MATHEMATICS

TOPIC : MATHEMATICAL INDUCTION,SEQUECES AND SERIES, MATHEMATICAL REASONING

1) Let P(n): $2^n < (1x\ 2\ x3\ x4\ x\ldots.x\ n)$. Then the smallest positive integer for which P(n) is true is

a) 1 b) 2 c) 3 d) 4

2) If P (n): "$2.4^{2n+1}+3^{3n+1}$ is divisible by λ for all $n \in N$ is true, then the value of λ is

a) 3 b) 11 c) 209 d) 5

3) If P (n): "$49^n + 16^n + k$ is divisible by 64 for $n \in N$" is true, then The least negative integral value of k is

a) -1 b) -2 c) -3 d)-4

4) The smallest positive integer n for which $n! < \left(\frac{n+1}{2}\right)^n$ holds is,

a) 1 b) 2 c) 3 d) 4

5) The greatest positive integer which divides (n+1) (n+2)….(n+r) for all $n \in N$ is,

a) r b) r! c) (n+r) d) (r+1)!

6) The inequality $n!>2^{n-1}$ is true

a) for all $n \in N$ b) for all n>1

c) for all n>2 d) for no $n \in N$

7) The sum of the cubes of three successive natural numbers is divisible by,

a) 6 b) 9 c) 27 d) 8

8) If $10^n+3.\ 4^{n+2} +k$ is divisible by 9 for all $n \in N$, then the least positive integer value of k is

a) 5 b) 3 c) 7 d) 1

9) The number $(49^2 - 4)\ (49^3 - 49)$ is divisible by

a) 5! b) 6! c) 9! d) 7!

10) If $x^n – 1$ is divisible by x-k , then the least positive integral value of k is

a) 1 b) 2 c) 3 d) 4

11) If P (n): 2n < n! , $n \in$ N, then P (n) is true for all $n \geq$

a) 1 b) 2 c) 3 d) 4

12) If (a-1) is the G.M between (a-2) and (a+1) then a =

a) 2 b) 3 c) 4 d) 1

13) $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+.........+\frac{1}{(3n-1)(3n+2)}=$

a) $\frac{n}{6n+3}$ b) $\frac{n}{6n-4}$ c) $\frac{n+1}{6n+4}$ d) $\frac{n}{6n+4}$

14) The 50^{th} term of the series 2+3+6+11+18+……… is

a) $(49)^2-1$ b) $(49)^2+2$ c) 50^2 d) $(50)^2-1$

15) The 12^{th} element from the end of AP 3, 8, 13… 253 is

a) 190 b) 194 c) 198 d) 200

16) The sum of all two digit numbers which when divided by 4, yield unity as remainder is

a) 1012 b) 1201 c) 1212 d) 1210

17) Sum of all integers between 100 and 200 which are not divisible by 2 is

a) 7000 b) 7550 c) 7500 d) 7250

18) The sum of the series $15^2 + 16^2 + \ldots\ldots\ldots\ldots + 30^2$ is

a) 7440 b) 8440 c) 6220 d) 4220

19) The nth term of 3 + 13 + 29 + 51 + 79 + …………. is

a) $3n^2 + n + 1$ b) $3n^2 + n - 1$ c) $4n^2 - 1$ d) $3n^2 - n + 1$

20) The sum $9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \times ....\infty$ is

a) 3 b) 6 c) 9 d) 12

21) If second, third and sixth terms of an AP are consecutive elements of a GP, then common ratio of the GP is

a) 1 b) -1 c) 3 d) -3

22) Let P (n) denote the statement that n^2+n is odd. It is seen that P(n) => P(n+1) . P (n) is true for all

a) n >1 b) n c) n>2 d) none of these

23) The sum to n terms of a series is $2^{n+1} + n - 2$, then the n^{th} term is

a) 2n + 1 b) $3^n - 1$ c) $2^n + 1$ d) $3^n + 1$

24) If m, n are any two odd positive integers with m > n then the largest positive integers which divides all the numbers of the type $m^2 - n^2$ is

a) 4 b) 6 c) 8 d) 9

25) $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...is \frac{n(n+1)^2}{2}$ where n is even . When n is Odd , the sum is

a) $\frac{n(n+1)}{2}$ b) $\frac{n^2(n+1)}{2}$ c) $\frac{n(n+1)^2}{2}$ d) $\left[\frac{n(n+1)}{2}\right]^2$

