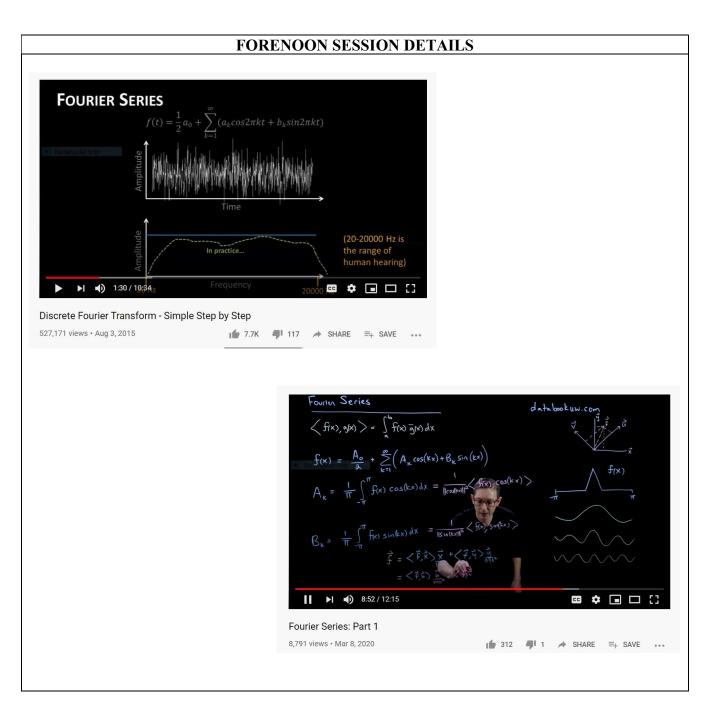
## **DAILY ASSESSMENT FORMAT**

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Course:	Digital Signal Processing	USN:	4AL17EC103
Topic:	Introduction to Fourier Series & Fourier Transform	Semester & Section:	6-B
Github Repository:	Sachin-Courses		



Fourier series AfundamentalresultinFourieranalysisisthatiff(x) is periodicand piecewise smooth, then it can be written in terms of a Fourier series, which is an infinite sum of cosines and sines of increasing frequency. In particular, if f(x) is  $2\pi periodic$ , it may be written as:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx)).$$

## FOURIER SERIES AND FOURIER TRANSFORMS

The coefficients ak and bk are given by

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx) dx$$
$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx) dx,$$

which may be viewed as the coordinates obtained by projecting the function onto the orthogonal cosine and sine basis  $\{\cos(kx), \sin(kx)\}$  k=0. In other words, the integrals in (2.6) may be re-written in terms of the inner product as:

$$a_k = \frac{1}{\|\cos(kx)\|^2} \langle f(x), \cos(kx) \rangle$$
$$b_k = \frac{1}{\|\sin(kx)\|^2} \langle f(x), \sin(kx) \rangle,$$

where  $k\cos(kx)k2 = k\sin(kx)k2 = \pi$ . This factor of  $1/\pi$  is easy to verify by numerically integrating  $\cos(x)2$  and  $\sin(x)2$  from  $-\pi$  to  $\pi$ . The Fourier series for an L-periodic function on [0,L) is similarly given by:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos\left(\frac{2\pi kx}{L}\right) + b_k \sin\left(\frac{2\pi kx}{L}\right) \right),$$

with coefficients ak and bk given by

$$a_k = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{2\pi kx}{L}\right) dx$$
$$b_k = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{2\pi kx}{L}\right) dx.$$

## Fourier series approximation to a hat function.

```
% Define domain dx = 0.001;
L = pi;
x = (-1+dx:dx:1)*L;
n = length(x);
nquart = floor(n/4);
% Define hat function f = 0*x;
f(nquart:2*nquart) = 4*(1:nquart+1)/n;
f(2*nquart+1:3*nquart) = 1-4*(0:nquart-1)/n;
plot(x,f,'-k','LineWidth',1.5), hold on
% Compute Fourier series CC = jet(20);
A0 = sum(f.*ones(size(x)))*dx;
fFS = A0/2;
for k=1:20 \text{ A}(k) = \text{sum}(f.*\cos(pi*k*x/L))*dx;
% Inner product B(k) = sum(f.*sin(pi*k*x/L))*dx;
fFS = fFS + A(k)*\cos(k*pi*x/L) + B(k)*\sin(k*pi*x/L);
plot(x,fFS,'-','Color',CC(k,:),'
LineWidth',1.2)
```