

DAILY ASSESSMENT FORMAT

Date:	26-05-2020	Name:	Sahana S R
Course:	Digital signal processing	USN:	4al17ec083
Topic:	<ul style="list-style-type: none"> •Fourier transform •Fourier transform and Derivatives •Fourier transform and Convolution integral •Transform and Laplace transform first order •Implementation of laplace transform using Matlab •Application of Z-Transform •Z-Transform of sequence using matlab 	Semester & Section:	6 th sem B sec
GitHub Repository:	sahanasr-course		

FORENOON SESSION DETAILS

Image of session

The Fourier Transform

Fourier Series \rightarrow Fourier Transform

$u_{tt} = C u_{xx}$ (PDE) $\xrightarrow{\mathcal{F}}$ $\hat{u}_{tt} = -\omega^2 \hat{u}$ (ODE)

$\mathcal{F}\left(\frac{df}{dx} f(x)\right) = \int_{-\infty}^{\infty} \frac{df}{dx} e^{-iwx} dx$

$= \left[f(x) e^{-iwx} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(x) (-iwe^{-iwx}) dx$

$= i\omega \int_{-\infty}^{\infty} f(x) e^{-iwx} dx = i\omega \mathcal{F}(f(x))$

$\hat{f}(\omega) = \mathcal{F}(f(x)) = \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$

$f(x) = \mathcal{F}^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{iwx} d\omega$

Fourier Transform Pair

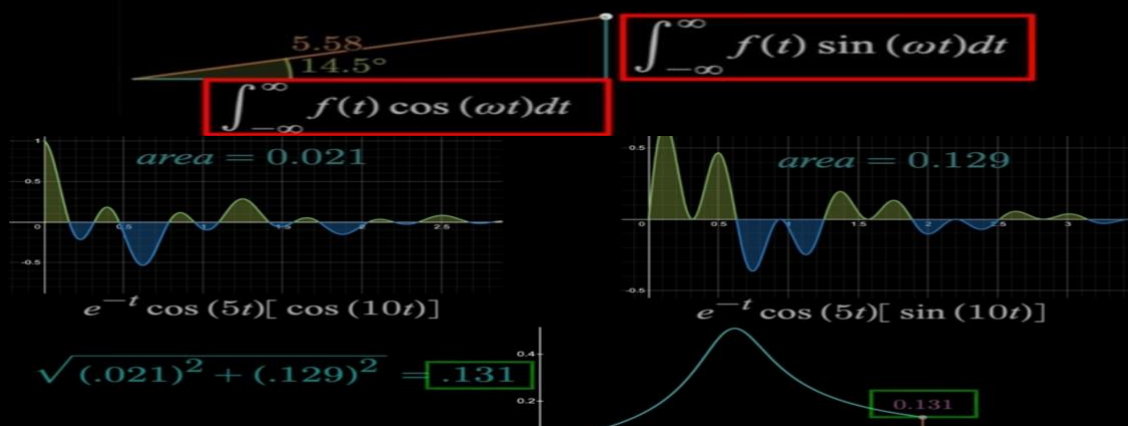
databookuw.com

The Fourier Transform and Convolution Integrals

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cos(\omega t) dt - i \int_{-\infty}^{\infty} f(t) \sin(\omega t) dt$$

$$\omega = 2.853$$



Laplace Transform: 1st order equation
The transforms of $f(t)$ and $y(t)$ are $F(s)$ and $Y(s)$
Definit $F(s) = \int_0^{\infty} f(t) e^{-st} dt$

Exa
 $f(t)$

Lec- 45

Applications of Z - transforms - I

Properties:

