Vaccination Effect in a Many-strain Model of Influenza Drift

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Abstract

By introducing the general vaccination strategy composing of new-born immunization(at rate ϕ_{new}) and the vaccination of currently susceptible (at rate ϕ_{pulse}) into the many-strain SIR epidemiological model (Gog and Grenfell 2002), a simple yet robust many-strain SIRV epidemiological model is developed for understanding the impact of vaccination at the many-strain antigenic evolution.

Keywords: Influenza, Vaccination, Immunity, Mutation

1. Introduction

Vaccination is one of the major medical advances in fighting various virus in recent centuries. Currently, influenza vaccination is a powerful tool in the global-health control arsenal, and allows for the mass prevention of infection. Various elements of mathematics are used throughout the vaccine development process. In this paper, we focus on the use of mathematics in influenza virus drift, understanding the impact of vaccination at the many-strain antigenic evolution.

2. Model

Mathematically, based on the many-strain SIR epidemiological model (Gog and Grenfell 2002), which specifies the dynamics of large number of

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antigenic types with cross-immunity interaction, and introduced the general vaccination strategy composing infants (a proportion ϕ_{new} of all children are assumed to be immunized) and the random vaccination of individuals in the population (at rate ϕ_{pulse}, although this vaccination will only affect those that are currently susceptible). We are able to developed a many-strain SIRV epidemiological model as the following:

$$\frac{\partial S_i(x,t)}{\partial t} = \mu N(1-\phi_{new}(t)\sum_{y=0}^{n}\sigma(x,y)) - \mu S_i(x,t) - \phi_{pulse}(t)\sum_{y=0}^{n}S_i(y,t)\sigma(x,y) - \beta S_i(x,t)\sum_{y=0}^{n}I_i(y,t)\sigma(x,y) \quad (1)$$

$$\frac{\partial I_i(x,t)}{\partial t} = \beta S_i(x,t)I_i(x,t) - \mu I_i(x,t) - \nu I_i(x,t) + m\frac{\partial^2 I_i(x,t)}{\partial x^2} \quad (2)$$

$$\frac{\partial R_i(x,t)}{\partial t} = \nu I_i(x,t) - \mu R_i(x,t) - \beta S_i(x,t)[I_i(x,t) - \sum_{y=0}^{n}I_i(y,t)\sigma(x,y)] \quad (3)$$

$$\frac{\partial V_i(x,t)}{\partial t} = \mu N\phi_{new}(t)\sum_{y=0}^{n}\sigma(x,y) + \phi_{pulse}(t)\sum_{y=0}^{n}S_i(y,t)\sigma(x,y) - \mu V_i(x,t) \quad (4)$$

where the cross-immunity kernel $\sigma(x,y)$ is an inverse form of the Monod equation:

$$\sigma(x,y) = 1 - \frac{|\frac{x-y}{r}|}{1+|\frac{x-y}{r}|} \quad (5)$$

3. Results

4. Analysis

5. Conclusion

In conclusion, exact.

