Ray D' Inverno

Introducing Einstein's Relativity

Non-official Solution Manual

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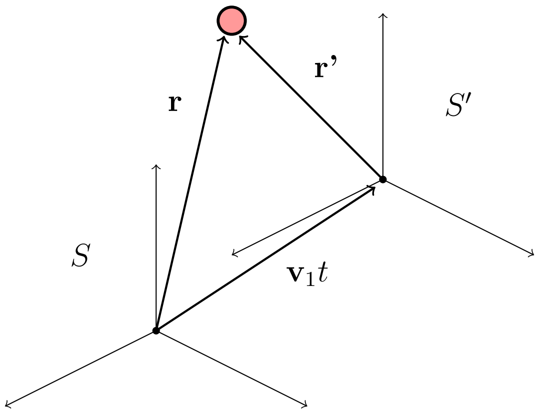
Conteúdo

1 Chapter 1

1.1 Exercise 1

We can draw the following diagram



and perform a simple vector addition to obtain $\mathbf{r} = \mathbf{r}' + \mathbf{v}_1 t$. In the unidimensional case we have $y = y'$ and $z = z'$, so the previous relation becomes:

$$x = x' + v_1 t, \;\; t = t' \tag{1.1.1}$$

If the frame frame S' moves away from the frame S with speed v_1 then we can also say that the frame S moves away from the frame S' with an opposite velocity $-v_1$:

$$x' = x - v_1 t$$

The second transformation is analogous:

$$x' = x'' + v_2 t, \;\; t' = t'' \tag{1.1.2}$$

Therefore, the transformations from S to S'' are:

$$\boxed{x = x'' + (v_1 + v_2)t, \;\; t = t''} \tag{1.1.3}$$

Note that we can write the first transformation in matrix for as:

$$\begin{pmatrix} x \\ t \end{pmatrix} = G(v)\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} x' \\ t' \end{pmatrix}\begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix} \tag{1.1.4}$$

Finally, we just have to prove that if we take two different matrices $G(v_1)$ and $G(v_2)$, corresponding to the velocities v_1 and v_2, respectively, their product commutes. This is quite easy and can be shown through direct calculation:

$$G(v_1)G(v_2) = \begin{pmatrix} 1 & v_1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & v_2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & v_1 + v_2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & v_2 + v_1 \\ 0 & 1 \end{pmatrix} = G(v_2)G(v_1)$$

Showing the other properties of a group is also easy: the inverse for example is given by $G^{-1}(v) = G(-v)$ and the identity element is simply $I = G(0)$.

1.2 Exercise 2

I think that the diagrams in the answer key are self-explanatory.

1.3 Exercise 3

We already know that $k = \sqrt{(1+v)/(1-v)}$. If we make the substitution $v \to -v$ we get:

$$\sqrt{\frac{1+(-v)}{1-(-v)}} = \sqrt{\frac{1-v}{1+v}} = \frac{1}{k} \tag{1.3.1}$$

Now, remember that:

$$v = \frac{k^2-1}{k^2+1}$$

If k is bigger than the unity, we have $v > 0$, which corresponds to a redshift. Whereas for $k < 1$ we have $v < 0$, which corresponds to a blueshift: B is approaching A.

1.4 Exercise 4

Equation (2.4) tells us that:

$$k = \sqrt{\frac{1+v}{1-v}}$$

Thus, $k_{AC} = k_{AB}k_{BC}$ implies:

$$\sqrt{\frac{1+v_{AC}}{1-v_{AC}}} = \sqrt{\frac{1+v_{AB}}{1-v_{AB}}}\sqrt{\frac{1+v_{BC}}{1-v_{BC}}} \implies \frac{1+v_{AC}}{1-v_{AC}} = \frac{1+v_{AB}+v_{BC}+v_{AB}v_{BC}}{1-v_{AB}-v_{BC}+v_{AB}v_{BC}}$$

which yields, after cross multiplying

$$\underbrace{v_{AC}(v_{AB}v_{BC})+v_{AC}}_{v_{AC}(1+v_{AB}v_{BC})} - v_{AB} - v_{BC} = -\underbrace{v_{AC}(v_{AB}v_{BC})+v_{AC}}_{v_{AC}(1+v_{AB}v_{BC})} + v_{AB} + v_{BC}$$

that leads us to the desired result:

$$\boxed{v_{AC} = \frac{v_{AB}+v_{BC}}{1+v_{AB}v_{BC}}} \tag{1.4.1}$$

If $0 < v_{AB} < 1$ and $0 < V_{BC} < 1$, if we add the two inequalities we get $0 < v_{AB} + v_{BC} < 2$. We may also multiply the second inequality by v_{AB} to find $0 < v_{AB}v_{BC} < v_{AB} < 1$, and then add 1 to find $1 < 1 + v_{AB}v_{BC} < 2$. Dividing the first result by the last one:

$$0 < \frac{v_{AB}+v_{BC}}{1+v_{AB}v_{BC}} = v_{AC} < 1$$

1.5 Exercise 5

1.6 Exercise 6

1.7 Exercise 7

1.8 Exercise 8

1.9 Exercise 9

1.10 Exercise 10