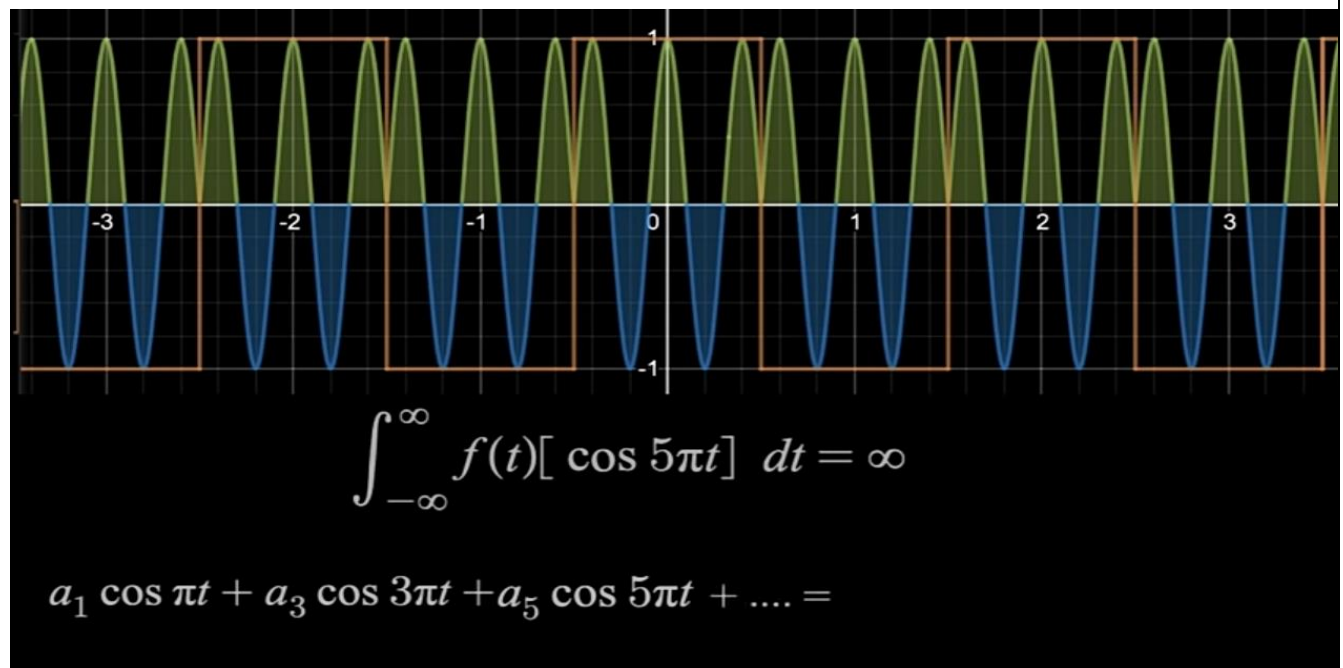
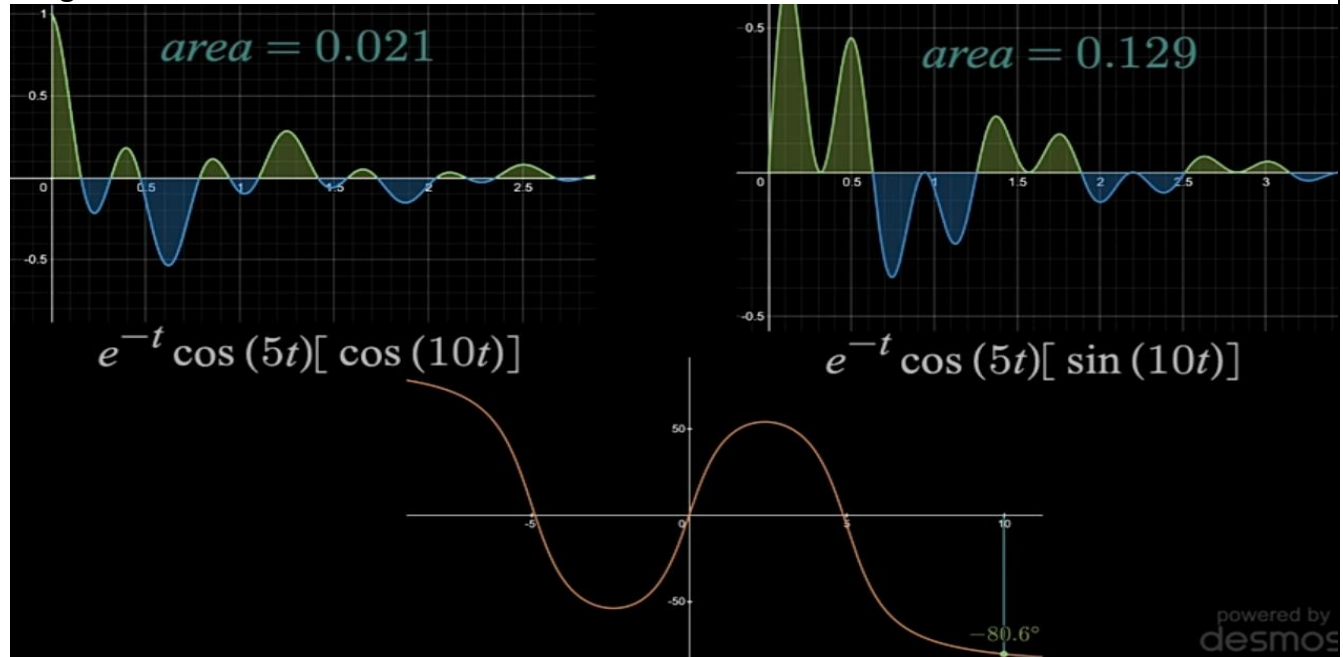


