

Assignment-1

1. what is Simulation? When Simulation is appropriate tool and when it is not Appropriate?  
⇒ A Simulation is the imitation of the operation of real world process or System overtime.
  - i) when Simulation is the Appropriate tool.
    - \* Study and experimentation with the internal interactions of a Complex System, & a Sub-System within a Complex System.
    - \* Informational, organizational and environmental changes can be simulated and the model's behaviour can be observed.
    - \* The knowledge gained in designing a simulation model can be of great value toward suggesting improvement in the system under investigation.
  - ii) when Simulation is Not Appropriate.
    - \* Simulation should not be used when the problem can be solved using common sense.
    - \* Simulation should not be used if the problem can be solved analytically.
    - \* Simulation should not be used if it is easier to perform direct experiments.
    - \* Simulation should not be used if the costs exceeds savings.
    - \* Simulation should not be used if the resources or time are not available.

## 2. Advantages and Disadvantages of Simulation-

3) Advantages

- a. New policies, operating procedures, decision rules, information, flow etc can be explored without disrupting the ongoing operations of the real system.

- b. New hardware designs, physical layouts, transportation systems can be used without committing resources for their acquisition.
- c. Hypotheses about how or why certain phenomena occur can be tested for feasibility.
- d. Time can be compressed or expanded allowing for a speedup or slowdown of a phenomenon under investigation.

### (e) Disadvantages

- a. Model building requires special training. It is an art that is learned over time and through experience.
- b. If 2 models are constructed by 2 component individuals, they may have similarities, but it is highly unlikely that they will be the same.
- c. Simulation results may be difficult to interpret. Since most simulation outputs are essentially random variables, it may be hard to determine whether an observation is a result of system interrelationships or randomness.
- d. Simulation modeling and analysis can be time consuming and expensive.

### 3. Areas of Application.

→ The application of SMS are as follows:-

#### i) Manufacturing Applications

- \* Analysis of electronics assembly operations
- \* Design and evaluation of a selective assembly station for high-precision scroll compressor shells.
- \* Comparison of dispatching rules for semiconductor manufacturing using large-facility models.

- ii) Semiconductor Manufacturing
  - \* Comparison of dispatching rules using large-facility models
  - \* The correction influence of variability.
  - \* A new lot-release rule for wafer fabs.
  - \* Assessment of potential gains in productivity due to proactive refile management.
- iii) Construction Engineering
  - \* Construction of dam embankment
  - \* Trenchless renewal of underground urban infrastructures.
  - \* Activity scheduling in a dynamic, multiproject setting.
  - \* Investigation of the structural steel erection process.
  - \* Special-purpose template for utility tunnel construction.
- iv) Military Application.
  - \* Modeling leadership effects and recruit type in a army recruiting station.
  - \* Design and test of an intelligent controller for autonomous underwater vehicles.
- v) Logistics, transportation and Distribution Applications.
  - \* Evaluating the potential benefits of rail traffic planning
  - \* Evaluating strategies to improve railroad performance.
  - \* Parametric modeling in rail-capacity planning.
  - \* Analysis of passenger + low in airport terminal.
- vi) Business Process Simulation.
  - \* Impact of connection bank realign on airport gate
  - \* Product development program planning.
  - \* Reconciliation of business and systems modeling.
- vii) Human System and Healthcare
  - \* Modeling human performance in complex systems
  - \* Studying the human element in air traffic control.
  - \* Modeling front office and patient care in ambulatory health care practices.

## 4. System and System Environment

### System

System is defined as a group of object that are joined together in some regular interaction or inter dependence toward the accomplishment of some.

### System environment

A System is often affected by changes occurring outside the System, such changes are said to occur in the System environment.

## 5. Components of a System

- Entity :- An entity of object of interest in a System
- Attribute :- An attribute denotes the property of an entity
- Activity :- Represent a time period of specified length.
- State of the System :- The State of a System is defined as the collection of Variables necessary to describe a System at any time, relative to objective of study.
- Events :- An event is defined as an instantaneous occurrence that may change the state of a System.

