Forenoon Session: Date: / /

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Course: Signals & System   USN : 4AL18EC042

Topic: Fourier series & Fourier Transform.   Sem & Sec: IV Sem & 'A' Sec.

Github Repository : Rakshita.

Report :

Introduction to Fourier series & Fourier Transform:-

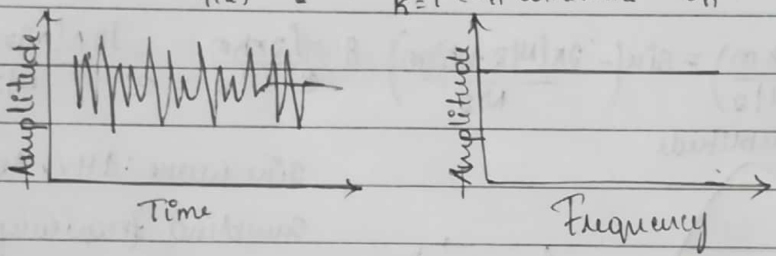
Coordinate transform.

$u_t = \alpha \nabla^2 u$.

SVD = Data - driven FFT (Fast fourier transform).

Fourier series :-

$$f(t) = \frac{1}{2} a_0 + \sum_{k=1}^{\infty} (a_k \cos 2\pi k t + b_k \sin 2\pi k t).$$



[20 - 20000Hz is the range of human hearing].

Fourier Transform:

$$x(F) = \int_{-\infty}^{\infty} \overbrace{x(t)}^{\text{function}} \cdot \underbrace{e^{-j2\pi F t}}_{\text{Analyzing func. sinusoids}} . dt.$$

Result : one complex cofficient per frequency.

$$x_a(F) = \int_{-\infty}^{\infty} x(t) \cos 2\pi F t . dt , \quad x_b(F) = \int_{-\infty}^{\infty} x(t) \sin 2\pi F t . dt.$$

Result : Two real coefficients per frequency.

Discrete fourier transform:

Continuous: $x(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi F t} . dt$; discrete : $X_k = \sum_{n=0}^{N-1} x_n . e^{\frac{-j2\pi k n}{N}}$

Fast fourier transform : frequency bin $f_k = \sum_{n=0}^{N-1} x_n . e^{\frac{-j2\pi k n}{N}}$

k: 0, # of samples. n: 0, ..., # of samples.

(1 operation) 1 Sample : $F_0 = x_0^*$ exponential.

(4 operation) 2 sample : $F_0 = x_0^*$ exponential + x_1^* exponenential.

$F_1 = x_0^*$ exponential + x_1^* exponential.

(9 operation) 3 Sample : $F_0 = x_0^*$ exponential + x_1^* exponential + x_2^* exponential.

$F_1 = x_0^*$ exponential + x_1^* exponential + x_2^* exponential.

$F_2 = x_0^*$ exponential + x_1^* exponential + x_2^* exponential

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Complexity : N^2 | Complexity : $N\log_2 N$

0-20000Hz, 1 Hz spacing | 0-20000Hz, 1 Hz spacing

20000^2 = 400 million operations | $20000 * \log_2 20000 = 283600$ operations

≈ 760 yrs. | ≈ 198 yrs.

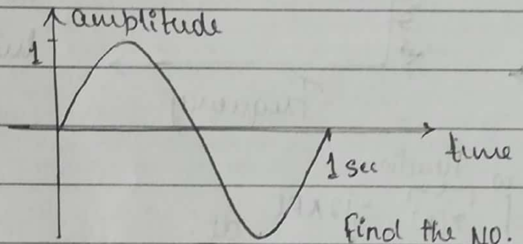
even index | odd index

$$F_k = \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{\frac{-j2\pi k(m)}{N/2}} + \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{\frac{-j2\pi k(m+1/2)}{N/2}}$$

$$\cos\left(\frac{-2\pi k m}{N/2}\right) + j\sin\left(\frac{-2\pi k m}{N/2}\right) \quad k:0,\ldots,N \text{ (integers)}$$

Symmetry identity: $\cos\left(\frac{-2\pi k m}{N/2}\right) = \cos\left(-\frac{2\pi(N/2+k)m}{N/2}\right)$

$\sin\left(-\frac{2\pi k m}{N/2}\right) = \sin\left(-\frac{2\pi(N/2+k)m}{N/2}\right)$ & $e^{\frac{-j2\pi k m}{N/2}} = e^{-\frac{j2\pi(N/2+k)m}{N/2}}$ $k:0,\ldots,N$.



