You may submit a separate typed document as an answer to Question 1.

1. Write about 3 different applications of Markov Decision Processes. You may write about any application from industry or research. Make sure to include the following information:

- How Markov Decision Process is used in the application
- Why the application is important
- What makes Markov Decision Process suited to be applied here

For Questions 2 to 6, write your answers by hand, then scan and submit them.

2. Two white and two black balls are distributed in two urns in such a way that each contains two balls. We say that the system is in state i, i = 0, 1, 2, if the first urn contains i white balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn. Let X_n denote the state of the system after the nth step. Calculate the transition probability matrix. What is the probability that the number of white balls in the first urn remains unchanged after 5 steps?

3. Suppose that whether or not it rains today depends on previous weather conditions through the last three days. Show how this system may be analyzed by using a Markov chain. How many states are needed? Suppose that if it has rained for the past three days, then it will rain today with probability 0.8; if it did not rain for any of the past three days, then it will rain today with probability 0.2; and in any other case the weather today will, with probability 0.6, be the same as the weather yesterday. Determine P for this Markov chain and draw the transition diagram.

4. Suppose that coin 1 has probability 0.7 of coming up heads, and coin 2 has probability 0.6 of coming up heads. If the coin flipped today comes up heads, then we select coin 1 to flip tomorrow, and if it comes up tails, then we select coin 2 to flip tomorrow. If the coin initially flipped is equally likely to be coin 1 or coin 2, then what is the probability that the coin flipped on the third day after the initial flip is coin 1? Suppose that the coin flipped on Monday comes up heads. What is the probability that the coin flipped on Friday of the same week also comes up heads?

5. A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or badly. Let p_i denote the probability that the class does well on a type i exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type i, i = 1, 2, 3?

6. A taxi driver provides service in two zones of a city. Fares picked up in zone A will have destinations in zone A with probability 0.6 or in zone B with probability 0.4. Fares picked up in zone B will have destinations in zone A with probability 0.3 or in zone B with probability 0.7. The driver's expected profit for a trip entirely in zone A is 6; for a trip entirely in zone B is 8; and for a trip that involves both zones is 12. Find the taxi driver's average profit per trip.