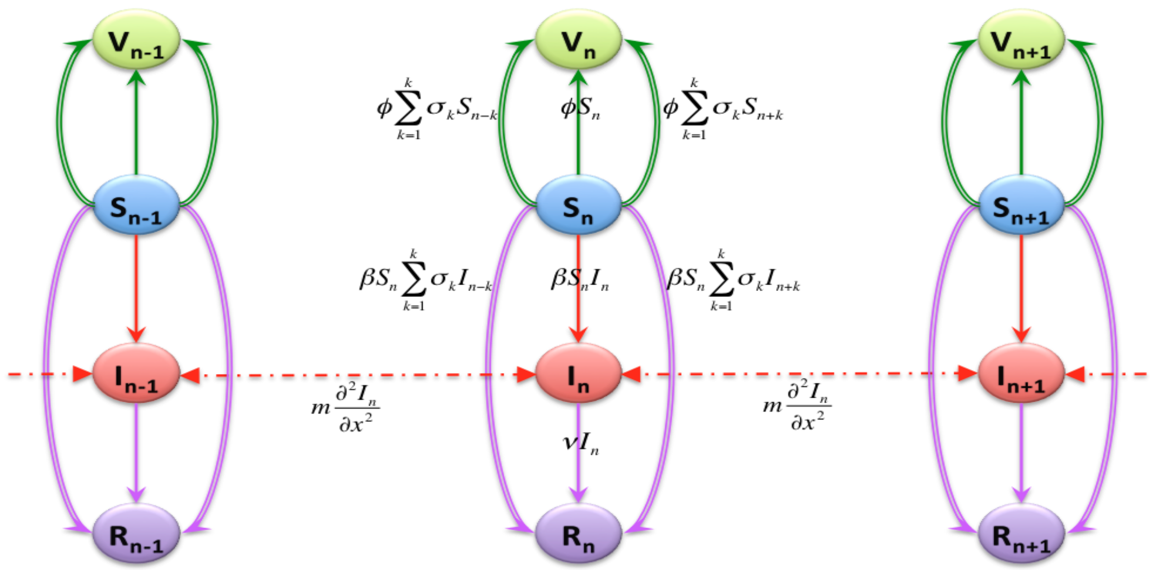

Figure 1: Schematic diagram of the many-stain SIRV model. The solid straight arrows indicate the classic single-strain SIRV relationship. The double-line arrows indicate the cross-immunity relationship with nearby strains. The dashed double-arrows indicate the mutation to nearby strains. For simplification, both the birth and death rate μ are ignored in this diagram.

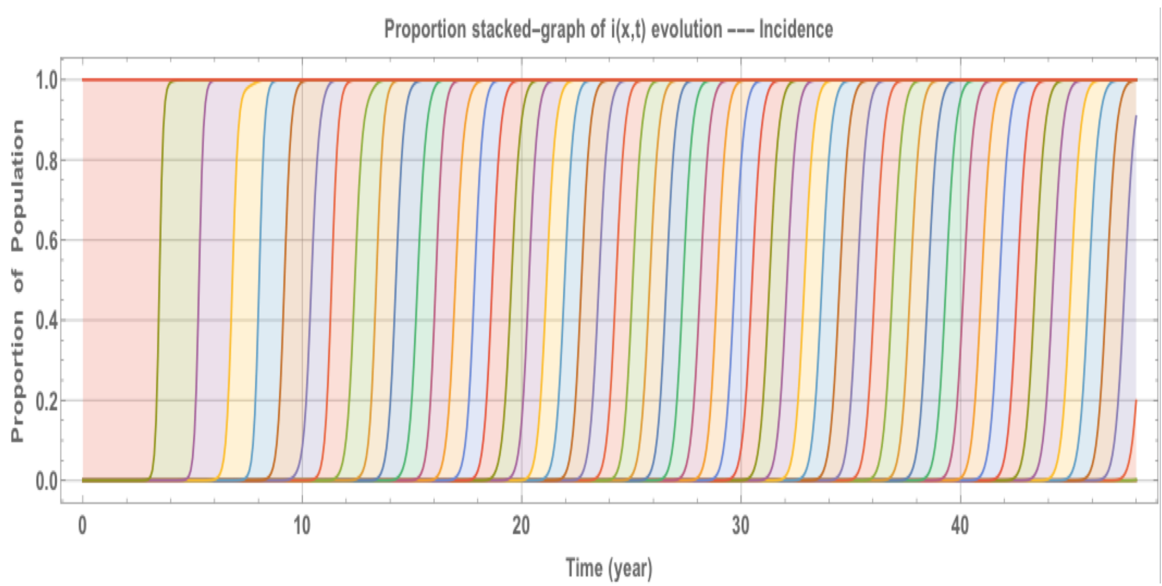

Figure 2: Proportion stacked graph indicated the building up process of steady traveling wave of influenza drift: initial single-strain-equilibrium-state is propagating into many-strain-equilibrium-state.

