DAILY ASSESSMENT FORMAT

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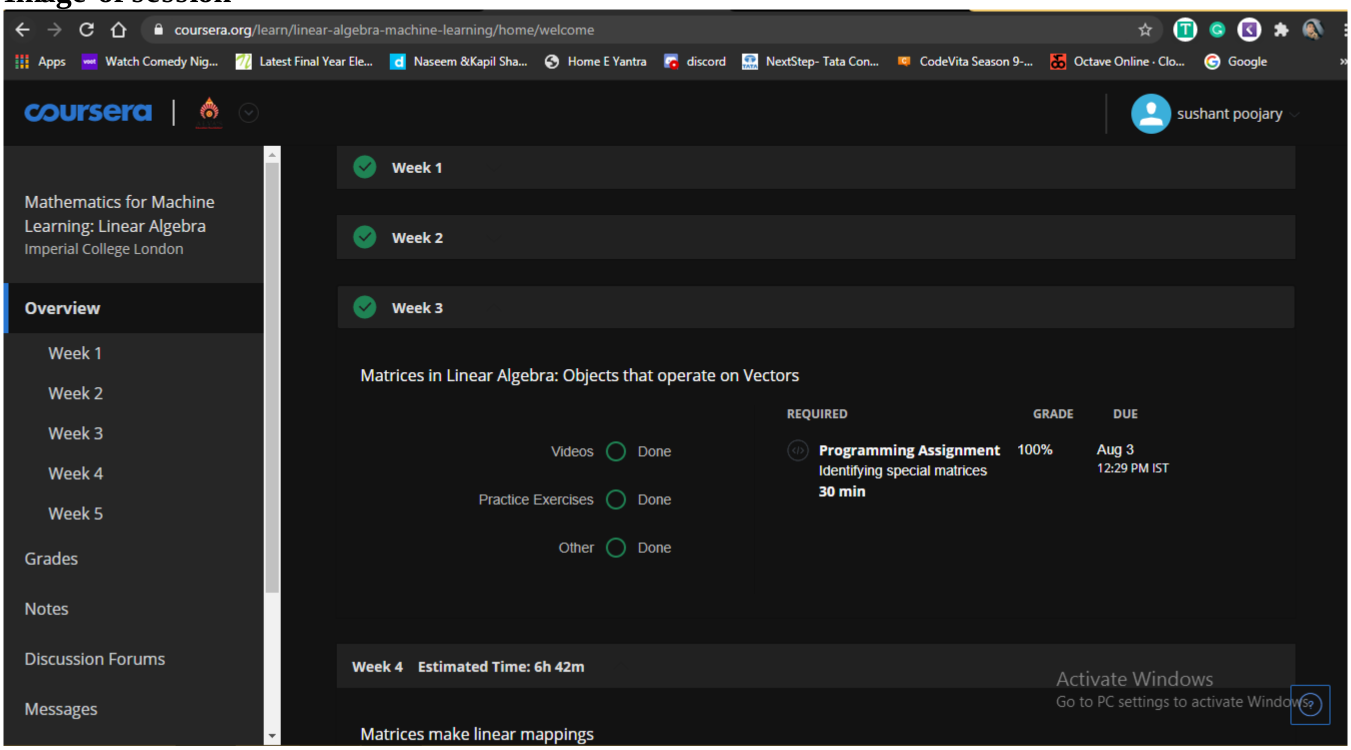
FORENOON SESSION DETAILS
Image of session



Report –

So what we've done in this last video in this module is look at the determinant, how much we grow space, the area change. We've also looked at the special case where the determinant is zero and found that that means that the basis vectors aren't linearly independent, and then that means that the inverse doesn't exist. So in this first module on matrices, what we've done is define what a matrix is, it's something that transforms space. We've looked at different archetypes of matrices, like rotations, and inverses, and stretches, and shears, and how to combine them by doing successive transformations, matrix multiplication or composition. Then, we've looked at how to solve systems of linear equations by elimination and how to find inverses. And then finally, we've looked at determinants and linear independence. And next week, we'll carry on and look at matrices in some more detail..

In the final video in this module, we're going to look at the property of a matrix called the determinant. More happens when the matrix doesn't have linearly independent basis vectors, picking up the basis vectors we looked at in the last module. Let's go back and look at a simple matrix like a,o, o, d. What this matrix does, is it scales space. So if we have our original basis vectors e1 hat and e2 hat, it scales them out by a factor of a in this direction to a,o, and by a factor of d in this direction to o,d, and we call those e1 prime and e2 prime. Now, see what I've done to the space. It was originally this size one-by-one. What I've done is I've scaled the space every vector in the space by a factor of a this way and by a factor of d this way and therefore, I have scaled the space by a factor of ad. All areas in the space everything's got bigger by a factor of ad. I call that the determinant of the transformation matrix. Now, if I instead have a matrix which is a,o b, d instead the same with a b, what that does is e1 hat stays as it was, it scaled by a factor of a, but e2 hat goes somewhere different, so e1 hat goes to a,o, but e2 goes to bd somewhere like this bd. Now, see what's happened to the space, I've gone from being something like this, to being something like this I've made a parallelogram, but the area of the parallelogram is still bad. It's base times its perpendicular height. So the determinant is still ad, of this transformation matrix. Now, if I have a general matrix a, c, b, d and it sounds the original unit square into a parallelogram like this. Where you saw this factor here. Then to find the area of this parallelogram I'm going to have to do a bit of maths. So I'm gonna do that and then we'll have a look at it in a moment.