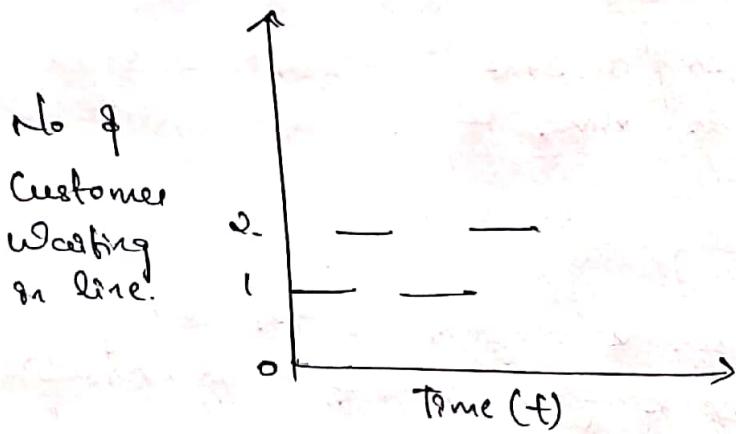
Endogenous : Is used to describe activities and events occurring within the System.

Exogenous : Is used to described activities and events in the environment that affect the System.

## 6. Discrete and Continuous System

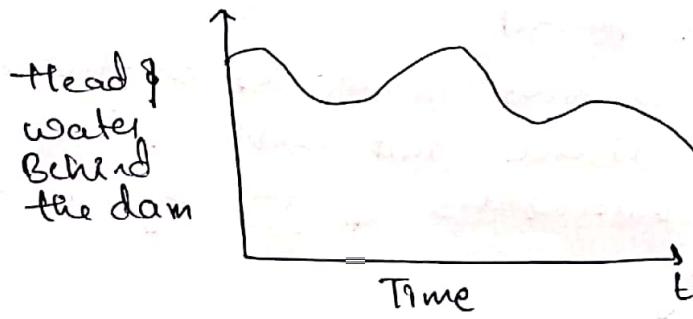
### Discrete

- \* Is one in which the state variable change only at a discrete set of points in time.
- + The bank is an example, since the state variable the number of cust in the bank changes only when a customer arrives or when the service provided a customer is completed.



### Continuous System.

- \* Is one in which the state variable change ~~continuous~~ continuous over time.
- \* Head of water behind a dam, during and for some time after a rain water flows into the tank behind the dam.



### 7. Model of a System

- \* Model of a System is defined as the representation of a system for the purpose of studying the system.
- \* It is necessary to consider only those aspects of the system that affect the problem under investigation.
- \* These aspects are represented in a model and by definition it is simplification of a system.

### 8. Types of Models.

#### i) Mathematical or physical model:-

Mathematical model uses symbolic notation and equations to represent a system.

## ii) Static model

A static simulation models represent a system at a particular point in time it is also called as montecarlo simulation.

## iii) Dynamic Model

A dynamic model represents system as the change over time, simulation of bank from a to b is an example

## iv) Deterministic Model

Simulation Model that contain no-random variable are classified as deterministic.

Deterministic Model have a known set of inputs which will result in a unique set of outputs.

## v) Discrete and Continuous Model

Used in analogous manner. Simulation models may be mixed both with discrete and continuous. The choice is based on the characteristics of the system and the objective of the study.

## 9. Discrete-Event Simulation.

Modeling of a system in which the state variable changes only at a discrete set of points in time. The simulation models are analyzed by numerical rather than by analytical methods. Analytical methods employ the deductive reasoning of mathematics to solve the model.

Eg:- Differential calculus can be used to determine the minimum cost policy for some inventory models. Numerical methods use computational procedures to solve mathematical models occurs, which is generated based on the model assumptions. and observation are collected to be analyzed and estimate the true system performance measures.

Real world simulation is so vast, whose runs are conducted with the help of computers. Much insight can be obtained by simulation manually which is applicable for small systems.

## 10. Steps in a Simulation Study

### i) Problem formulation

Every study begins with a statement of the problem, provided by policy makers. Analyst ensures its clearly understood that it is developed by analyst policy makers should understand and agree with it.

### ii) Setting objectives and overall project plan.

The objectives indicates the questions to be answered by simulation. At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming it is appropriate, the overall project plan should include.

- \* A statement of the alternative systems.
- \* A method for evaluating the effectiveness of these alternatives.

- \* Plans for the study in terms of the number of the people involved.

### iii) Model Conceptualization

The construction of a model of a system is probably as much art as science. The art of modeling is enhanced by ability.

Modeling is enhanced by ability.

- \* To abstract the essential features of a problem
- \* To select and modify basic assumptions that characterizes the system.

