The Stefan-Boltzmann Radiation Law

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Abstract

In this experiment, we aimed to observe Stefan-Boltzmann and Inverse Square Laws. We performed the experiment in four parts using Stefan-Boltzmann lamp and radiation sensor. We managed to observe clearly the Inverse Square Law, yet, we mostly failed to confirm the relation R $\propto T^4$.

1 Introduction

1.1 The Stefan Boltzmann Radiation Law

The Stefan-Boltzmann radiation law states that the power per unite area is proportional to the fourth power of the temperature[1]. It is formulated by Josef Stefan in 1879 and Ludwig Boltzmann in 1879[1]. The relation between power and temperature can be obtained using Planck's radiation formula[2] which is

$$\frac{dP}{A} = \frac{2\pi hc^2}{\lambda^5(e^{\frac{hc}{\lambda kT}} - 1)} d\lambda \tag{1}$$

where P is the power, λ is the wavelength, A is the area, h is the Planck's constant, k is the Boltzmann constant, c is the speed of light, and T is the temperature. Integrating both sides for all wavelengths the equation becomes

$$\frac{P}{A} = 2\pi hc^2 \int_0^\infty \frac{d\lambda}{\lambda^5(e^{\frac{hc}{\lambda kT}} - 1)}. \tag{2}$$

Evaluating the integral[2] and inserting the universal constants the relation is found to be

$$\frac{P}{A} = \frac{2\pi^5 k^4}{15h^3c^2} T = \sigma T^4 \tag{3}$$

where

$$\sigma = 5.670 \times 10^{-8} \frac{Watts}{m^2 K^4}. \tag{4}$$

1.2 The Inverse Square Law

It is also possible to observe the inverse square law by using similar setup used in the Stefan-Boltzmann experiment. The law states that the intensity (I) of a spherical surface which is created by a point source and an object is inversely proportional to the square of the distance between them (D)[3]. Since intensity is

$$I \equiv \frac{P}{A} \tag{5}$$

where P is the total power radiated from the source and A is the spherical area created by the source and the object, the equation can be written as

$$I = \frac{P}{4\pi D^2}. \tag{6}$$

Hence,

$$I \propto \frac{1}{D^2}. \tag{7}$$

2 Apparatus

- Stefan-Boltzmann Lamp
- Voltage Amplifier
- Optical Bench
- Radiation Sensor
- Electrical Oven
- Voltmeter
- Ammeter
- Blackbody accessories
- Power Supply
- Connecting leads

3 Procedure

3.1 Part 0: Determination of R_{300}

1. Twenty different voltage (in 0-20mV range) and current data sets are obtained in order to determine the resistance of the Stefan-Boltzmann lamp at 300°K.

3.2 Part 1: The Inverse Square Law

1. Fourteen different distance and voltage data sets are recorded altering the distance between the lamp and the sensor.

3.3 High Temperature Measurement

1. About ten data sets of V_{sensor}, $V_{applied}$, and I_{sensor} are recorded while the distance between the lamp and the sensor is fixed.
2. The previous step is repeated for two other distance values.

3.4 Low Temperature Measurement

1. Temperature and V_{sensor} data sets are recorded as the oven is heated up to 400°C.

4 Data Analysis

4.1 Part 0

We plot the voltage and current data sets and applied a line fit in order to calculate the R_{300} value. It is found to be

$$R_{300} = (2.93 \pm 0.01) \times 10^{-1}\Omega.$$

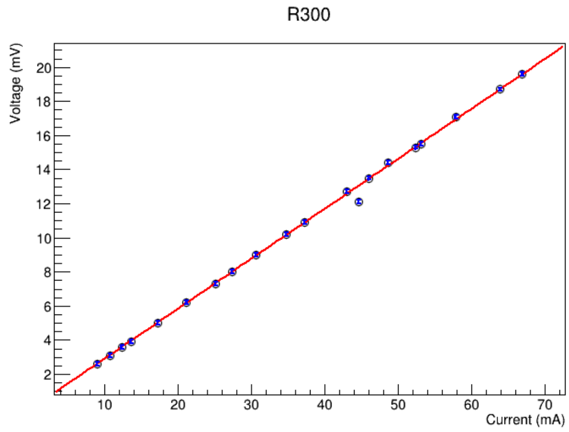

Fig 1.1: Current versus voltage plot and the line fit applied in order to obtain R_{300}.