Sin wave : 1Hz, Amplitude = 1

Sampling frequency : 4Hz

samples (N) : 4.

Find the No. of samples we have.

```
int N = samples.size();
if (N==1): {return samples}
int M = N/2;
for (int i=0; i!=M; i++)
{ Xeven (i) = samples [2*i];
  Xodd (i) = samples [2*i+1];
vector <complex<double>> Feven (M,0);
Feven = FFT {Xeven};
Vector <complex<double>> Fodd (M,0);
Fodd = FFT (Xodd);
Vector <complex <double>> freqbins (N,0);
for (int k=0; k!=N/2; k+1)
Complex<double> complrexponential = polar (1.0, -2*pi*k/N)*Fodd(k);
Freqbins[k] = Feven[k] + complrexponential;
Freqbins [k+N/2] = Feven[k] - complrexponential;
}
return freqbins;
}
```

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$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} \left(A_k \cos(kx) + B_k \sin(kx)\right)$$

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx. \qquad B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx.$$

$$= \frac{1}{\|\cos(kx)\|^2}\langle f(x), \cos(kx)\rangle \qquad = \frac{1}{\|\sin(kx)\|^2}\langle f(x), \sin(kx)\rangle.$$

$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} \left(A_k \cos\left(\frac{2\pi k x}{L}\right) + B_k \sin\left(\frac{2\pi k x}{L}\right)\right)$$

$$A_k = \frac{2}{L}\int_{0}^{L} f(x)\cos\left(\frac{2\pi k}{L}x\right)dx. \qquad B_k = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(\frac{2\pi k}{L}x\right)dx.$$

Inner products in Hilbert space:-

Complex Fourier Series:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{ikx}. \qquad e^{ikx} = \cos(kx) + i\sin(kx). = \psi_k.$$

$$= \sum_{k=-\infty}^{\infty} (\alpha_k + i\beta_k)\left(\cos(kx) + i\sin(kx)\right). \qquad (c_k = \bar{c}_{-k} \text{ if } f(x) \text{ is real}).$$

$$\langle \psi_j, \psi_k\rangle = \int_{-\pi}^{\pi} e^{ijx}\, e^{-ikx}\,dx. = \int_{-\pi}^{\pi} e^{i(j-k)x}\,dx. = \frac{1}{i(j-k)}\left[e^{i(j-k)x}\right]_{-\pi}^{\pi}.$$

$$= \begin{cases} 0, & \text{if } j \neq k. \\ 2\pi, & \text{if } j = k. \end{cases}$$

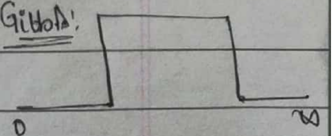
$$= \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \underbrace{\langle f(x), \psi_k\rangle}_{c_k} \underbrace{\psi_k}_{e^{jkx}}.$$

Fourier Series for MATLAB:

Fourier series using python:-

Fourier series & Gibbs phenomena :-

Gibbs:



using MATLAB $f(x) = \sum_{k=0}^{20} a_k \cos\left(k \cdot \frac{2\pi x}{L}\right) + b_k \cdot \sin\left(k \cdot \frac{2\pi x}{L}\right)$

$$a_k = \left\langle f(x), \cos\left(k \cdot \frac{2\pi x}{L}\right)\right\rangle, \quad b_k = \left\langle f(x), \sin\left(k \cdot \frac{2\pi x}{L}\right)\right\rangle$$