- \* To enrich and elaborate the model until a useful approximation results.

#### VII) Data Results.

There is a constant interplay b/w the construction of model and collection of needed input data, done in the early stages. Objective kinds of data are to be collected

#### VIII) Model translation.

Real-World System result in models that require a great deal of information storage and computation, the model must be entered into computer program and checking the performance. If the inputs parameters and logical structure are correctly represented, verification is completed.

#### IX) Verified?

It pertains to the computer program and checking the performance. If the input parameters and logical structure are correctly represented, verification is completed.

#### X) Validated?

It is the determination that a model is an accurate representation of the real system. Achieved through calibration of the model, an iterative process of comparing the model to actual system behaviour.

#### XI) Experimental Design

The alternative that are to be simulated must be determined which alternatives to simulate may be function of runs. For each system design, decisions need to be made concerning.

- \* Length of the initialization period.
- \* Length of simulation runs.

\* Number of replications to be made of each run.

#### ix). Production runs and analysis

They are used to estimate measure of performance for the system designs that are being simulated.

#### x) More runs?

Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.

#### xii) Documentation and Reporting.

Two types of documentation:-

- \* Program documentation
- \* Process documentation

⇒ Program documentation is necessary for numerous reasons. If the program is going to be used again by the same or different analysts.

⇒ Program reports that provide the important written history of a simulation report. Project reports give a chronology of work done and decisions made. This can prove to be great value in keeping the project on course.

#### xiii) Implementation

Success depends on the previous steps. If the model user had been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

## II. Review of terminology and concepts

### i) Discrete random variable

Let  $x$  be a random variable of the number of possible values if  $x$  is finite or countable infinite and called a discrete random variable.

### ii) Continuous random Variables

If the range space  $R_x$  of the random variable  $x$  is an interval or a collection of intervals  $x$  is called a continuous random variable. For continuous random variables  $x$ , the prob that  $x$  lies in the interval  $[a, b]$  is given by

$$P(a \leq x \leq b) = \int_a^b f(x) dx$$

The function  $f(x)$  is called the prob density of the random variables  $x$ . The probability density function satisfies the following conditions:

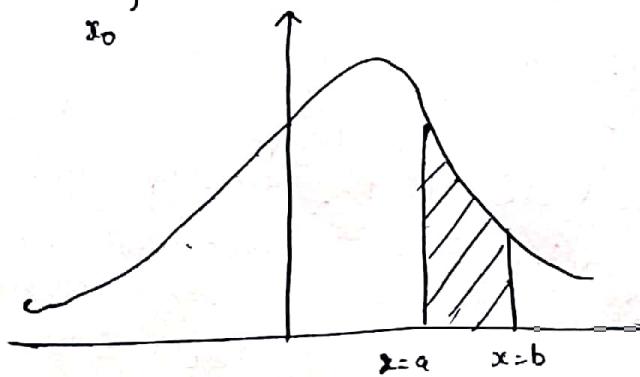
a)  $f(x) > 0$  for all  $x$  in  $R_x$

b)  $\int_{R_x} f(x) dx = 1$

c)  $f(x) = 0$  if  $x$  is not in  $R_x$

As a result of equation (1), for any specified value  $x_0$ ,  $P(x = x_0) = 0$  because

$$\int_{x_0}^{x_1} f(x) dx = 0$$



$P(x = x_0) = 0$  also means that the following eqn hold  
 $P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x \leq b) = P(a < x < b)$

### iii) Cumulative distribution function

The Cumulative distribution function (cdf) is denoted by  $f(x)$ , where  $F(x) = P(X \leq x)$

\* If  $X$  is discrete, then  $F(x) = \sum_{x_i < x} P(x_i)$

\* If  $X$  is continuous, then  $F(x) = \int_{-\infty}^x f(t) dt$ .

All Prob questions about  $X$  can be answered in terms of the Cdf:

$$P(a \leq X \leq b) = F(b) - F(a), \text{ for all } a \leq b$$

### iv) Expectation and Variance.

\* Expectation essentially is the expected Value of a random Variable.

\* Variance is a measure how a random variable varies from its expected Value.

The expected Value of  $X$  is denoted by  $E(X)$

\* If  $X$  is discrete  $E(X) = \sum_{\text{all } i} x_i p(x_i)$

\* If  $X$  is Continuous  $E(X) = \int_{-\infty}^{\infty} x \cdot f(x) dx$

\*  $a, k, \alpha$  the mean, m, u, or the 1<sup>st</sup> moment of  $X$ .

