Assignment 3

Problem 1

If A has row 1 + row 2 = row 3, show that A is not invertible:

1. Explain why Ax = (1,0,0) cannot have a solution.
2. Which right sides (b_1 , b_2 , b_3) might allow a solution to $Ax = b$?
3. What happens to row 3 in elimination?

Problem 2

Change I into A^{-1} as you reduce A to I (by row operations).

$$[\ A\ \ I\ ] = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 7 & 1 & 0 \end{bmatrix} \text{ and } [\ A\ \ I\ ] = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 1 & 0 \end{bmatrix}$$

Problem 3

Find A^{-1} and B^{-1} (if they exist) by elimination on [A I] and [B I]:

$$\text{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix} \text{ and B} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 2 \\ -1 & -1 & 2 \end{bmatrix}$$

Problem 4

For which three numbers C is this matrix not invertible, and why not?

$$\begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

Problem 5

Suppose the matrices P and Q have the same rows as I but in any order. They are ”permutation matrices”. Show that P - Q is singular by solving $(P - Q)x = 0$.

Problem 6

A is a 4 by 4 matrix with 1's on the diagonal and -*a*, -*b*, -*c* on the diagonal above. Find A^{-1} for this bidiagonal matrix.

Problem 7

What three elimination matrices E_{21}, E_{31}, E_{32} put A into its upper triangular form $E_{32}E_{31}E_{21}A = U$? Multiply by E_{32}^{-1},E_{31}^{-1},E_{21}^{-1} to factor A into L times U:

$$A= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

Problem 8

What are L and D (the diagonal pivot matrix) for this matrix A? What is U in A = LU and what is the new U in A = LDU?

$$\textbf{Already Triangular},\ A= \begin{bmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{bmatrix}$$

Problem 9

Compute L and U for the symmetric matrix A:

$$A= \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

Find four conditions on *a, b, c, d* to get $A = LU$ with four pivots.

Problem 10

Factor these symmetric, matrices into $A = LDL^T$. The pivot matrix D is diagonal:

$$\text{A}=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$$

$$\text{and A}=\begin{bmatrix}1 & b\\ b & c\end{bmatrix} \text{ and A}=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$$

Problem 11

Find a 4 by 4 permutation matrix (call it A) that needs 3 row exchanges to reach the end of elimination. For this matrix, what are its factors P, L, and U?