26) The 10^{th} common term between the series 3+7+11+…… and 1+6+11+…. is

a) 191 b) 193 c) 211 d) 181

27) If a clock strikes appropriate number of times at each hour . Then the number of times it strikes in one full day is

a)78 b)156 c) 144 d) 72

28) $\frac{1.2^2 + 2.3^2 + ... + n.(n+1)^2}{1^2.2 + 2^2.3 + ... + n^2.(n+1)} =$

a) $\frac{(n+1)}{n}$ b) $\frac{3n+4}{3n+1}$ c) $\frac{3n+7}{3n+2}$ d) $\frac{3n+5}{3n+1}$

29) The minimum value of $4^{x} + 4^{1-x}$, x €R is

a) 2 b) 4 c) 1 d) 0

30) "The diagonals of a rhombus are perpendicular" The contra positive of this statement is

a) If the figure is not a rhombus ,then its diagonals are not perpendicular

b) If the diagonals are perpendicular ,then the figure is a rhombus

c) If the diagonals are not perpendicular ,then the figure is a rhombus

d) If the diagonals are not perpendicular ,then the figure is not a rhombus

31) The inverse of "if $x \in A \text{I} B$ then x ЄA or xϵ B" is

a)if $x \notin A \cap B$ then $x \notin A$ & $x \notin B$ b) if $x \notin A \cap B$ then $x \notin A$ or $x \notin B$

c) if $x \notin A$ or $x \notin B$ then $x \notin A \cap B$ d)none

32) The contra-positive of the inverse of $p \rightarrow \sim q$

a) $\sim q \rightarrow \sim p$ b) $\sim p \rightarrow \sim q$ c) $\sim q \rightarrow p$ d) $p \rightarrow q$

33) Negate the following proposition :" If it rains heavily , the college is closed But the students do not go home"

a) It rains heavily and either the college is not closed or the students go home

b) It does not rain heavily and the college is closed or the students go home

c) It does not rain heavily , the college is neither closed nor the students go home

d) None of these

34) If p ,q ,r have truth values T,F,T respectively , which of the following is true ?

a)$(p \Rightarrow q) \wedge r$ b) $(p \Rightarrow q) \wedge \sim r$ c)$(p \wedge q) \wedge (p \vee r)$ d) $q \Rightarrow (p \wedge r)$

35) The truth value of the contra positive of the statement " If $x \in A$, $x \in B$ then $x \in A \cap B$" is

a) T b) F c) no conclusion d) None

36) Let p:2+3=5 ; q= $\sqrt{2}$ is irrational. The symbolic form of the statement " It is not true that 2+3=5 iff $\sqrt{2}$ is irrational" ,is

a) $\sim p \leftrightarrow q$ b) $\sim p \leftrightarrow \sim q$ c) $\sim(p \leftrightarrow \sim q)$ d) $\sim(p \leftrightarrow q)$

37) Which of the following is not logically equivalent to the proposition:

"A real number is either rational or irrational"

a) If a number is neither rational nor irrational then it is not real

b) If a number is not rational or not an irrational then it is not real

c) If a number is not real , then it is neither rational nor irrational

d) If a number is real , then it is rational or irrational

38) Negation of "2+3=5 and 8<10" is

a) $2 + 3 \neq 5$ and $8 < 10$ b) $2 + 3 = 5$ and $8 \not< 10$

c) $2 + 3 \neq 5$ or $8 \not< 10$ d) None of these

39) The contrapositive of $(p \vee q) \Rightarrow r$ is

a) $r \Rightarrow (p \vee q)$ b) $\sim r \Rightarrow (p \vee q)$

c) $\sim r \Rightarrow (\sim p \wedge \sim q)$ d) $p \Rightarrow (q \vee r)$

40) If $(p \wedge \sim r) \rightarrow (\sim p \vee q)$ is false , then truth values of p , q , r are respectively

a) F ,F & T b) T, F & F c) T, F & T d) F, T & T

41) The negation of $p \vee (q \wedge \sim r)$ is

a) $p \wedge (\sim q \vee r)$ b) $\sim p \wedge (\sim q \wedge r)$ c) $\sim p \wedge (\sim q \vee r)$ d) None

