

Imagine you have a circle of people and you go around the circle removing every second person until one person is left. If you have 3 people in the circle, then the 3rd person will be the last one remaining. If you have 4 people then the 1st person will be the last one remaining. If you have 11 people then the 7th person will be the one remaining. If you have N people in the circle, who will be the last one remaining? Show off your work.

First, I created a quick Java algorithm to be able to see if I find some quick patterns:

```
public class PuzzleHelper {
    private final static int SIZES = 300; //Try group sizes from 1 to SIZES
    public static void main(String[] args) {
        for(int i = 1; i < SIZES; i++) {
            System.out.println("Group size: " + i + ". " +
                               "Last standing: " + lastPerson(i));
        }

        /* Find the latest person left in the circle given a circle of size n: */
        public static int lastPerson(int n) {
            if (n <= 0) return 0; //Invalid input
            boolean[] circle = new boolean[n]; //true if removed
            int index = -1;
            while (n > 1) {
                index = nextAvailable(index, circle); //Jump one.
                index = nextAvailable(index, circle);
                circle[index] = true; //remove the person
                n--;
            }
            return lastStanding(circle);
        }

        /* Get the next person available starting from a given index: */
        private static int nextAvailable(int index, boolean[] circle) {
            do {
                index = next(index, circle.length); //index++ in the circle
            } while (circle[index]); //ignore those who were already removed
            return index;
        }

        /* Get the next index in a circular array: */
        private static int next(int index, int size) {
            return (index + 1) % size;
        }

        /* Return the first true boolean in the array: */
        private static int lastStanding(boolean[] circle) { //O(n)
            for (int i = 0; i < circle.length; i++)
                if (!circle[i]) return i + 1;
            return 0; // No one left :(
        }
    }
}
```

```

    }
}

```

And yes, there is a pattern. It's a sequence of even numbers that grows.

1 1 3 1 3 5 7 1 3 5 7 9 11 13 15 1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 1 3 5 7 ...

Let $f(N)$ be a function that returns a list of the first N odd numbers. For example, $f(3)$ returns 1, 3, 5. Thus, the result is the same as $f(1)$, $f(2)$, $f(4)$, $f(8)$, $f(16)$, $f(32)$, $f(64)$, ... That means the size of the sequence is doubling!

Since it's doubling, we can take the $\lg(N)$. If $\lg(N)$ is a whole number, then the person to be eliminated is the first one. Otherwise, we can simply remove the biggest multiple of 2 from N and then get the K th odd element where K is what was left.

Thus, one answer for the challenge is the $NthOdd(n - LargestMultiple2(n))$.

Here's a Java implementation of the solution:

```

public static int lastPerson2(int n) {
    return getNthOdd(n - LargestMultiple2(n));
}

private static int largestMultiple2(int n) {
    int largest = 1;
    do
        largest *= 2;
    while (largest <= n);
    return largest /= 2;
}

/* Get the Nth odd number of a sequence: */
private static int getNthOdd(int n) {
    return n * 2 + 1;
}

```

To make sure this formula gives the right solution, I've compared it with my previous algorithm for 3000 group sizes. They all match. Here's my test in Java:

```

private final static int SIZES = 3000; //Try group sizes from 1 to SIZES
public static void main(String[] args) {
    for(int i = 1; i < SIZES; i++) {
        int firstAlgOutput = lastPerson(i);
        int secondAlgOutput = lastPerson2(i);

        if (firstAlgOutput != secondAlgOutput) {
            System.out.println("Houston, we have a problem.");
            break;
        }
    }
}

```