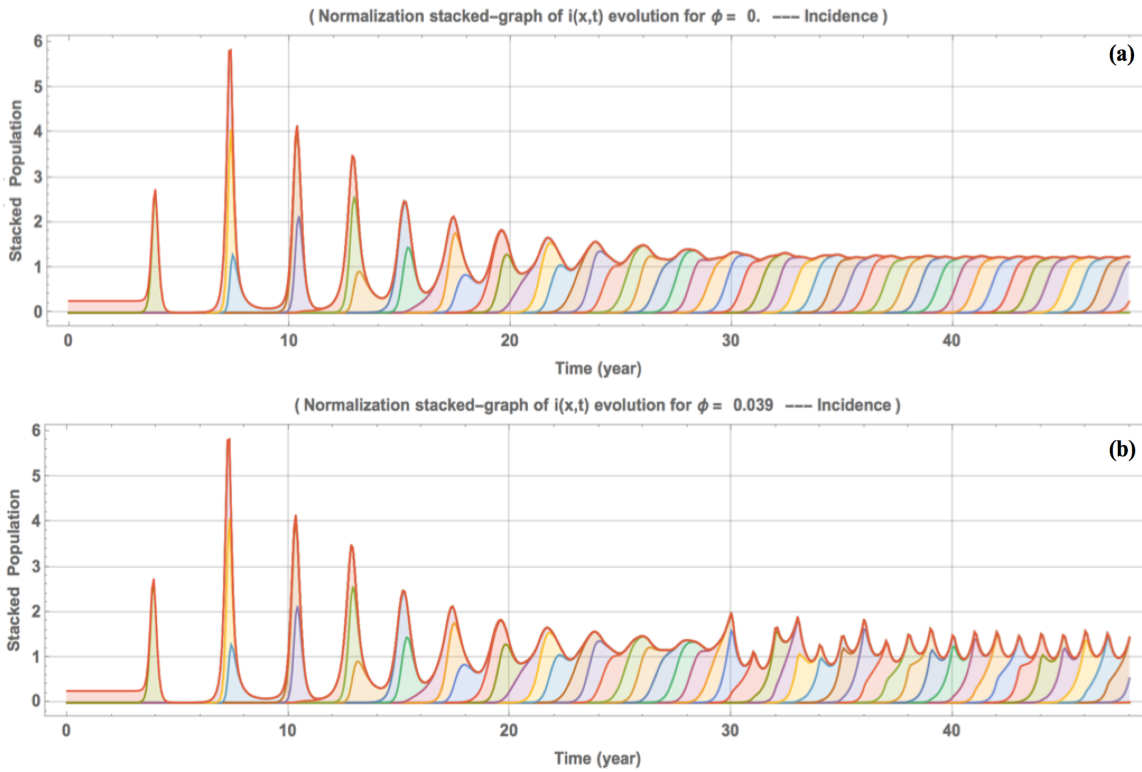

Figure 3: Comparison of incidence rate between non-vaccination and under-vaccination. (a) is a stain evolutionary map without vaccination (vaccination rate $\phi = 0$), in which a single-strain-equilibrium-state is evolving into many-strain-equilibrium-state in a half century time frame. (b) is the strain evolutionary map with 20 years vaccination: at the beginning 30 years, a single-strain-equilibrium-state is approaching into many-strain-equilibrium-state, however, starting from the 30th year, an annual vaccination strategy (vaccination rate $\phi = 0.039$) is applied. After 20 years of vaccination pressure, the evolution is evolving and approaching into vaccination-equilibrium state.

