Assignment 2

Problem 1

Reduce this system to upper triangular form by two row operations:

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 7y + 5z &= 20 \\ -2y + 2z &= 0 \end{aligned}$$

Circle the pivots. Solve by back substitution for z, y, x.

Problem 2

Which number d forces a row exchange, and what is the triangular system (not singular) for that d? Which d makes this system singular (no third pivot)?

$$\begin{aligned} 2x + 5y + z &= 0 \\ 4x + dy + z &= 2 \\ y - z &= 3. \end{aligned}$$

Problem 3

Find the pivots and the solution for both systems (Ax = band Kx = b):

$$\begin{aligned} 2x + y &= 0 \\ x + 2y + z &= 0 \\ y + 2z + t &= 0 \\ z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= 0 \\ y + 2z - t &= 0 \\ -z + 2t &= 5. \end{aligned}$$

Problem 4

For which three numbers b will elimination fail to give three pivots?

$\begin{bmatrix} b & 2 & 3 \\ b & b & 4 \\ b & b & b \end{bmatrix}$, is singular for three values of b.

Problem 5

What 3 by 3 matrix E_{21} subtracts 4 times row 1 from row 2? What matrix P_{32} exchanges row 2 and row 3? If you multiply A on the right instead of the left, describe the results AE_{21} and AP_{32}.

Problem 6

Write down the 3 by 3 matrices that produce these elimination steps:

1. E_{21} subtracts 5 times row 1 from row 2.
2. E_{32} subtracts -7 times row 2 from row 3.
3. P exchanges rows 1 and 2, then rows 2 and 3.

Problem 7

If every column of A is a multiple of (1,1,1), then Ax is always a multiple of (1,1,1). Do a 3 by 3 example. How many pivots are produced by elimination?

Problem 8

This 4 by 4 matrix will need elimination matrices E_{21} and E_{32} and E_{43}. What are those matrices?

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Problem 9

If E adds row 1 to row 2 and F adds row 2 to row 1, does EF equal FE?