\* A Measure of the Central tendency.

### v) The Mode

The Mode is used to describe most frequently occurred values in discrete random variable, or the maximum value of a continuous random variables.

## 12. Useful Statistical Models

In this Section, statistical models appropriate to some application areas are presented.

The areas include:

### i) Queuing System

Simple statistical models for interarrival & service time distribution.

- \* Exponential distribution: if service times are completely random.
- \* Normal distribution: fairly constant but with some random variability.
- \* Truncated normal distribution: similar to normal distribution but with restricted values.

### ii) Inventory and Supply chain

In realistic inventory and supply chain system, there are at least random variables:

- \* The no of units demanded per order.
- \* The time b/w demands
- \* The lead time = Time b/w placing an order and the receipt of that order.

### iii) Reliability and maintainability

Time to failure (TTF)

- \* Exponential: failures are random.
- \* Gamma: for standby redundancy where each component has an exponential TTF.
- \* Weibull: failure is due to most serious of a large no of defects in a system of components
- \* Normal: failures are due to wear.

iv) Limited data: Simulation begin before the data.  
Collection phase has been completed.

### 13. Discrete distribution.

#### i) Bernoulli Trials and Bernoulli distribution.

An experiment consists of  $n$  trials, each trial being a success or a failure. Define  $x_j=1$  if the  $j^{\text{th}}$  expt was a success and  $x_j=0$  if the  $j^{\text{th}}$  expt was a failure. The  $n$  Bernoulli trials are called a Bernoulli process if they are independent.

$$P(x_1, x_2, \dots, x_n) = P_1(x_1) \cdot P_2(x_2)$$

where

$$P_j(x_j) = \begin{cases} p & \text{if } x_j=1, j=1 \dots n \\ 1-p = q & \text{if } x_j=0, j=1 \dots n \\ 0 & \text{otherwise.} \end{cases}$$

#### ii) Binomial Distribution

The random variable  $X$  that denotes the number of successes in Bernoulli's trials has a binomial distribution and given by  $P(x)$ , where,

$$P(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x=0, 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

#### iii) Geometric and Negative Binomial distribution

The geometric distribution is related to a sequence of Bernoulli's trials, the random variable of instance  $x$ , is defined by the number of trials to achieve the first success. The distribution of  $x$  is given by

$$P(x) = \begin{cases} 2^{x-1} p, & x=0, 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

## 14. Continuous Distribution

### i) Uniform distribution.

A random variable  $x$  is uniformly distributed on the interval  $(a, b)$  if its pdf is given by

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

The cdf is given by

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

### ii) Exponential distribution

A random variable  $x$  is said to be exponentially distributed with parameter  $\lambda > 0$  if its pdf is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

### iii) Gamma distribution

This function used in defining the gamma distribution is the gamma function, which is defined as  $\beta > 0$  as

$$f(\beta) = \int_0^\infty x^{\beta-1} e^{-x} dx$$

## 15. Poisson Process

$N(t)$  is a counting function that represents the number of events occurred in  $[0, t]$ . A counting process  $\{N(t), t \geq 0\}$  is a Poisson Process with mean rate  $\lambda$  if:

\* Arrivals occur one at a time

\* Arrivals have stationary increments

\*  $\{N(t), t \geq 0\}$  has independent increments

Number of arrivals in  $[t, t+s]$  depends only on  $s$ , not on starting point  $t$ .

\* Arrivals are completely random.

- \*  $\{N(t), t \geq 0\}$  has independent increments.
- Number of arrivals during non overlapping time intervals are independent.
- future arrivals occur completely random.

### Properties

$$P(N(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \text{ for } t \geq 0 \text{ and } n=0, 1, 2, \dots$$

- \* Equal mean and Variance  $E[N(t)] = V[N(t)] = \lambda t$

- \* Stationary increment.

The number of arrivals in time  $s$  to  $t$  with  $s < t$ , is also Poisson distribution with mean  $\lambda(t-s)$

### 16. Empirical distribution

A distribution whose parameters are the observed values in a sample of data.

May be used when it is impossible or unnecessary to establish that a random variable has any particular parametric distribution.

An empirical distribution may be used when it is possible or unnecessary to establish that a random variable has any particular parametric distributions.

