

## DAILY ASSESSMENT FORMAT

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Course:	Digital signal processing	USN:	4a117ec083
Topic:	Fourier series	Semester & Section:	6 <sup>th</sup> sem B sec
Github Repository:	sahanasr-course		

### FORENOON SESSION DETAILS

Image of session

## Complex Fourier Series

Fourier Series  $e^{ikx} = \cos(kx) + i\sin(kx) = \psi_k$  databookuw.com

$$\langle f(x), \psi_k \rangle = \int_{-\pi}^{\pi} f(x) \bar{\psi}_k(x) dx$$

$$f(x) = \sum_{k=-\infty}^{\infty} C_k e^{ikx} = \sum_{k=-\infty}^{\infty} (a_k + i b_k) \left( \cos(kx) + i \sin(kx) \right)$$

$(C_k = \bar{C}_{-k} \text{ if } f(x) \text{ real})$

$$\langle \psi_j, \psi_k \rangle = \int_{-\pi}^{\pi} e^{ijx} e^{-ikx} dx = \int_{-\pi}^{\pi} e^{i(j-k)x} dx = \frac{1}{i(j-k)} e^{i(j-k)x} \Big|_{-\pi}^{\pi}$$

$$= \begin{cases} 0 & j \neq k \\ 2\pi & j = k \end{cases}$$

## Fourier Series and Gibbs Phenomena

### [ MATLAB ]

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1 clear all, close all, clc
2
3 figure
4 set(gcf, 'Position', [1500 200 2000 1200])
5
6 % Define domain
7 L = pi;
8 N = 1024;
9 dx = 2*L/(N-1);
10 x = -L:dx:L;
11
12 % Define function
13 f = 0*x;
14 f(N/4:N/2) = 4*L/4*(1-1/N);
15 f(N/2+1:3N/4) = 3-4*(8/N/4-1)/N;
16 plot(x, f, 'k', 'LineWidth', 3.5), hold on
17
18 % Compute Fourier series
19 CC = zeros(1,N);
20 AB = sum(f.*cos((1:N)/2*pi))./(dx*pi);
21 fFS = AB/2;
22 for k=1:N
23     AK = sum(f.*cos(k*x))./(dx*pi); % inner product
24     BK = sum(f.*sin(k*x))./(dx*pi);
25     fFS = fFS + AK*cos(k*x) + BK*sin(k*x);
26     plot(x, fFS, 'r', 'Color', 'C(1,1)', 'LineWidth', 2);
27     pause(1)
28 end
29
30 % Plot approximation
31 figure; set(gcf, 'Position', [1500 200 2000 1200]);
32 clear fFS;
33 close;
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**Report – Report can be typed or hand written for up to two pages.**

1. Introduction.
2. Fourier Series part-1,2.
3. Inner product in Hilbert Transform.
4. Complex Fourier Series.
5. Fourier Series using Mat Lab.
6. Fourier Series using Python.
7. Fourier Series and Gibbs phenomena.

## Introduction:

1. Fourier Series and Wavelets.
2. Coordinate Transform-used for Image Compression.
3. Hilbert Transform.
4. Fast Fourier Transform(FFT).

## Discrete Fourier Transform:

- It converts a finite sequence of equally spaced samples of a function into a same length sequence of equally -spaced samples of DTFT.

Analyzing the Functions.

### Fourier Series:

- A Fourier series is a way of representing a periodic function as a (possibly infinite) sum of sine and cosine functions. It is analogous to a Taylor series, which represents functions as possibly infinite sums of monomial terms. A sawtooth wave represented by a successively larger sum of trigonometric terms.

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} s(x) \cos(nx) dx = 0, \quad n \geq 0.$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} s(x) \sin(nx) dx \\ &= -\frac{2}{\pi n} \cos(n\pi) + \frac{2}{\pi^2 n^2} \sin(n\pi) \\ &= \frac{2(-1)^{n+1}}{\pi n}, \quad n \geq 1. \end{aligned}$$

### Inner Product in Hilbert Space:

- A Hilbert space H is a real or complex inner product space that is also a complete metric space with respect to the distance function induced by the inner product. A real inner product space is defined in the same way, except that H is a real vector space and the inner product takes real values.

Sampling.

### Complex Fourier Series:

- The complex Fourier series is presented first with period  $2\pi$ , then with general period.
- Using Mat Lab.

