**Chapter 11: AC Power Analysis**

Table of Contents

[11.1 Introduction 2](#_Toc44584738)

[11.2 Instantaneous and Average Power 3](#_Toc44584739)

[11.3 Maximum Average Power Transfer 6](#_Toc44584740)

[11.4 Effective or R.M.S. Value 8](#_Toc44584741)

[11.5 Apparent Power and Power Factor 9](#_Toc44584742)

[11.6 Complex Power 10](#_Toc44584743)

[11.8 Power Factor Correction 12](#_Toc44584744)

## 11.1 Introduction

Power is the rate of energy transferred per unit time.

For DC circuits, .

However, resistors are energy dissipating elements, whereas capacitors and inductors are energy storing elements. Thus, they must be dealt with differently.

## 11.2 Instantaneous and Average Power

An instantaneous value is the value at a specific moment in time. The average value is the algebraic summation of all instantaneous values, divided by the total time.

Thus, for a circuit with voltage and current , the instantaneous power is given by

The average voltage would be

Similarly, the average power is given by

In more detail,

Thus, the instantaneous value of power is given by

From the formula ,

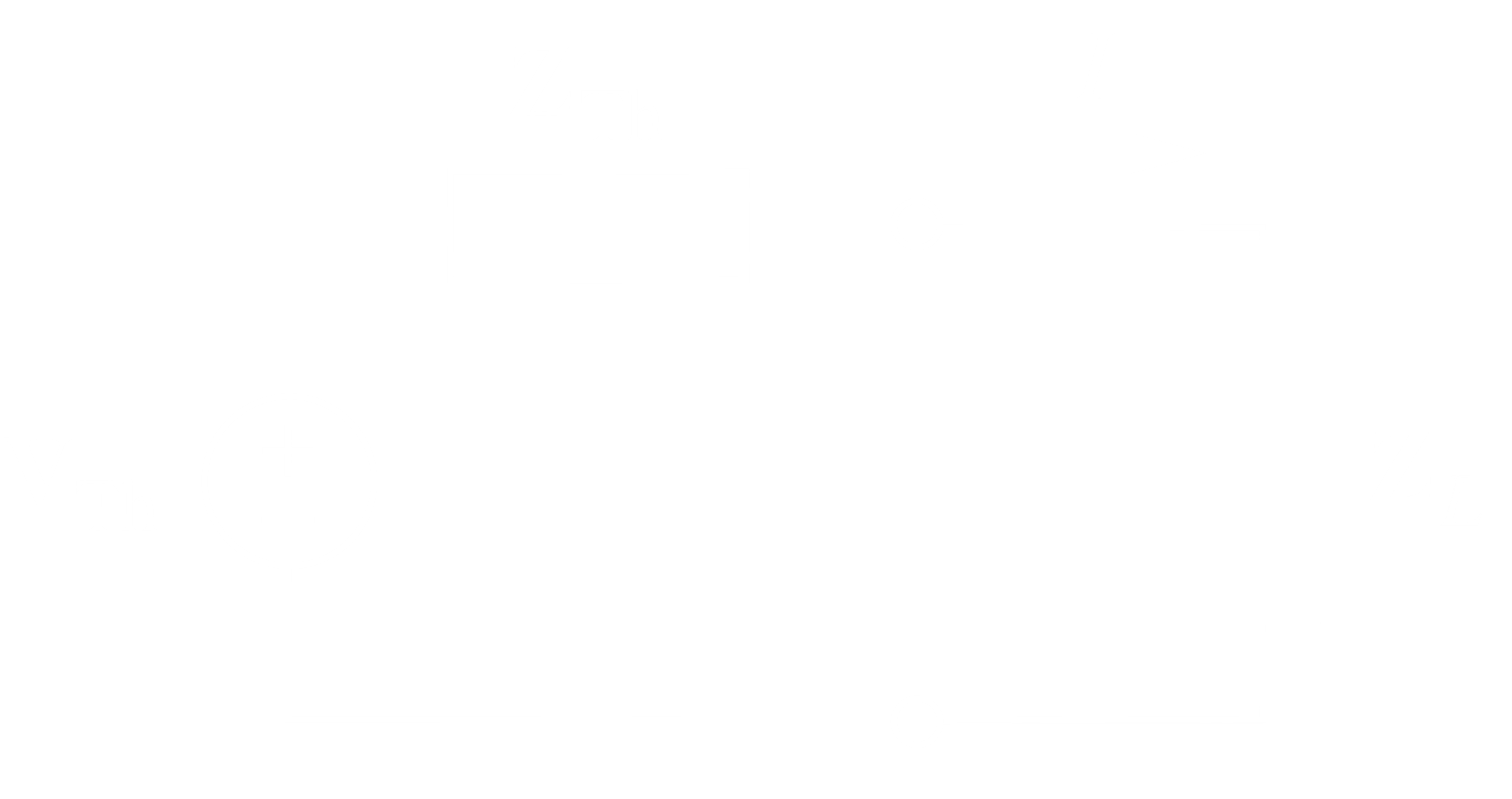
The average value of power is given by

Here, is a constant, and the average value of is . The only integrable value is , which does not have an average value of since it does not have the term .

Thus,

Example 11.1

## 11.3 Maximum Average Power Transfer



We know, for , . Thus,

Thus,

Thus,

Here, is the real part of and is from .

If the load is purely real, , thus

As such, where is from and is from .

## 11.4 Effective or R.M.S. Value

The Root Mean Square (R.M.S.) Value of a periodic current is the DC current that delivers the same average power to a resistor as the periodic current.

