**III. Methods:**

**Estimation**

**Life Cycle Human Capital Model**

The human capital life cycle investment model’s earnings function is highly nonlinear. From this function, it is possible to identify three ability parameters from the production of human capital. Heckman et. al. (1998) used the terms to describe these parameters. In order to be consistent, I follow Heckman et. al. (1998)’s definitions and terminology. Two of these parameters are the representation of the “ability to learn” or more precisely how quickly an individual produces human capital from existing human capital. The third ability parameter is the “ability to earn” and represents earnings power independent of human capital investments. This can be thought of as someone’s ability or lack thereof to generate earnings above or below what his human capital would produce on the average. Each individual also has a depreciation rate and a discount rate. The depreciation rate can be interpreted as forgetfulness and the discount rate as the representation of time preference.

Given these parameters, individuals try to maximize their discounted earnings over the course of their lifetimes. Individuals invest in human capital in the form of schooling and then use their human capital to obtain a return on their investments in the form of earnings. Creating the necessary human capital requires existing human capital and time. The greater an individual’s ability the faster an individual creates human capital.

The life cycle human capital model offers many benefits. It has been used to explain occupational segregation and why schooling increases earnings. Most importantly for this context and unlike other labor economics models, the life cycle human capital model estimates parameters that other models cannot estimate.

**Polachek, Das, and Thamma-Apiroam (2015)**

This paper follows the Polachek, Das, and Thamma-Apiroam (2015) framework. The analysis assumes perfect capital markets to allow people to invest in their human capital. The objective of an individual is to maximize the following expression:

(1) .

Utility is U, the discount rate is represented by r, is earning’s potential at time and the number of years someone is expected to work is . People work for T years with certainty.

Disposable income can be decomposed further and this leads to the following equation:

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is the rental rate of human capital, and is the amount of human capital in existence at time . Additionally, individuals know their individual parameters and these parameters are constant. R denotes the amount of currency someone receives per unit of human capital. R varies across countries because R is denominated in a nation’s currency and because of institutional factors in across countries. Human capital is obtained throughout one’s life but is also lost overtime through depreciation. Human capital times the rental rate of human capital is a person’s earnings. Incomplete information does not exist. The rental rate of capital is constant throughout the population. This process of accumulating human capital follows a law of motion. The mathematical representation of human capital’s law of motion is defined by the following equation:

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is the rate of change in human capital stock at time t. In equation (3) is the production of human capital at time and δ is the depreciation rate of human capital in the model. The production function is defined by the following equation:

(4)

In equation (4) and represents how quickly a person develops human capital. is the human capital that an individual uses for the production of more human capital at time . Time and past human capital are the only inputs to create human capital. β is the total factor productivity parameter. b and β are ability parameters in this model.

Subject to the constraints (3) and (4) and the additional constraint that .

(5) .

This produces the following equation:

(6) .

This is a dynamic optimization problem in which a person will choose how many years of schooling to receive, how many years to work, and consequently when to retire based on numerous constraints. This study is interested in the years working because they provide information on observed earnings. The following equation is estimated:

Where

is the age at which the individual finishes his schooling; N is the anticipated retirement age, which I choose to be 65 to be consistent among the various countries; and is the human capital stock when training begins. I assume period 0 is when an individual starts formal schooling because once someone enters schooling an individual begins to study full time in an organized structured setting. is the human capital stock when training ends; I account for measurement error by including a stochastic error term . Equation (7) is an earnings function. The paper uses nonlinear least squares to estimate the earnings function.

Identification follows from Polachek, Das, and Thamma-Apiroam (2015). This paper uses a genetic algorithm to obtain convergence. The genetic algorithm is provided by Czarnitzki and Doherr (2009). This technique is more efficient than the traditional Newton-Raphson method, a more common maximum likelihood technique used to obtain convergence (Ryaben'kii et. al. 243).

As stated previously, this paper estimates R, β, b, δ, r, and for several countries and then uses these parameter estimates to test the homogeneity of human capital. There are several parameters that are dimensionless, r and δ. The parameter b is the output elasticity for the human capital model and R is denoted in the each country’s currency; in this case I use Swiss Francs, British Pounds, US Dollars, South Korean Won, and German Marks and later Euros. and β are unobservable; consequently, each will be treated as one parameter in the estimation. I also take into account dataset specific issues e.g. payment periods. For each individual i is written as

(8) and

For the estimation equation (8) becomes the following:

(8’)

After cleaning the data, the analysis implements the genetic algorithm to estimate the following parameters: b, W, E, δ, and r in equation (7’) using nonlinear least squares.

Equation (7’) is the econometric model and unlike equation (7) has an error term. The analysis then proceeds to run this regression for each of the remaining individuals within the dataset. The time spent in school in all of these models is set at the maximum amount of education a person receives. This assumes that there is no school intermittency in the data.

I fit (7’) for each individual in each dataset to estimate W, b, δ, r, and E. The dependent variable is an individual’s monthly earnings adjusted for inflation. t represents age and is the age at which an individual finished schooling. I only use data from individuals who did not engage in any school intermittency (Polachek 1975). I do not attempt to solve for using the Ben-Porath parameters because it would complicate the model and omit relevant information that is available for individuals. By treating as a constant for each individual I also avoid any issues of endogeneity that may arise from estimating it directly in the model.

**Equation to Estimate**