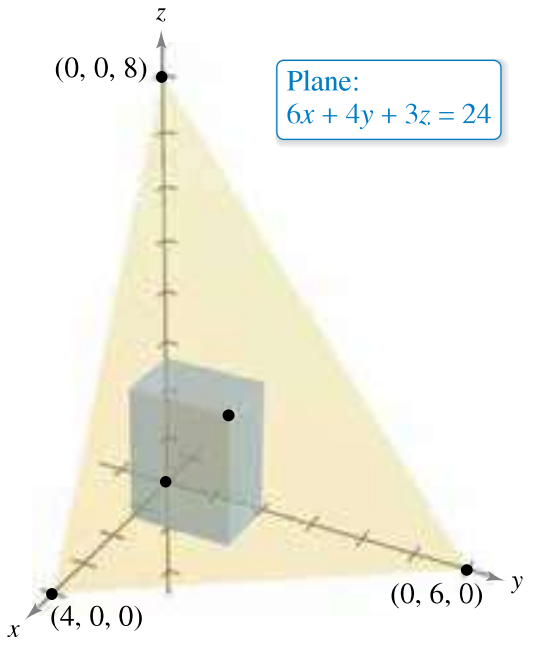
**13.9 Applications of Extrema**

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Consider that we have a **rectangular box** that is resting on the  **plane**, with one vertex at the **origin** and the opposite vertex on the **plane** .



We want to find the **maximum possible volume** of the box.

Let , and represent the length, width and height of the box. We can write the equation as:

Then, the **volume** of the box can be written as:

From here,

,

,

Thus, there are four possible **critical points**, , , , , from the four possible combinations of the above results.

Since the volume is at the first three of these points, those cannot be critical points. Thus, the **maximum value** must occur at . The volume at this point is

## Hessian Matrix

Consider that we have an equation . This equation has two possible derivatives, and . It also has four possible **second derivatives**, , , and .

We can create a **matrix** using the **four second derivatives** as follows:

This is called the **Hessian Matrix**. It is used to determine **maxima** and **minima** for multivariable equations.

We will be taking **partial matrices**, which are just submatrices.

If and , we have a **minima**.

If and , we have a **maxima**.

If , we have a **saddle point**, which is a point which has neighbouring points that are less and also neighbouring points that are greater.

We can extend this to accommodate **three variables** as well:

where if , and are all positive, we have a **minima** and if and are negative but is positive, we have a **maxima**.

Going back to the example we did earlier, we simply declared that we had a maximum pointer, but we should also prove this using the **Hessian Matrix**.

and , which makes this a **maximum point**.

## Least Squares Method

We have previously used the **Least Squares Method** to determine the **line of best-fit** for several points on a graph. We were trying to reduce the **distance** between the given points and the line.

For a given set of points, , we can determine the line of best fit as , where