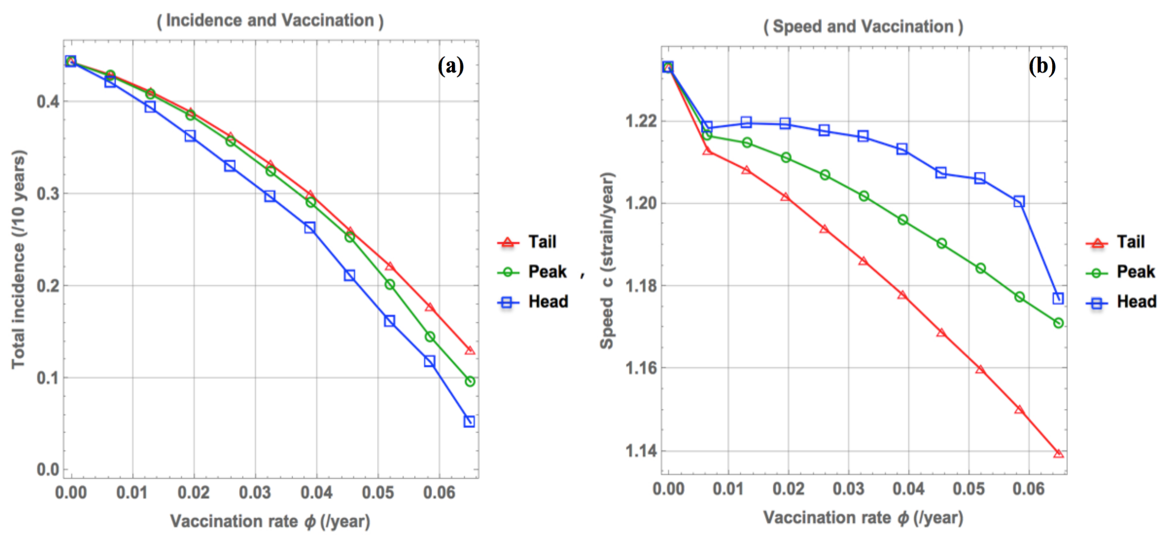

Figure 4: Vaccination effects. (a) Total incidence rate per 10 years as a function of vaccination rate ϕ. (b) Speed as a function of vaccination rate ϕ.

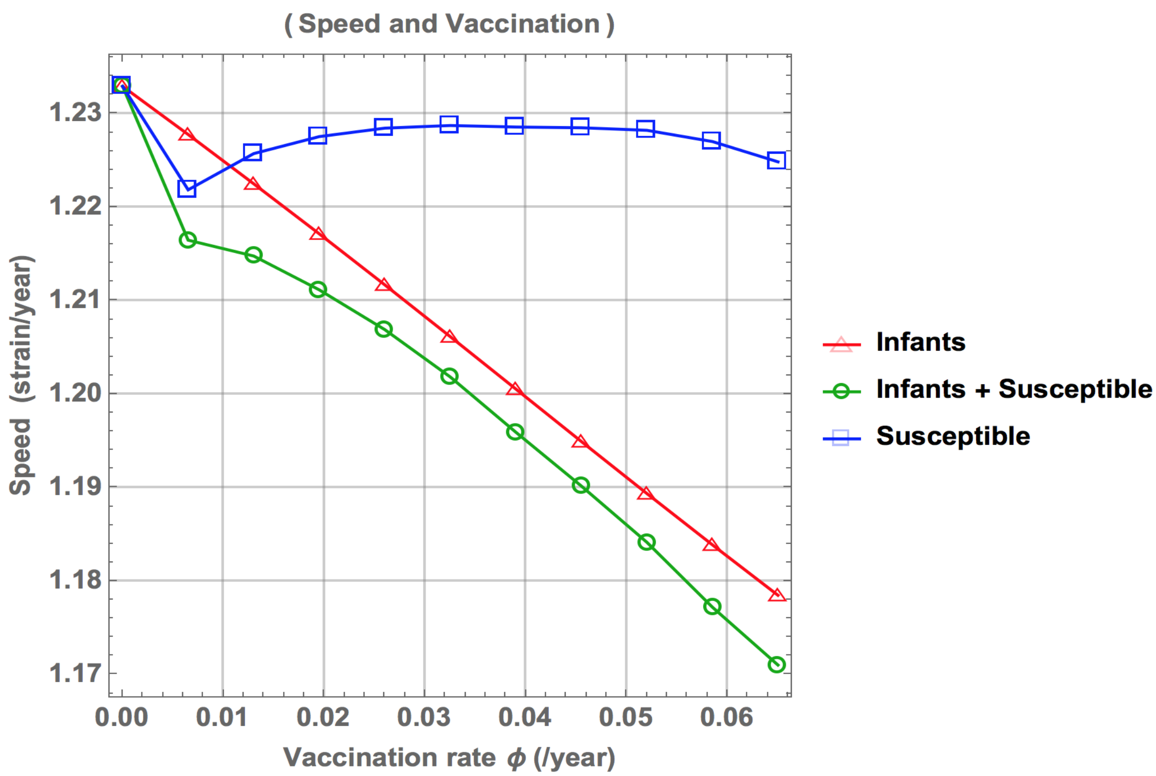

Figure 5: Contribution from two vaccination strategies. The triangle-marker red line is the strain evolutionary speed as a function of infant-vaccination-rate only, while the rectangle-marker blue line is the strain evolutionary speed as a function of susceptible-vaccination-rate only. The circle-marker green line is the composition of both vaccination strategy.