DAILY ASSESSMENT FORMAT

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|----------------|----------------------------|--------------------------------|---------------------|
| Date: | 21 st July 2020 | Name: | Soundarya NA |
| Course: | Coursera | USN: | 4AL16EC077 |
| Topic: | Digital Signal Processing | Semester & Section: | 8 th - B |

Image:



Report:

The problem:

Given the Fourier transform of a general function, find the Fourier transform of its derivative. Use this result to find the Fourier transform of a window function out of the Fourier transform of an antisymmetric pair of delta functions.

The solution:

We are given the following:

$$F.T[f(t)] = F(\omega),$$

and we take into account that:

$$\lim_{t \rightarrow \pm\infty} f(t) \rightarrow 0.$$

We begin by writing explicitly:

$$F.T[f'(t)] = \int_{-\infty}^{\infty} f'(t) e^{i\omega t} dt.$$

Integration by parts gives us:

$$f(t)e^{i\omega t} \Big|_{-\infty}^{+\infty} - i\omega \int_{-\infty}^{\infty} f(t)e^{i\omega t} dt = -i\omega F(\omega),$$

and we get:

$$F.T[f'(t)] = -i\omega F(\omega).$$

Let us represent a window function in the region $[-a, a]$ as a sum of two step functions:

$$\Pi(t) = \Theta(t + a) - \Theta(t - a).$$

We also note that $\delta(t + a) = \frac{d}{dt}\Theta(t + a)$.

Now, using what we have derived earlier we find:

$$F.T[\delta(t + a) - \delta(t - a)] = F.T\left[\frac{d}{dt}\Theta(t + a) - \frac{d}{dt}\Theta(t - a)\right] = -i\omega F.T[\Theta(t + a) - \Theta(t - a)] = -i\omega F.T[\Pi(t)], \text{ and,}$$

$$F.T[\delta(t + a) - \delta(t - a)] = \int_{-\infty}^{\infty} \delta(t + a)e^{i\omega t} dt - \int_{-\infty}^{\infty} \delta(t - a)e^{i\omega t} dt = e^{-i\omega a} - e^{i\omega a} = -2i\sin(\omega a).$$

$$\Rightarrow F.T[\Pi(t)] = 2a\text{sinc}(\omega a)$$

In the remainder of the course, we'll study several methods that depend on analysis of images or reconstruction of structure from images:

- Light microscopy (particularly fluorescence microscopy)
- Electron microscopy (particularly for single-particle reconstruction)
- X-ray crystallography

The computational aspects of each of these methods involve Fourier transforms and convolution. These concepts are also important for:

- Some approaches to ligand docking (and protein-protein docking)
- Fast evaluation of electrostatic interactions in molecular dynamics
- (You're not responsible for these additional applications)

Calculate the Laplace Transform using Matlab Calculating the Laplace $F(s)$ transform of a function $f(t)$ is quite simple in Matlab.

First you need to specify that the variable t and s are symbolic ones. This is done with the command `>> syms t s` Next you define the function $f(t)$.

The actual command to calculate the transform is `>> F=laplace(f,t,s)` To make the expression more readable one can use the commands, `simplify` and `pretty`. here is an example for the function $f(t)$, $f(t) = -1.25 + 3.5t \exp(-2t) + 1.25 \exp(-2t)$

```
>> syms t s
>> f=-1.25+3.5*t*exp(-2*t)+1.25*exp(-2*t);
>> F=laplace(f,t,s)
F =
-5/4/s+7/2/(s+2)^2+5/4/(s+2)
>> simplify(F)
ans =
(s-5)/s/(s+2)^2
>> pretty(ans)
s - 5 ----- 2 s (s + 2)
```

which corresponds to

$$F(s) = \frac{s-5}{s(s+2)^2} = \frac{1}{s} - \frac{5}{s+2} + \frac{5}{(s+2)^2}$$

Alternatively, one can write the function $f(t)$ directly as part of the laplace command:

```
>> F2=laplace(-1.25+3.5*t*exp(-2*t)+1.25*exp(-2*t))
```