Then, using the table[1], the relation between R/R300 and temperature is obtained by fitting a function to the data.

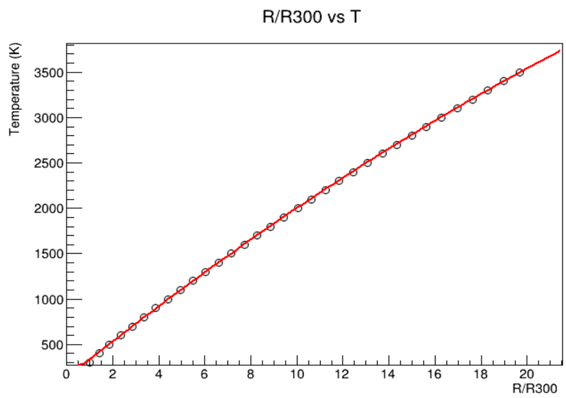

Fig 1.2: The applied fit on R/R300 and temperature plot.

The fit function and the parameters are found as

$$T = p_0 + p_1\left(\frac{R}{R_{300}}\right) + p_2\left(\frac{R}{R_{300}}\right)^2$$

$$p_0 = (1.26 \pm 0.05) \times 10^2 K$$

$$p_1 = (2.03 \pm 0.01) \times 10^2 K$$

$$p_2 = (-1.65 \pm 0.05) K.$$

[1]Some data sets of R/R300 and temperature are taken from Erhan Gulmez's book[4].

4.2 Part 1

We plot the distance and voltage data sets and applied some fit function on it. The fit function applied is

$$V = p_0 d^{p_1}$$

where p_0 and p_1 are the fit parameters we aimed to obtain and V and d are the voltage and distance values respectively.

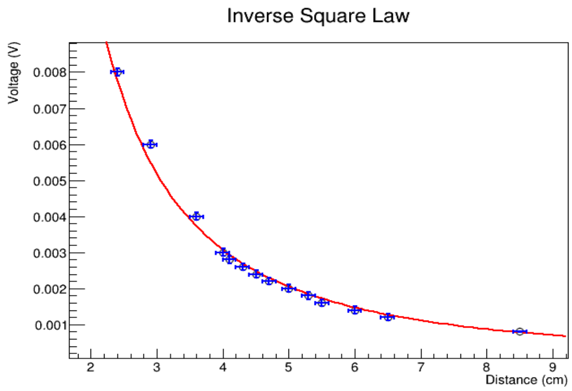

Fig 1.3: The best fit to the distance versus voltage plot

Applying the fit function to the data we obtained p_1.

$$p_1 = -1.82 \pm 3.68 \times 10^{-2}$$

4.3 Part 2

In this part of the experiment, we obtained three different changing the data sets of V_{app}, I_{app}, and V_{sensor}. We obtain the resistances for each data sets dividing V_{app} into I_{app}. Then, using the earlier R_{300} value and the relation between $\frac{R}{R_{300}}$ and temperature the temperature values can be obtained. Finally applying some fit function to the temperature vs V_{sensor} plot the power of temperature in the Stefan-Boltzmann equation can be determined.

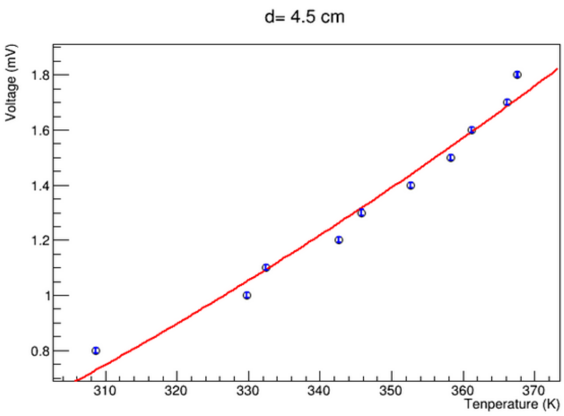

Fig 1.4: The temperature vs V_{sensor} plot at 4.5 cm distance and the function of $V = p_0 T^{p_1} + p_2$ fit