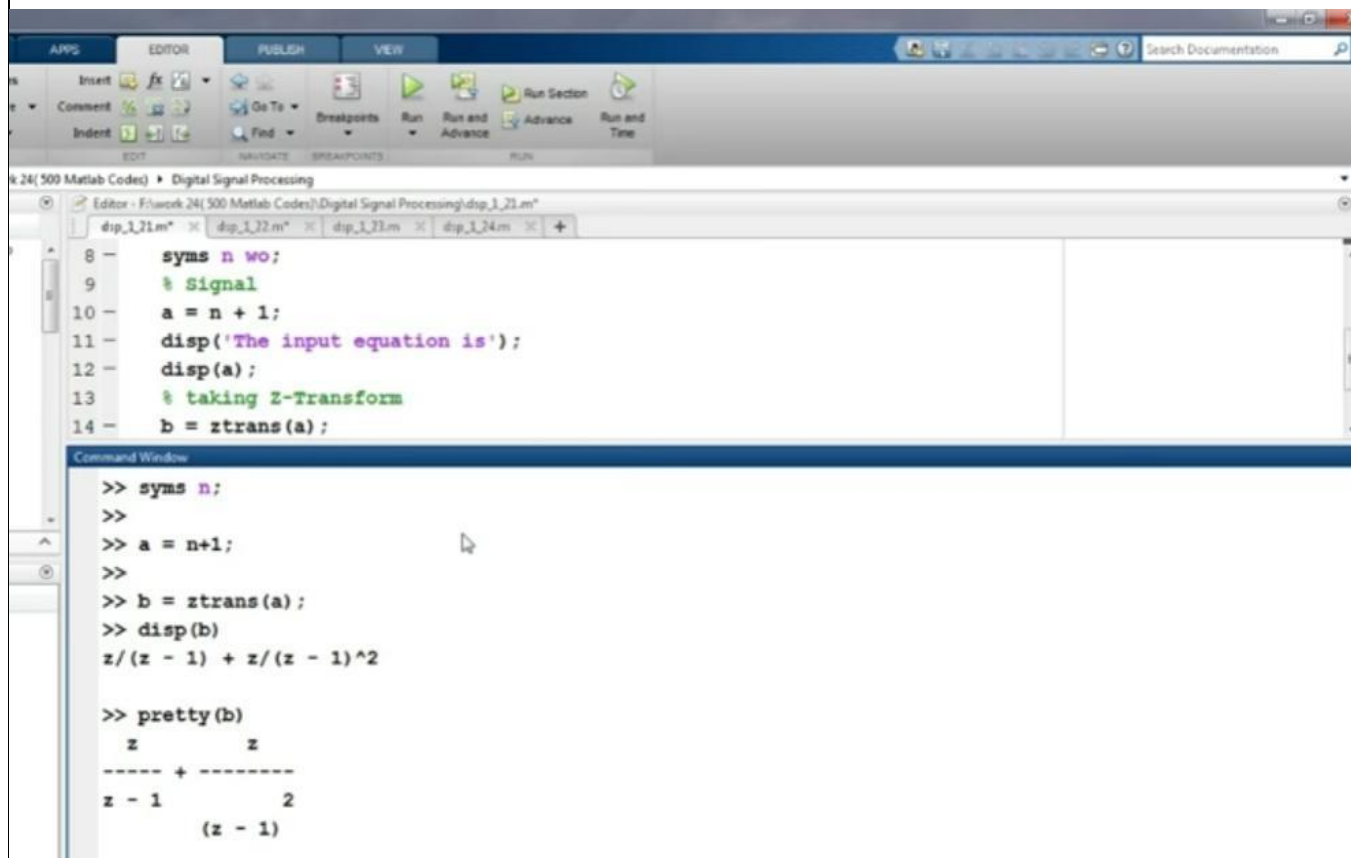
If $u_1(n), u_2(n), \dots, u_k(n)$ be k independent solutions of the equation $y_{n+k} + a_1 y_{n+k-1} + \dots + a_k y_n = 0$,

then $U_n = c_1 u_1(n) + c_2 u_2(n) + \dots + c_k u_k(n)$,

is called **complete solution**.

If V_n is a particular solution of equation (1), then the complete solution of (1) is $y_n = U_n + V_n$.

The part U_n is called the complementary function and the part V_n is called the particular integral.



The screenshot shows the MATLAB environment. The Editor window displays a script named `dsp_1_21.m` with the following code:

```
8 - syms n wo;  
9 - % Signal  
10 - a = n + 1;  
11 - disp('The input equation is');  
12 - disp(a);  
13 - % taking Z-Transform  
14 - b = ztrans(a);
```

The Command Window shows the execution results:

```
>> syms n;  
>>  
>> a = n+1;  
>>  
>> b = ztrans(a);  
>> disp(b)  
z/(z - 1) + z/(z - 1)^2  
  
>> pretty(b)  
      z      z  
----- + -----  
z - 1      (z - 1)^2
```

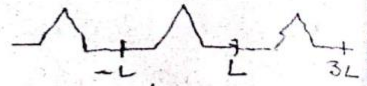

Report – Report can be typed or hand written for up to two pages.