Similarly,

From here, the average power is given by

## 11.5 Apparent Power and Power Factor

It is expected that multiplying any voltage and current would result in some power. This, . is known as the apparent power and is measured in (volt amperes). Apparent power is a combination of two types of power, real power, which gets consumed in the circuit, and reactive power, which will be discussed later.

Thus,

This suggests that there is a difference between the actual power being consumed by the circuit (given by ) and the apparent power being supplied to the circuit (given by ). This makes the system inefficient. The ratio of real power to apparent power is known as the power factor.

Power Factor ()

Ideally, the power factor should be , i.e. the apparent power being delivered to the system is what is actually being consumed by the system.

Formally, the power factor is the cosine of the phase difference between voltage and current. is also known as the power factor angle.

For a system where and ,

Thus, the angle of load impedance is the same as the power factor angle. This would lead to the conclusion that for an ideal system, where the power factor is (and thus the power factor angle is ), the load is purely resistive.

## 11.6 Complex Power

Given the phasor form and , the complex power absorbed by an AC load is given by

This can also be expressed as , where is the real power and is the reactive power. Note that the is **bold** to represent complex power.

We already know that apparent power is the combination of real and reactive powers. An easier way to define it is that apparent power is the magnitude of complex power. Thus, apparent power .

We can derive a few formulas from this.

Real Power:

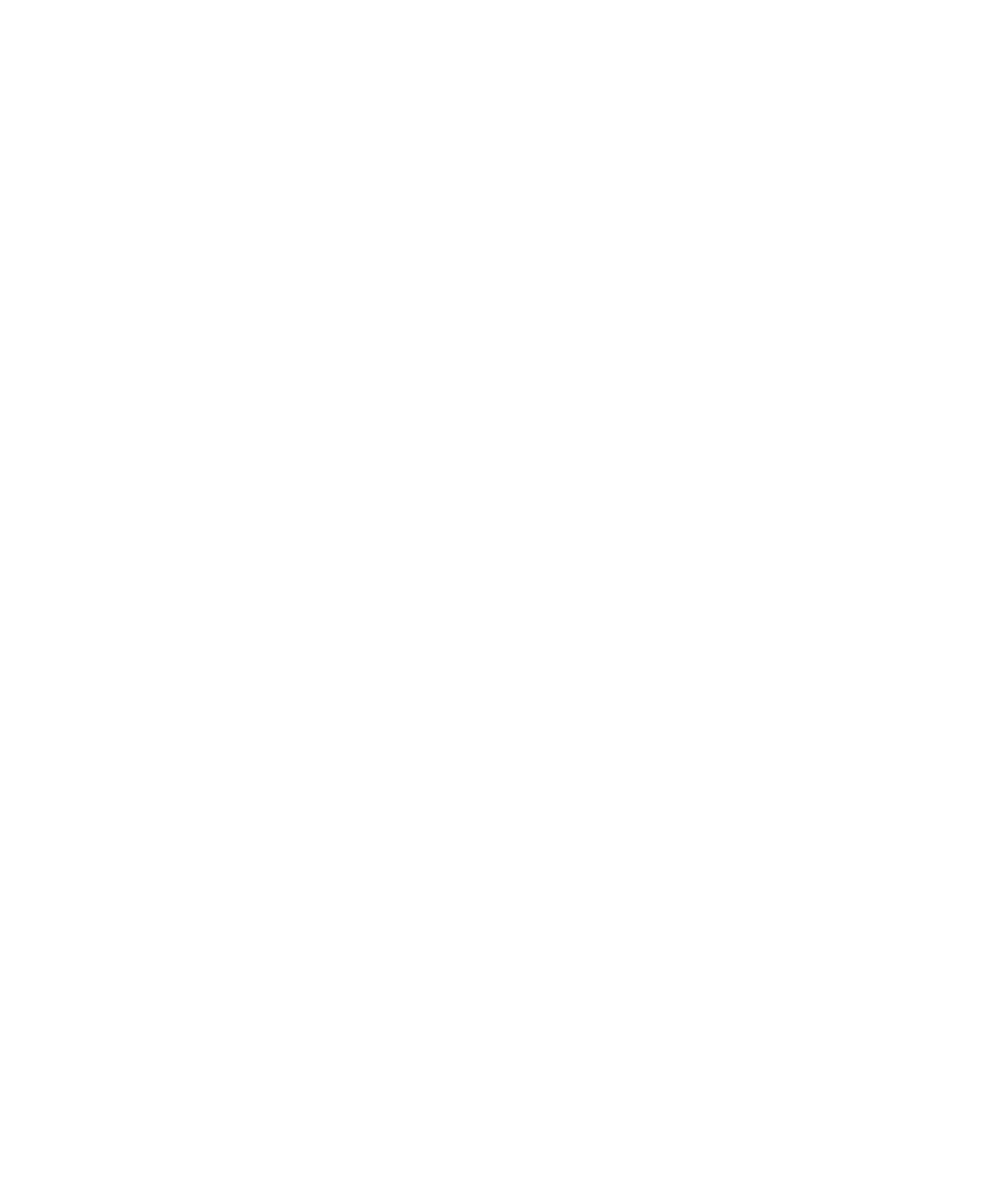
Reactive Power:

Power Factor:

Average Power: [No clue why this is here. 😐]

Finally, reactive power is the power that relates to the reactance of a circuit instead of the resistance. Reactive power is stored, not used. For example, if of power is generated and is actually used by the consumer, is stored in the inductive load.

Power Triangle:



The power triangle gives us a simple way to look at all the types of powers. From here, we can derive all the formulas given previously. Again, note that the in the triangle is in bold, representing complex power.

Note that is the power factor angle, i.e. .

The power triangle shows the importance of complex power. contains all the information required about the power in the system. Its real part is , its imaginary part is , its magnitude is and the cosine of its phase angle is the power factor.

## 11.8 Power Factor Correction

The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

For a given load, the real power is given by and the reactive power is given by .

The power factor is to be increased from to without altering the real power, i.e. . Thus, the new reactive power must be .

The reduction in reactive power can be achieved by a capacitor. The reactive power across the capacitor is thus given by

Thus, the value of capacitance required to reduce the reactive power can be found.

Commonly, inductive loads are used. However, it is possible for the load to be capacitive as well. In that scenario an inductor must be used instead of a capacitor for power factor correction. The reactive power across the inductor would be given by

Example 11.8