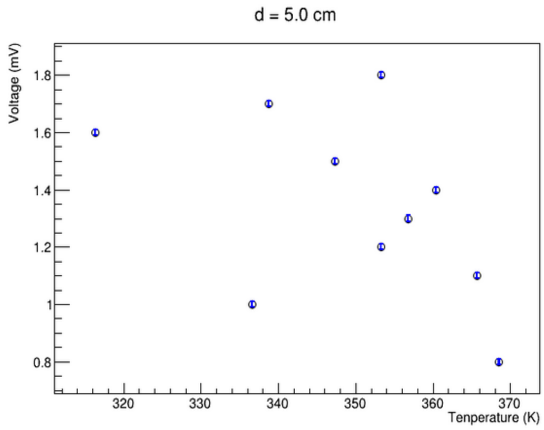

Fig 1.5: The temperature vs V_{sensor} plot at 5.0 cm distance

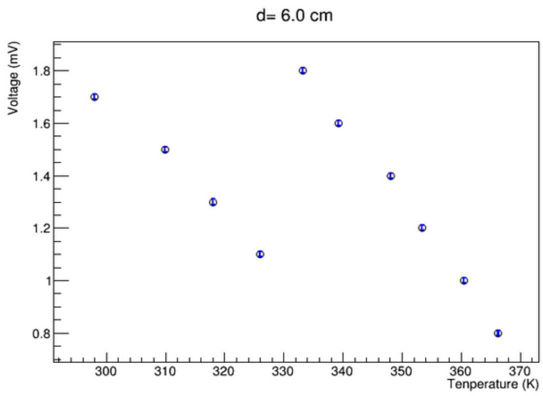

Fig 1.6: The temperature vs V_{sensor} plot at 6.0 cm distance

The fit parameter of the temperature and V_{sensor} plot at 4.5cm distance is found to be

$$p_1 = 2.58 \pm 7.01 \times 10^{-5}$$

Unfortunately it is not possible to apply proper fit functions to the data sets of 5.0 and 6.0 cm distances [2].

[2]Possible reasons are discussed in the Conclusion section.

4.4 Part 3

We plot the data sets of temperature and sensor voltage and applied some fit function on it. The fit function applied is

$$V = p_0 T^{p_1} + p_2$$

where p_0, p_1, and p_2 are the fit parameters we aimed to obtain. Applying the fit function to the data we obtained p_0 and p_1.

$$p_0 = (6.39 \pm 0.18) \times 10^{-8}$$

$$p_1 = 3.15 \pm 1.13 \times 10^{-3}$$

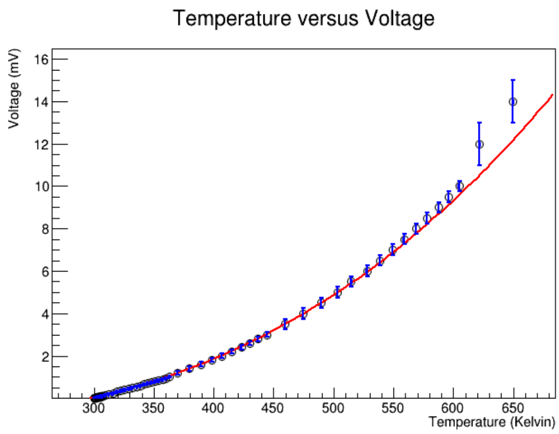

Fig 1.7: The best fit to the temperature versus voltage plot

We alternatively make the previous plot with their logarithmic values and applied another fit function to the plot. Once their logarithmic values are taken, then the characteristics of the plot will be a linear line. Hence, the fit function is

$$lnV = p_0 + p_1 lnT$$

where p_0 and p_1 are the fit parameters we aimed to obtain. Applying the fit function to the data we find p_1.

$$p_1 = 6.89 \pm 2.02 \times 10^{-2}$$

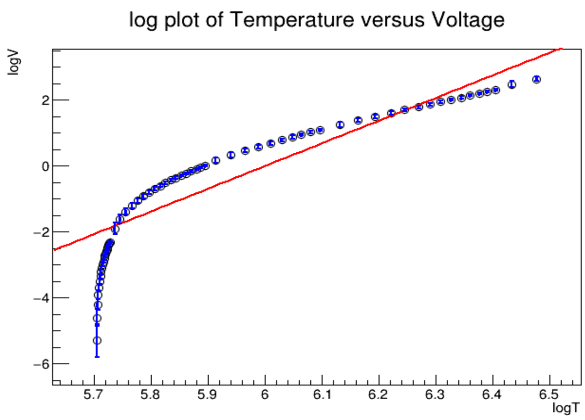

Fig 1.8: The best fit to plot of the logarithmic values of temperature and voltage values

We also suggest that it might lead better results for some reason if the fit function is applied to the region where linear behaviour is dominant.

