sinusoid.

Result → one complex co-efficient per frequency.

$$X_a(F) = \int_{-\infty}^{\infty} x(t) \cos 2\pi Ft\, dt, \quad X_b(F) = \int_{-\infty}^{\infty} x(t) \sin 2\pi Ft\, dt$$

Result: Two result co-efficients / frequency.

Discrete Fourier transform.

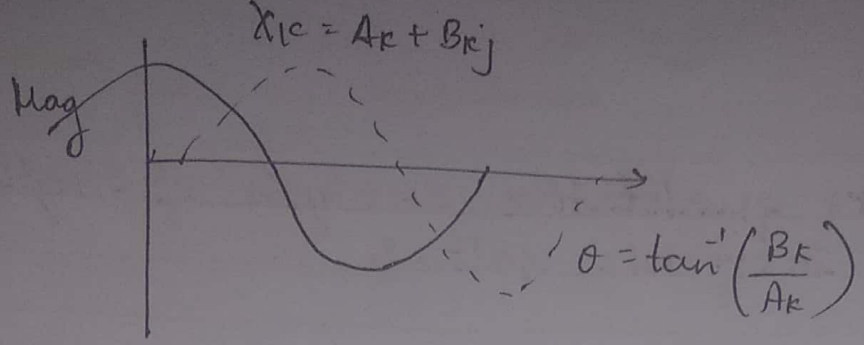
$$X_K = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi Kh}{N}}$$

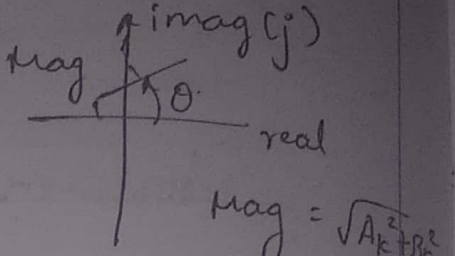
Euler's formula.

$$e^{jx} = \cos x + j \sin x.$$

$$\Rightarrow X_K = x_0[\cos(-b_0) + j \sin(-b_0)] + \cdots$$

$$X_K = A_K + B_K j$$





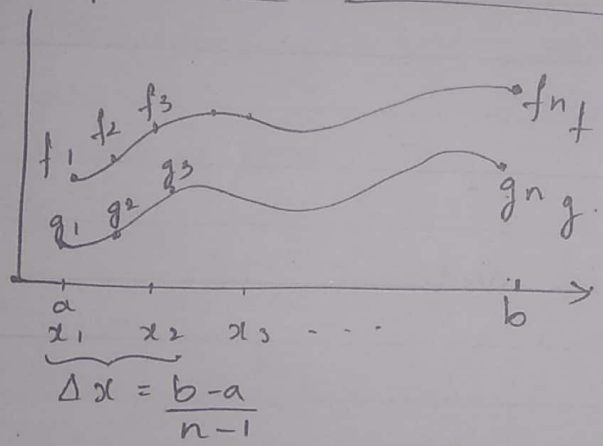
Fourier series

$$\langle f(x), g(x) \rangle = \int_a^b f(x)\,\bar{g}(x)\,dx.$$

$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} \left(A_k \cos(kx) + B_k \sin(kx)\right)$$

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx. \qquad B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx.$$

Inner products in Hilbert space



$$\langle f(x), g(x) \rangle = \int_a^b f(x)\,\bar{g}(x)\,dx.$$

$$f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} \quad g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \Rightarrow \langle f, g \rangle = g^T f = \boxed{g}\,\boxed{f}$$

$$= \sum_{k=1}^{n} f_k g_k.$$

$$\langle f, g \rangle \Delta x = \sum_{k=1}^{n} f(x_k)\, g(x_k)\, \Delta x.$$

Complex Fourier Series

$$\langle f(x) \cdot g(x) \rangle = \int_{-\pi}^{\pi} f(x)\, \bar{g}(x)\, dx.$$

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}.$$

$$e^{ikx} = \psi_k = \cos(kx) + i\sin(kx)$$

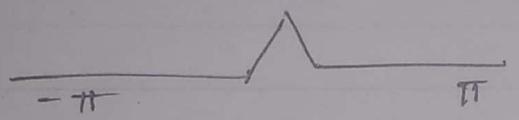
$c_k = \bar{c}_{-k}$ if $f(x)$ is real.

$$f(x) = \sum_{k=-\infty}^{\infty} (\alpha_k + i\beta_k)(\cos(kx) + i\sin(kx))$$

$$\langle \psi_j, \psi_k \rangle = \int_{-\pi}^{\pi} e^{ijx} e^{-jkx}\, dx = \int_{-\pi}^{\pi} e^{i(j-k)x}\, dx = \frac{1}{i(j-k)} \left[e^{i(j-k)x} \right]_{-\pi}^{\pi}$$

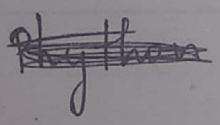
$$= \begin{cases} 0 & \text{if } j \neq k \\ 2\pi & \text{if } j = k. \end{cases}$$

Fourier series [MATLAB]



$$f(x) = \sum_{k=0}^{\infty} a_k \cos\left(k\frac{2\pi x}{L}\right) + b_k \sin\left(k\frac{2\pi x}{L}\right)$$

$$a_k = \left\langle f(x), \cos\left(k\frac{2\pi x}{L}\right) \right\rangle \qquad b_k = \left\langle f(x), \sin\left(k\frac{2\pi x}{L}\right) \right\rangle$$



Python:

* Fixing programming errors:

- Syntax error.

```
print (1)
int(9)
int 9
print (2)
print 3
```

o/p: file "errors.py", lin 3
int 9
invalid syntax.

* int is a function in python, therefore the 9 should be enclose in bracket & even print.

→ Exceptions:

```
a=1
b="2"
print [int 2.5]
print (a+b)
```

Error due to previous line i.e, print (int (2.5)) ~~has~~ missing the print closing bracket.

If we don't understand the message just copy the instruction & search in google.

Build a websit blocker.

program architecture.

windows: c:\windows\system 32\drivers\etc
host file