42) Contrapositive of the inverse of the proposition "if I am Ok, then everybody is Ok" is

a) If everybody is Ok ,then I am Ok

b) If everybody is not Ok , I am not Ok

c) If I am not Ok, then everybody is not Ok

d) If I am Ok, then everybody is Ok

43) The negation of the proposition " If $x \in A \cap B$, then $x \in A$ and $x \in B$" is

a) $x \notin A \cap B, then\ x \notin A$ and $x \notin B$ b) $x \in A \cap B$ and $x \notin A$ or $x \notin B$

c) If $x \in A \cap B, then\ x \notin A$ and $x \notin B$ d) $x \in A \cap B$ and $x \notin A$ and $x \notin B$

44) The third term of a GP is 4 , the product of the first five terms is

a) 64 b)1024 c)256 d) 512

45) The sum to n terms of an AP is n(n+3), the common difference is

a) 1 b) 2 c) 3 d) 4

46) The nth term of the series $2\cdot1^2\cdot3^3+3\cdot2^2\cdot4^3+\ldots$ Is

a) $(n^2+3n+2)(n^2+2n)^2$ b) $(n^2+3n+2)(n^2+2n+1)^2$

c) $(n^2+3n)(n^2+2n+1)^2$ d) None of these

47) Sum to n terms of the series is $1-\frac{1}{(n+1)!}$ The 20^{th} term is

a) $\frac{20}{21!}$ b) $\frac{19}{20!}$ c) $\frac{21}{22!}$ d)None of these

48) If the sum of n terms of the series $2^3+4^3+6^3+\ldots\infty$ is 3528 then n=

a) 6 b) 7 c)8 d)9

49) $\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+....n \text{ terms}=$

a) $\frac{2}{n+1}$ b) $\frac{2n}{n+1}$ c) $\frac{n}{n+1}$ d) $\frac{1}{n+1}$

50) $1^2+1+2^2+2+3^2+3+\ldots n^2+n=$

a) $\frac{n^2(n+1)^2}{4}$ b) $\frac{n(n+1)(n+2)}{6}$ c) $\frac{n(n+1)(n+2)}{3}$ d) $\frac{n(n+1)}{2}$

51) If $\left(\sum n^3\right)\left(\sum n\right)=\left(\sum n^2\right)^2$ then

a) n=3 b) n=-1 c) $n^2=3$ d) n=1

52) If the sum of first n terms of a series is $5n^2+2n$ then its second term is

a) 7 b) 27 c) 24 c) 17

53) The fourth ,seventh and tenth terms of a G.P are p, q and r respectively then Which one of the following is true ?

a)$P^2=q^2+r^2$ b) $p^2= qr$ c) $q^2=pr$ d) $r^2=p^2+q^2$

54) 7th term of an A.P is 40 . The sum of the first 13 terms is

a) 520 b) 53 c) 2080 d) 1040

55) If p(n) = 2+4+6+…….+2n ,n€N ,then P(k) = K(K+1)+2 =>P(K+1) = (K+1)(K+2)+2 For all n€N .So we can conclude that P(n)=n(n+1) +2 for

a) All n€ N b) n>1 c) n>2 d) nothing can be said

56)The sixth term of a H.P is 1/61 and 10th term is 1/105 . Then the first term of that H.P is

a) 1/17 b) 1/6 c) 1/39 d) 1/28

57) The sum to infinity of the progression $9 - 3 + 1 - \frac{1}{3} + \ldots\ldots$ is

a) 27/4 b) 15/2 c) 9 d) 9/4

58) $S_n = \frac{1}{6.11}+\frac{1}{11.16}+\frac{1}{16.21}+\ldots\ldots\ldots$ .to n terms , then $6S_n=$

a) $\frac{n}{5n+6}$ b) $\frac{5n-4}{5n+6}$ c) $\frac{1}{5n+6}$ d) $\frac{2n-1}{5n+6}$

59) The sum to n terms of the series $1^2+(1^2+2^2)+(1^2+2^2+3^2)+\ldots\ldots$

a) $\frac{n(n+1)(n+2)}{6}$ b) $\frac{n(n+1)^2(n+2)}{12}$ c) $\frac{n(n+1)(n+2)^2}{12}$ d) $\frac{n(n+1)}{2}$ 60)

60) The 99^{th} term of the series 2 + 7 + 14 + 23 + 34 + ……………. is

a) 9999 b) 9998 c) 10000 d) None

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Solutions:

1) Ans: d P(1),P(2),P(3) all are false. P(4) is true
2) Ans:b for n =1, P(n)= 209 , For n =2 , P(n) = 4235
   HCF of 209 and 4235 is 11

