**Tangent Planes and Normal Lines**

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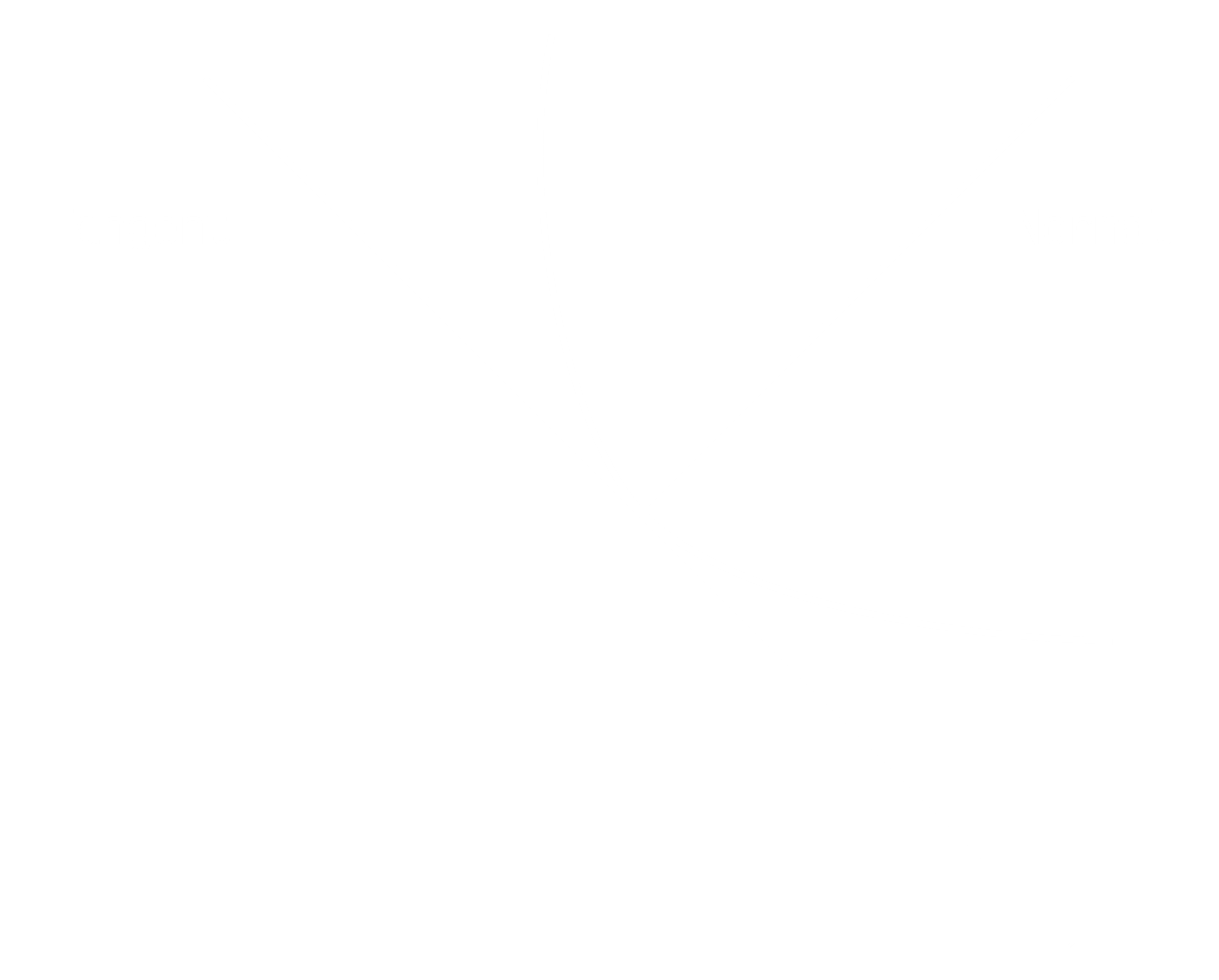
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## Two Dimensions

For the equation , the **tangent** at the point has a slope , while the **normal** at the point has a slope .