One advantage of a empirical distribution is nothing is assumed beyond the obtained values in the sample.

### 17. Characteristics of Queuing System.

#### 1) The Calling Population

The population of potential customers, referred to as the calling population, may be assigned to be finite or infinite. In system with a large population of potential customer, the calling population is usually

Seemed to be infinite.

### qii) System Capacity:

In many Queueing Systems, there is a limit to the no of customers that may be in the waiting line or System. As arriving customer who finds the System full does not enter but returns immediately to the calling process.

### qiii) The arrival Process:

The arrival process for infinite-population model is usually characterized by intervals of interval time of successive customers. Arrivals may occur at scheduled times or at a random time. When at random times, the interarrival times are usually characterized by a prob distribution.

### qiv) Queue Behavior and Queue discipline:

Queue Behavior refers to the actions of customer while in Queue waiting for service to begin. There is a possibility that incoming customer will break.

Queue discipline refers to the logical ordering of customers in a Queue and determines which customer will be chosen.

## 18. Long run measures of performance of queuing Systems

The primary long-run measures of performance of queuing systems are the long run-time avg no of customers in the system ( $L$ ) and in the queue ( $L_q$ ).

The long-run avg time spent in system ( $w$ ) and in the queue ( $w_q$ ) per customer, and the server utilization,  $\rho$  proportion of time that server is busy ( $P$ ).

The term System usually refers to waiting time plus the service mechanism. It is general, can refer to any Subsystem of the Queueing system, on the other hand the term queue rules to the waiting line along the measures of performance of interest include the long-run proportion of customers.

## 19. Queueing Notation.

Recognizing the diversity of Queueing Systems, Kendall proposed a notational system in parallel servers system which has been widely adopted. An abridged version of this convention is based on the format  $A|B|c|n|k$ . These letters represent the following system characteristics:

A - represents the interarrival time distribution

B - represents the service-time distribution.

C - represents the no of parallel Servers.

N - represents the System Capacity.

K - represents the size of the calling population.

## 20. Steady state behaviour M|G|1 Queue

For any M|G|1 Queue, if lines are too long, they can be reduced by decreasing the server utilization or by decreasing the service time variability +2. These demands hold for almost all Queue not just the M|G|1 Queue. The utilization factor  $\rho$  can be reduced by decreasing the arrival rate  $\lambda$ , by increasing the service rate  $\mu$ , or by increasing the no of servers in general,  $\rho = \lambda / (C\mu)$  where C is no of parallel servers.

## 21. Networks of Queues-

Many systems are naturally modeled as Networks of Single-Queue in which customers departing from one queue may be routed to another. The following results assume a stable system with an infinite calling population and no limit on system capacity.

- i) Provided that no customers are created or destroyed in the queue, then the arrival rate from queue  $i$  to queue  $j$  is  $\lambda_j p_{ij}$ , over the long run.
- ii) If customers arrive to queue  $i$  at rate  $\lambda_i$  and a fraction  $0 \leq p_{ij} \leq 1$  of them are routed to queue  $j$  upon departure, then the arrival rate from queue  $i$  to queue  $j$  is  $\lambda_i p_{ij}$ , over the long run.
- iii) The overall arrival rate into Queue  $j$ ,  $\lambda_j$  is the sum of the arrival rate from all sources, if customers arrive from outside the network at rate  $a_j$ , then,

$$\lambda_j = a_j + \sum_{i \neq j} \lambda_i p_{ij}$$

- iv) If queue  $j$  has  $g_{j\infty}$  parallel servers, each working at rate  $\mu_j$ , then the long run utilization of each server is  $P_j = \frac{\lambda_j}{C_j \mu_j}$

## 22. Models, Optimization Via Simulation

Optimization via Simulation to refer to the problem of maximizing or minimizing the expected of a discrete event, stochastic systems that is represent by a computer simulation model.

Optimization usually deals with problems with certainty but in stochastic discrete-event simulation the result of any simulation run is a random variable.

Let  $x_1, x_2 \dots x_n$  be the  $m$  controllable design variables and  $y(x_1, x_2 \dots x_m)$  be the observed simulation output performance on one run.

To optimize  $y(x_1, x_2 \dots x_m)$  with respect to  $x_1, x_2 \dots x_m$  is to maximize or minimize the mathematical expectation of performance  $E[y(x_1, x_2 \dots x_m)]$ .