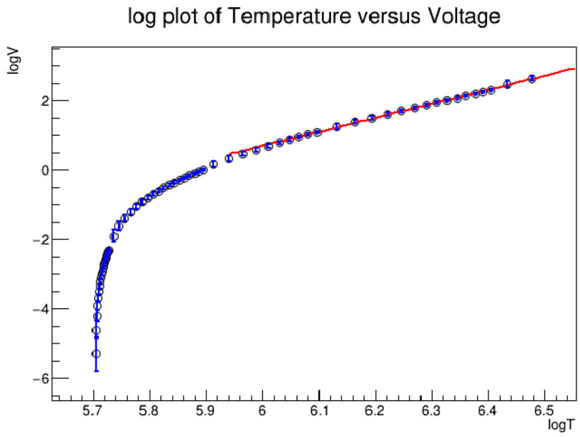

Fig 1.9: The best fit to the specific region of the plot of the logarithmic values of temperature and voltage values

Then we calculated the slope as

$$p_1 = 4.04 \pm 0.06$$

Finally we calculate the weighted average of p_1 values we obtain. Those values correspond to the power of temperature in Stefan-Boltzmann law.

n	σ_n
6.89E+00	2.02E-02
4.04E+00	6.00E-02
3.15E+00	1.13E-03

Table: Calculated n values and their uncertainties.

The weighted average[3] of n is

$$n = 3.16 \pm 6.31 \times 10^{-4}.$$

5 Conclusion

In the zeroth part of the experiment, we determined the resistance value at 300K by fitting a second order polynomial to the plot. It is seen that the fit functions match with the data. Then, in the first part of the experiment we aimed to observe the inverse square law by applying power fit to the distance versus voltage plot. We calculated the value 4.89σ away from the theoretical value of 2.

$$\frac{|-2.00 + 1.82|}{3.68 \times 10^{-2}} = 4.89$$

Better results can be obtained with more sensitive measurements since the geometry of the glass of the Stefan-Boltzmann lamp is a considerably big error source in centimeter scale.
In the second part of the experiment, we aimed to calculate the power factor of temperature in the Stefan-Boltzmann equation by taking measurements for three different distances. However, we are unable to apply a proper fit function to the two of the plots. There might exist blunders made in this part of the experiment such as taking wrong measurements since no possible error source explain this type of mistake. We also excluded the only result we managed to obtain in this part since it is very far from the theoretical value and it is highly possible that this part may also contain wrong measurements. In the third part of the experiment, we applied power and line fits to the plots we have. We calculated the weighted average of those values. Note that since the power fit has ten times smaller uncertainty than the others do, the weighted average is very close to that value. Despite the weighted average and the values we calculated are very far from the theoretical value, we noticed that it may be caused because of the data sets lower than 400K. We could not give better explanations other than the fact that the fit function is affected by those data

[3]The result obtained in Part 2 is excluded. See Conclusion section for details.

sets. On the other hand, we additionally applied line fit to the region where the linear behaviour is dominant and we manage to obtain very close value within the range of 0.67σ of the theoretical value. It is unnecessary to calculate how much sigma away the experimental results are found since they are extremely far from the theoretical value.

$$\frac{|4.04-4|}{0.06} = 0.67$$

References

1.https://www.britannica.com/science/Stefan-Boltzmann-law
Obtained on December 2, 2019.
2.http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/stefan2.html
Obtained on December 2, 2019.
3.http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/isq.html
Obtained on December 2, 2019.
4. 4.Advanced Physics Experiments. Erhan Gülmez, 1998, pg 97.

Appendix

- The analysis is made by using pyROOT 6.16.

- Weighted average and its standard deviation formula are given below.

$$\lambda_{weighted} = \frac{\sum_{i}^{N} \lambda_i/\sigma_i^2}{\sum_{i}^{N} 1/\sigma_i^2} \tag{8}$$

$$\sigma^2_{\lambda_{weighted}} = \frac{1}{\sum_{i}^{N} 1/\sigma_i^2} \tag{9}$$

- The error propagation formula used is given below.

$$\sigma_f = \sqrt{\sum_{i}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2} \tag{10}$$