**Chapter 04: Complex Integration and Cauchy’s Theorem**

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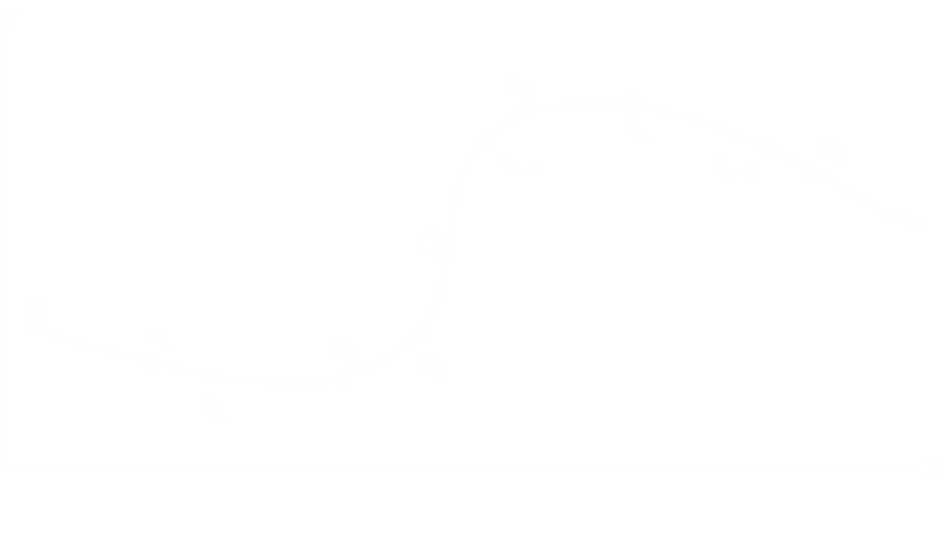
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## 4.1 Complex Line Integrals

Say we have a **curve**, , and a function , which is **continuous** at all points of that curve. We can **subdivide** the curve into parts, with the th part being from the point to the point . Thus, the points would be from to .



For a particular subdivision, if we want to find the **area** of the curve beneath that subdivision, we can approximately say that it is equal to , provided the subdivisions are sufficiently small to make this approximation.

Thus, if we want to know the area beneath the curve between to ,

can also be written as . Thus,

If we now **increase**  in such a way that even the largest possible value of begins to approach , we can essentially say

where and .

This is called the **complex line integral** of along the curve or the **definite integral** of from to along the curve . In this case, is said to be **integrable** along . If is analytical at all points of a region , and lies within , then is continuous and therefore integrable along .

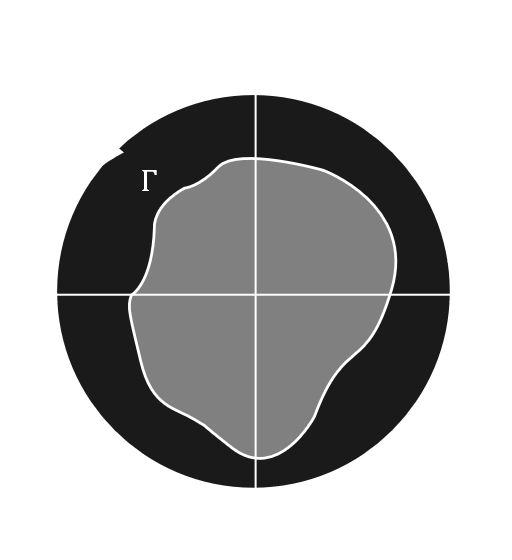
## 4.3 Connection Between Real and Complex Line Integrals

Suppose . Then, we can express the complex line integral in terms of the **real line integrals**.

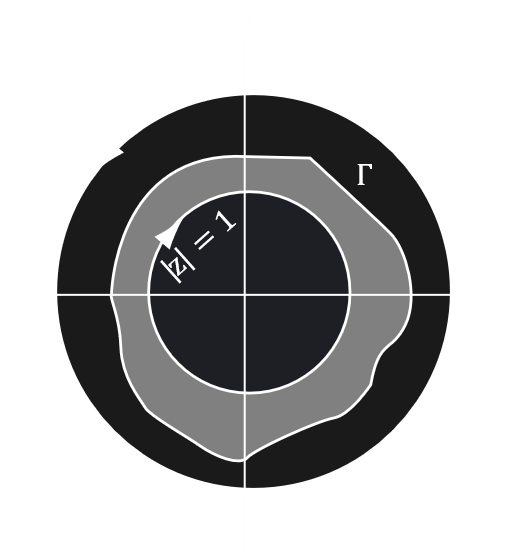
## 4.6 Simply and Multiply Connected Regions

Say we have a region and a **simple closed curve**, , which lies entirely within .

The region is said to be **simply connected** if the curve can be shrunk to a point without leaving . is said to be **multiply connected** if this is not true.



In the figure above, if the darkly shaded region is and the lightly shaded region is , then can be shrunk to a point and will still be within . Thus, is simply connected.



In this second graph, the darkly shaded region has a hole in the centre. Thus, cannot be shrunk to a point while still remaining within , making multiply connected.

In simpler terms, is simply connected if it does not have any **holes** in it and multiply connected otherwise.