Definition 5.2 Linear Combinations and Span in $\mathbb{R}^n$

The set of all such linear combinations is called the **span** *of the* $\mathbf{x}_i$ *and is denoted*

$$\text{span}\{\mathbf{x}_1,\ \mathbf{x}_2,\ \dots,\ \mathbf{x}_k\} = \{t_1\mathbf{x}_1 + t_2\mathbf{x}_2 + \dots + t_k\mathbf{x}_k \mid t_i \text{ in } \mathbb{R}\}$$

If $V = \text{span}\{\mathbf{x}_1,\ \mathbf{x}_2,\ \dots,\ \mathbf{x}_k\}$*, we say that* V *is* **spanned** *by the vectors* $\mathbf{x}_1,\ \mathbf{x}_2,\ \dots,\ \mathbf{x}_k$*, and that the vectors* $\mathbf{x}_1,\ \mathbf{x}_2,\ \dots,\ \mathbf{x}_k$ **span** *the space* V*.*

$\mathbb{R}^n = \text{span}\{\mathbf{e}_1,\ \mathbf{e}_2,\ \dots,\ \mathbf{e}_n\}$ where $\mathbf{e}_1,\ \mathbf{e}_2,\ \dots,\ \mathbf{e}_n$ are the columns of I_n.

Definition 5.3 Linear Independence in $\mathbb{R}^n$

With this in mind, we call a set $\{\mathbf{x}_1,\ \mathbf{x}_2,\ \dots,\ \mathbf{x}_k\}$ *of vectors* **linearly independent** *(or simply* **independent***) if it satisfies the following condition:*

$$\text{If } t_1\mathbf{x}_1 + t_2\mathbf{x}_2 + \dots + t_k\mathbf{x}_k = \mathbf{0} \text{ then } t_1 = t_2 = \dots = t_k = 0$$

As for square matrices, if $A = [a_{ij}]$ is an $m \times n$ matrix, the elements $a_{11},\ a_{22},\ a_{33},\ \dots$ form the **main diagonal** of A. Then A is called **upper triangular** if every entry below and to the left of the main diagonal is zero. Every row-echelon matrix is upper triangular, as are the matrices

Definition 3.4 Eigenvalues and Eigenvectors of a Matrix

If A *is an* $n \times n$ *matrix, a number* λ *is called an* **eigenvalue** *of* A *if*

$$A\mathbf{x} = \lambda\mathbf{x} \text{ for some column } \mathbf{x} \neq \mathbf{0} \text{ in } \mathbb{R}^n$$

In this case, $\mathbf{x}$ *is called an* **eigenvector** *of* A *corresponding to the eigenvalue* λ*, or a* λ**-eigenvector** *for short.*

Definition 1.3 Row-Echelon Form (Reduced)

A matrix is said to be in **row-echelon form** *(and will be called a* **row-echelon matrix***) if it satisfies the following three conditions:*

1. *All* **zero rows** *(consisting entirely of zeros) are at the bottom.*
2. *The first nonzero entry from the left in each nonzero row is a* 1*, called the* **leading 1** *for that row.*
3. *Each leading* 1 *is to the right of all leading* 1*s in the rows above it.*

A row-echelon matrix is said to be in **reduced row-echelon form** *(and will be called a* **reduced row-echelon matrix***) if, in addition, it satisfies the following condition:*

4. *Each leading* 1 *is the only nonzero entry in its column.*

Definition 1.4 Rank of a Matrix

The **rank** *of matrix* A *is the number of leading* 1*s in any row-echelon matrix to which* A *can be carried by row operations.*

A square matrix is called a **diagonal matrix** if all the entries off the main diagonal are zero.