   So, P(n) is divisible by 11. Hence $\lambda = 11$

3) Ans:a for n=1 ,p(1) :65 +k is divisible by 64
   Thus,k should be -1

4) Ans: c
5) Ans: b Product of r consecutive integers is divisible by r!
6) Ans: c Put n=1,2,3,…. Verify
7) Ans: b
   Let the numbers be n, n+1, n+2. Sum of the cubes of successive natural Numbers is $n^3 + (n+1)^3 + (n+2)^3$. Put n=1,2,…. And verify
8) Ans:a put n=1 ,202+k is divisible by 9 if k = 5
9) Ans : a product of 5 consecutive numbers
10) Ans : a $x^n - 1^n$ is a multiple of x-1
11)Ans: d
12Ans: b $(a-1)^2=(a-2)(a+1)$ => a=3
13)Ans:d put n=1
14) Ans:b successive differences are in AP .
   using method of differences solve it .
15) Ans: c a=253 , d = -5 ,t_{12}=253+12x-5=198
16) Ans:d
17) Ans:c sum =101+103+105+….+199= 50/2[101+199]
18) Ans:b $\sum 30^2 - \sum 14^2$
19) Ans:b put n=2

20) Ans:a $9^{\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots\infty} = 9^{\frac{1/3}{1-1/3}}$

21) Ans:c a+d, a+2d, a+5d are in GP => d= -2a

22) Ans: d

23) Ans:c $T_n=S_n-S_{n-1}$

24) Ans:c $m^2 - n^2 = (2n+3)^2-(2n+1)^2$

25) Ans:b put n=3

26) Ans:a first common term = 11=a , d=LCM(4,5) =20 , T_{10} =11+9x20=191

27) Ans:b 2(1+2+3+….12) = $2\sum 12$

28) Ans:d put n=2

29) Ans:b AM $\geq$ GM

30) Ans:d

31) Ans:a

32) Ans:c

33) Ans:a

34) Ans:d

35) Ans:a

36) Ans:d

37) Ans:b

38) Ans:c

39) Ans:c

40) Ans:b

41) Ans:c

42) Ans: a

43) Ans: b

$p: x \in A \cap B$, $q: x \in A$, $r: x \in B$

Then given statement is $p \to (q \wedge r)$

Then $\sim [p \to (q \wedge r)] = p \wedge (\sim q \vee \sim r)$

44) Ans:b $ar^2=4$, $a.ar.ar^2.ar^3.ar^4 = 4^5$

46) Ans: a

nth term = (n+1) $n^2(n+2)^3 = (n^2+3n+2)(n^2+2n)^2$

47) Ans: a

$S_n = 1 - \frac{1}{(n+1)!}$, $S_{n-1} = 1 - \frac{1}{n!}$ and $t_n = S_n - S_{n-1} = \frac{n}{(n+1)!}$ and $t_{20} = \frac{20}{21!}$

48) Ans: a

$t_n = (2n)^3 = 8n^3; S_n = 8\sum n^3 = \frac{8n^2(n+1)^2}{4} = 2n^2(n+1)^2$

$2n^2(n+1)^2 = 3528, n^2(n+1)^2 = 1764$, n(n+1)=42 ,

$n^2 + n - 42 = 0$ => n=6 or n= -7 , but n≠-7

49) Ans: b

Put n=1, n=2 and verify

When n=1 , $\frac{1}{1^3}$=1 2) gives answer 1

When n=2 , $\frac{1}{1^3}+\frac{1+2}{1^3+2^3}=1+\frac{1}{3}=\frac{4}{3}$ option 2) gives $\frac{4}{3}$

50) Ans: c

(1+2+3+……+n)+($1^2 + 2^2$+…..+n^2)= $\frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(n+2)}{3}$

51) Ans: d

$(\sum n^3)(\sum n) = (\sum n^2)^2$,simplifying we get $\frac{n(n+1)}{2} = \frac{(2n+1)^2}{9}$, clearly this holds good for n=1

52) Ans:c $T_2 = S_2 - S_1$

53) Ans:c $ar^3=p$,$ar^6=q$, $ar^9=r$ => $pr= a^2r^{12}=q^2$

54)Ans:a a+6d =40 S_{13}=13/2[2a+12d]

55) Ans:d

56) Ans:b a+5d =61 and a+9d =105 , solve it

57) Ans:a

58) Ans:a use result: $\frac{1}{a(a+d)} + \frac{1}{(a+d)(a+2d)} + ...to\, n\, terms = \frac{n}{a(a+nd)}$

59) Ans:b put n=2

60) Ans:b using method of differences

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