

## Assignment 01

# Graph Traversal

Please submit your solutions in PDF format. The PDF must be typed, NOT handwritten. Solution for each problem must start on a new page. The solutions should be concise; complicated and half-witted solutions might receive low marks, even when they are correct. Solutions should be submitted on the course website.

### Problem 1: Collaborators

[2 points]

List the name of the collaborators for this assignment. If you did not collaborate with anyone, write “None” (without the quotes).

**Problem 2: Welcome to the World of Graphs**

[17 points]

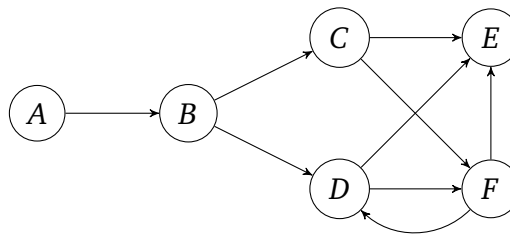
**(a) [4 points] Representation to Graph**

Using the following graph representations, draw the directed graph associated with them. Here,  $G_1$  is represented by an adjacency matrix.

	0	1	2	3	4
0	0	1	0	1	0
1	1	0	0	0	1
2	1	0	0	0	0
3	1	0	1	0	1
4	1	0	0	1	0

**(b) [3 points] Graph to Representation**

Write down the adjacency list representation of the graph shown below. The vertices should be listed in lexicographical order.

**(c) [6 points] Traversal**

Run both BFS and DFS on the graph from part (b). Take A as your source node. While performing each search, each outgoing neighbors will be visited in lexicographical order. For each search, list the vertices in the order in which they were first visited.

**(d) [4 points] DAG**

Removing one single edge from the graph in part (b) can make it a DAG. Find out all such possible edges with this property, and for each, state the topological sort order of the resulting DAG.

**Problem 3: Diameter of a Tree**

[5 points]

The diameter  $d(u)$  of a node  $u \in V$  denotes the distance to the farthest node  $v \in V$ . The diameter  $d(G)$  of an undirected graph  $G = (V, E)$  is the largest diameter of any nodes. Consider that,  $G$  is a tree, that means any two vertices are connected by exactly one path. Describe a  $O(V + E)$ -time algorithm to compute the diameter of a given connected undirected graph.

**Problem 4: Properties of a Graph**

[16 points]

(a) [8 points] **Pickle Mick**

Mick has gotten into a pickle. His neighbors do not get along with each other. You see, Mick's grandson, Rorty wants to invite all their neighbors to a party, but that would cause chaos. That's why Mick advised Rorty to throw two separate parties: one on Saturday and another on Sunday. Rorty wants to invite all his neighbors to at most one of the parties. He is worried that if two neighbors who don't like each other are invited to the same party, there will be too much drama. To solve the problem, Mick has created a list of conflicting pairs of neighbors. Given the list, you need to come up with a linear time algorithm to determine whether Mick and Rorty can invite all their neighbors to the parties so that two conflicting neighbors do not come to the same party.

(b) [8 points] **Mucced**

BaceFook is a new social network. Every pair of users in BaceFook is either friends with each other or not. Two users  $u_1$  and  $u_2$  are considered mutual if there exists a sequence of users starting from  $u_1$  and ending with  $u_2$  where each adjacent pair of users in the sequence are friends. Zark Muckerberg has recently joined BaceFook and is not friends with anyone. Given a list of all pairs of users that are mutual on BaceFook, describe an efficient algorithm to determine the minimum number of users with whom Zark needs to become friends with to be mutual with every user in BaceFook.