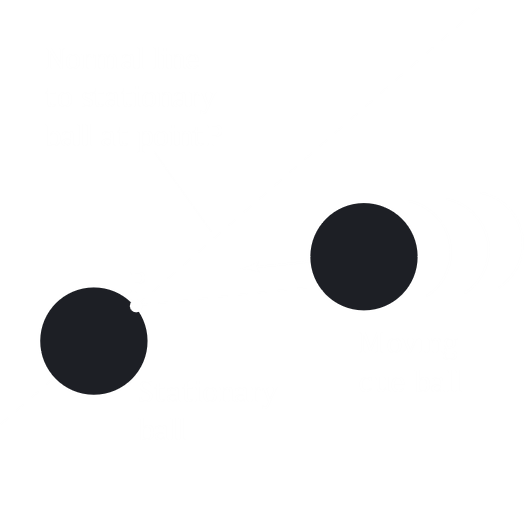


From this, we can derive that the tangent has the equation:

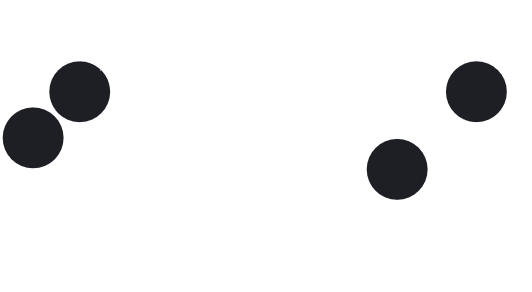
and the normal has the equation:

## Three Dimensions

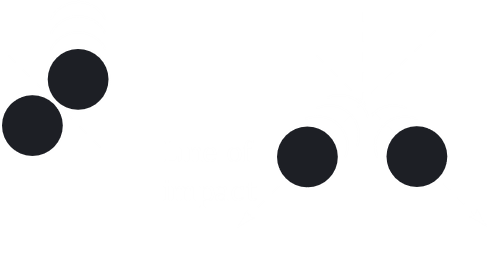
Normals and tangents become even more important for practical life if we move to **three dimensions**. Consider that one billiard ball is hitting another, stationary billiard ball. If the stationary ball is hit at the **point**  on its surface, it will move in the direction of the **normal** at that point. The normal in this situation can also be called the **line of impact**.



The impact can occur in two ways. One is if the **moving ball** is also moving exactly along the **line of impact**. In this case, the moving ball **stops dead** and transfers all its **momentum** to the stationary ball.



If the moving ball is not moving along the line of impact, it transfers part of its momentum to the stationary ball and gets **deflected** to one side. The direction in which it ends up moving is the **tangent** at the point of the stationary ball. Regardless, the stationary ball still moves along the line of impact.



So far, we have used equations for 3 dimensions as . Here however, it is easier to use the format , where .

Let be a **point** on the **surface** , defined by . At this point, we can determine , and .

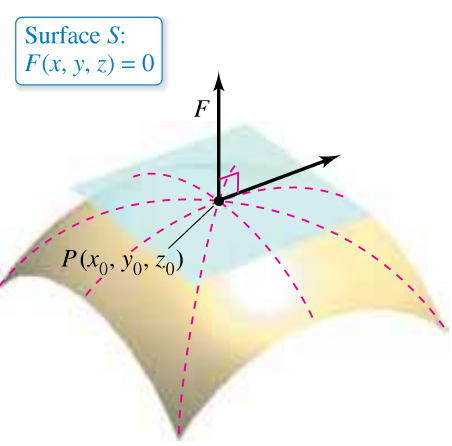
The next thing we need is a **direction** in which we want to determine the tangent. Let this be define as , a **curve** on through that has the equation given by the **vector-valued function** . Thus, for the part of the surface that overlaps with the curve, .

From the **chain rule**, we can differentiate as

The equivalent **vector form** is

Here, is the **gradient** and is the **tangent vector**.

Since , we can say that the **gradient** at is **orthogonal** to the **tangent vector**. This also means that all tangent lines on lie in a plane that is normal to and contains .



In conclusion, at a **point** on the **surface** which has the equation , where and is differentiable:

* The **plane** through that is **normal** to is the **tangent** plane to at .
* The **line** through that has the **same direction** as is the **normal** line to at .

If we want to determine the tangent and normal at a point , and all we know is that is an **arbitrary point** in the **tangent plane**, then the equation of the vector will be

Since is normal to the tangent plane at , it must be **orthogonal** to every vector on the tangent plane. Thus,

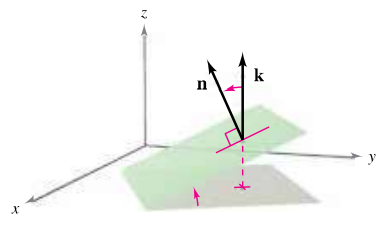
This gives us another equation for the **tangent plane** to the surface given by at .

If we use , this equation becomes:

## Angle of Inclination of a Plane

Another use of the gradient, , is to determine the **angle of inclination** of the tangent plane to a surface.

The angle of inclination is the **angle**, , between **the plane** and **the plane**.



Since is the **normal to the plane**,

where is the **normal to the plane**.

Example

Say and we want to find the angle of inclination at the point .