So I've done the maths and it's here, you can do geometry and puzzle out for yourself and pull it if you'd like to verify if I'm correct, but the area of the parallelogram here is actually ad minus bc. I'm going to denote the operation of finding that area with vertical lines like this and we call that finding the determinant of a, which you probably saw in school. In school, when you looked at matrices you probably saw that you could find the inverse of a matrix like a, b, c, d, by flipping the terms on the leading diagonal, and taking the minus of the off-diagonal terms for two by two. So let's do that and see what we get. So when we multiply that out, we get ad minus bc, interesting. Then on this element here is a minus b plus ba. So I've got a minus b plus ba so that's zero. When we do this term we get cd minus cd. When we do this term here, we get cb times minus b times minus bc plus ad. So that's ad minus bc, interesting. So that's the determinant. So if I multiply by 1 over the determinant ad minus bc, then these terms will become one and I'll get the identity matrix. So if this was matrix a, and I think if this and this together as being a_-1, then I've got this is in fact when I pre-multiply by one over the determinant, is in fact the inverse of a. So we proved here that the inverse you learned in school, is in fact correct. But the interesting thing is that this determinant there, this now that it scales space. If we then take this matrix when we do the flipping around we haven't changed it's scaling of space. We need to undo that scaling and bring it back down to a scale of one. So the determinant here is what we need to divide the inverse matrix by in order for it to probably be an inverse. Now we could spend

another video looking at the extension of the idea of well actually the form to find out how to find the determinants in the general case computationally. But this is both tricky to show and it's pointless. Knowing how to do the operations isn't a useful skill anymore because we just type det A into our computer, and polyphenol MATLAB will do it for us.
From a learning perspective, it doesn't add much to the val echelon. Val echelon does actually we slowly went through it. I'm not going to teach you how to find determinants in the general case. If you want to know, lookup QR decomposition then follow that through and that's how computation you go and find it out, over the linear algebra test book is the other place to look. That's how you do it in the general case. Now let's think about this matrix A here. It transforms e1 and e2 hats to two points on the same line. It transforms e1 hat to 1,1, and it transforms e2 hat from there over to 2,2. They're both points on the same line. They are just a multiple of each other; they're not linearly independent. What this matrix in fact does, is it transforms every point in space on a line. Notice that the tone of that matrix is going to be 0, if I take a, d minus b, c determinant of a is nought because the any area was gone onto a lot and therefore that area is nought. So having computed by the geometrically or computationally, you get a determinant of nought. So if I had a three by three matrix with a similar situation describing a 3D space, and if I had the same position where one of the new basis vectors was just a linear multiple of the other two, it wasn't linearly independent, then now would mean the new space was either a plane or if there was only one independent basis vector, a line like we have here. In either case the volume enclosed would be zero, so the determinant would be zero. Now, let's turn back to our val echelon form and apply that idea. Let's take this set of simultaneous equations, say.
You can see that the third row is just the sum of the first two. So row three is equal to row one plus row two. You can see that column three is just given by two of column one plus column two. If you want to pause for a moment to verify that it really is true. So this transformation matrix doesn't describe three independent basis vectors. One of them is linearly dependent on the other two. So this doesn't describe any 3D space it collapses it into a 2D space. So let's see what happens when I try to reduce this val echelon form. If I take off the first row from the second, okay so far so good, I've got that my a,b,c stays the same, and I take the first one off the second one. So I've got 12, take 12 of 17, I get five. If I then take the first and second ones off the third one, I've now got zeros everywhere here. If I do that on here I take 12 and 17 of 29 I get zero here. So now I've got zero C equals zero, which is sort of true but not useful. Then an infinite number of solutions for C in effect, any value of C would work. So now I can't solve my system of equations anymore, I don't have enough information. So if we think about this from solving simultaneous equations point of view, my mistake was when I went into the shop to buy apples, bananas, and carrots, the third time I went in I just ordered a copy of my first two orders. So I didn't get any new information and I don't have enough data therefore to find out the solution for how much I individual apples and bananas and carrots cost. So what we've shown is that where the basis vectors describing the matrix are linearly independent, then the determinant is zero, and that means I can't solve the system of simultaneous equations anymore. Which means I can't invert the matrix because I can't take one over the determinant either. That means I'm stuck this matrix has no inverse. So there are situations where I might want to do a transformation that collapses the number of dimensions in the space but that will come at a cost. Another way of looking at this is that the inverse matrix lets me undo my transformation, it lets me get from the new vectors to the original vectors. If I've done two dimensions by turning a 2D space into a line, I can't undo that anymore, I don't have enough information because I've lost some of it during the transformation, I've lost that extra dimension. So in general, it's worth checking before you propose a new basis vector set and then use a matrix to transform your data vectors, that this is a transformation you can undo, by checking that the new basis vectors are linearly independent. So what we've done in this last video in

this module is look at the determinant, how much we grow space. We've also looked at the special case where the determinant is zero, which means that the basis vectors are linearly independent, which means the inverse doesn't exist. In this first module on matrices what we've done so far is define what a matrix is as something that transformed space. We've looked at different arch terms of matrices like rotations and inverse [inaudible] and shears and how to combine them by doing successive transformations. We've looked at how to solve systems of linear equations, by elimination and how to find inverses. Then finally, we've looked at determinants and linear independence.

Summary :
So what we've done in this last video in this module is look at the determinant, how much we grow space, the area change. We've also looked at the special case where the determinant is zero and found that that means that the basis vectors aren't linearly independent, and then that means that the inverse doesn't exist. So in this first module on matrices, what we've done is define what a matrix is, it's something that transforms space. We've looked at different archetypes of matrices, like rotations, and inverses, and stretches, and shears, and how to combine them by doing successive transformations, matrix multiplication or composition. Then, we've looked at how to solve systems of linear equations by elimination and how to find inverses. And then finally, we've looked at determinants and linear independence. And next week, we'll carry on and look at matrices in some more detail.