Fourier Transform

$$\omega_L = \frac{k\pi}{L} = k\Delta\omega \quad \Delta\omega = \frac{\pi}{L}$$

$$f(x) = \sum_{k=-\infty}^{\infty} C_k e^{ik\pi x/L}$$

$$C_k = \frac{1}{2\pi} \langle f(x) | \psi_k \rangle = \frac{1}{2L} \int_{-L}^L \frac{f(x)}{\psi_k} e^{-jk\pi x/L} dx$$



$$\begin{aligned} f(x) &= \lim_{\Delta\omega \rightarrow 0} \sum_{k=-\infty}^{\infty} \frac{\Delta\omega}{2\pi} \int_{-\pi/\Delta\omega}^{\pi/\Delta\omega} f(\xi) e^{-ik\Delta\omega\xi} d\xi \\ &= \int_{-\infty}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\xi) e^{-i\omega\xi} d\xi e^{i\omega x} d\omega \end{aligned}$$

$$\begin{aligned} f(\omega) &= F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx \\ f(x) &= F(f(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{i\omega x} d\omega \end{aligned}$$

Fourier Transform and Derivative

$$U_{tt} = (U_{xx}) \Rightarrow \begin{cases} \text{ODE} \\ \text{PDE} \end{cases} \quad \hat{U}_{tt} = -\omega^2 \hat{U}$$

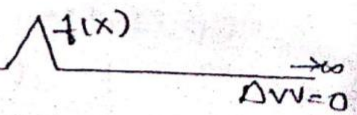
$$\begin{aligned} \mathcal{F}\left(\frac{d}{dx} f(x)\right) &= \int_{-\infty}^{\infty} \frac{d}{dx} f(x) e^{i\omega x} dx \\ &= \left[f(x) e^{i\omega x} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(x) (-i\omega e^{i\omega x}) dx \\ &= i\omega \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx = i\omega \mathcal{F}(f(x)) \end{aligned}$$

Fourier Transform and Convolution Integrals

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x-z)g(z) dz$$

Fourier Transform

$$\hat{f}(\omega) = F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx$$



$$f(x) = \mathcal{F}^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x} d\omega$$

$$F(f * g) = F(f) F(g) = \hat{f} \hat{g}$$

$$\begin{aligned} \mathcal{F}^{-1}(\hat{f} \hat{g})(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \hat{g}(\omega) e^{i\omega x} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \left(\int_{-\infty}^{\infty} g(y) e^{-i\omega y} dy \right) e^{i\omega x} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} g(y) \left(\int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega(x-y)} d\omega \right) dy \end{aligned}$$

Intuition of Fourier Transform & Laplace Transform

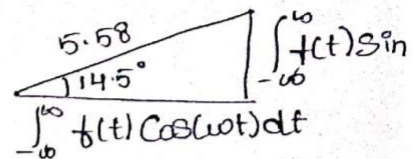
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cos(\omega t) dt - i \int_{-\infty}^{\infty} f(t) \sin(\omega t) dt$$

$$\omega = 2.853$$

Fourier: $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$

Laplace: $F(s) = \int_0^{\infty} f(t) e^{-(\sigma + i\omega)t} dt$



$$F(s) = \int_0^{\infty} f(t) e^{-i\omega t} e^{-\sigma t} dt$$

Transform and Laplace Transform first order

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Eg: $F(s) = \int_0^{\infty} e^{at} e^{-st} dt = \left[\frac{e^{(a-s)t}}{a-s} \right]_0^{\infty} = \frac{1}{s-a} = F(s)$

Implementation of Laplace Transform using matlab

Application of Z-Transform

$$y_{n+1} + y_n = 2$$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta y_{n+1} = y_{n+2} - y_{n+1}$$

$$\Delta y_{n+1} + y_n = 2$$

$$y_{n+2} - y_{n+1} + y_n = 2$$

$$\Delta y_{n+1} + \Delta^2 y_{n-1} = 1$$

$$y_{n+2} - y_{n+1} + \Delta(\Delta y_{n-1}) = 1$$

$$y_{n+2} - y_{n+1} + y_{n+1} - 2y_n + y_{n-1}$$

Find the Z-Transform of Sequence using Matlab

