**Chapter 6: Applications of Definite Integrals**

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## 6.1 Volume Using Cross-Sections

Volume,

For rotation about -axis, volume by disks is used.

Volume

For rotation about -aaxis,

Example 4

The region between the curve , , and the -axis is revolved about the -axis to generate a solid. Find its volume.

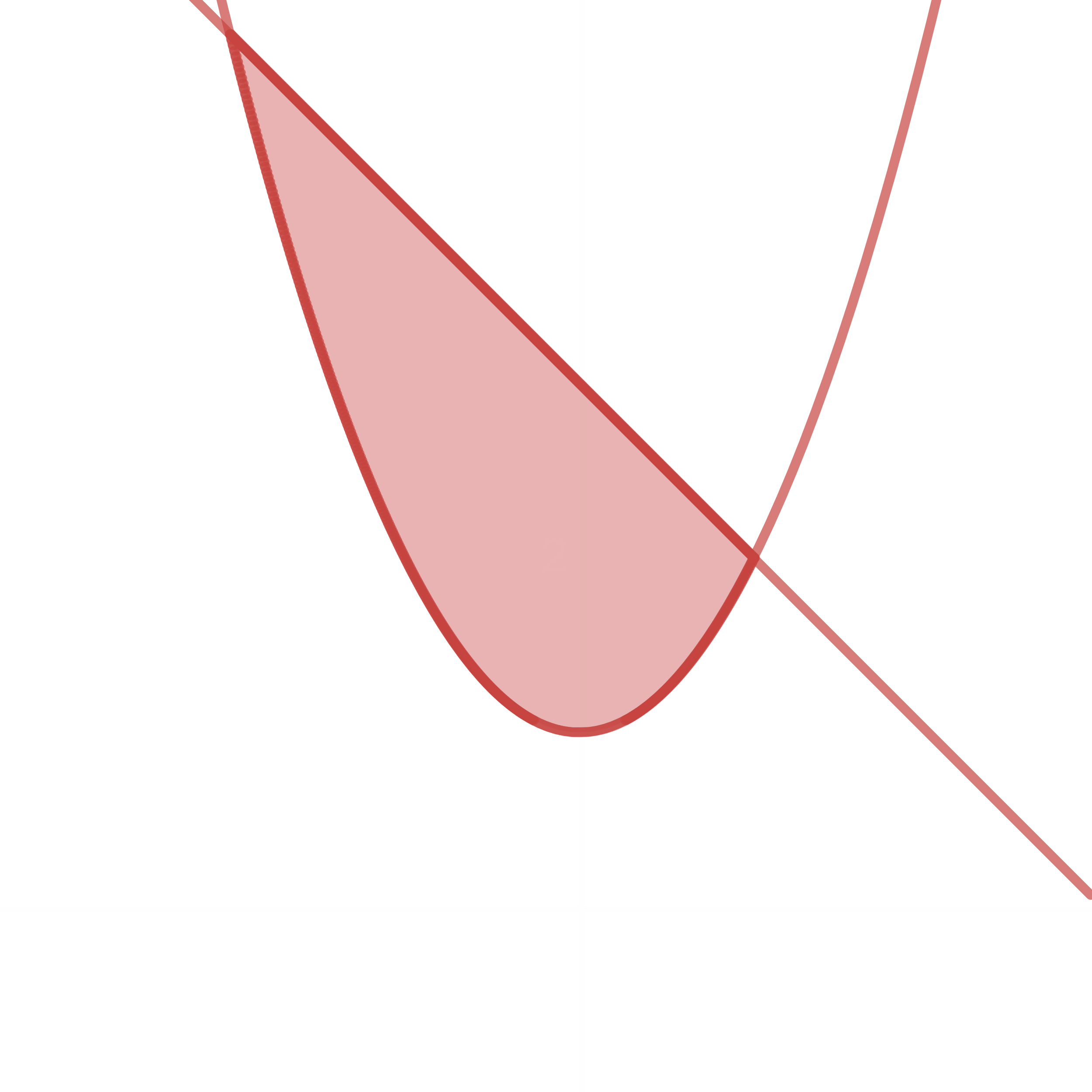
Example 5

The circle is rotated about the -axis to generate a sphere. Find its volume.

Volume by Washers for Rotation About the -Axis

Example 9

The region bounded by the curve and the line is revolved about the -axis to generate a solid. Find the volume of the solid.



The region is bounded above by the line , below by the curve , on the left by and on the right by . Thus, the outer radius is and the inner radius is .

Volume

Exercise 6.1

Find the volume of the solid generated by revolving the region bounded by

Exercise 19 – 28

Volume by Disk

Exercise 19

The volume of the region bounded by , and and rotated about the -axis is

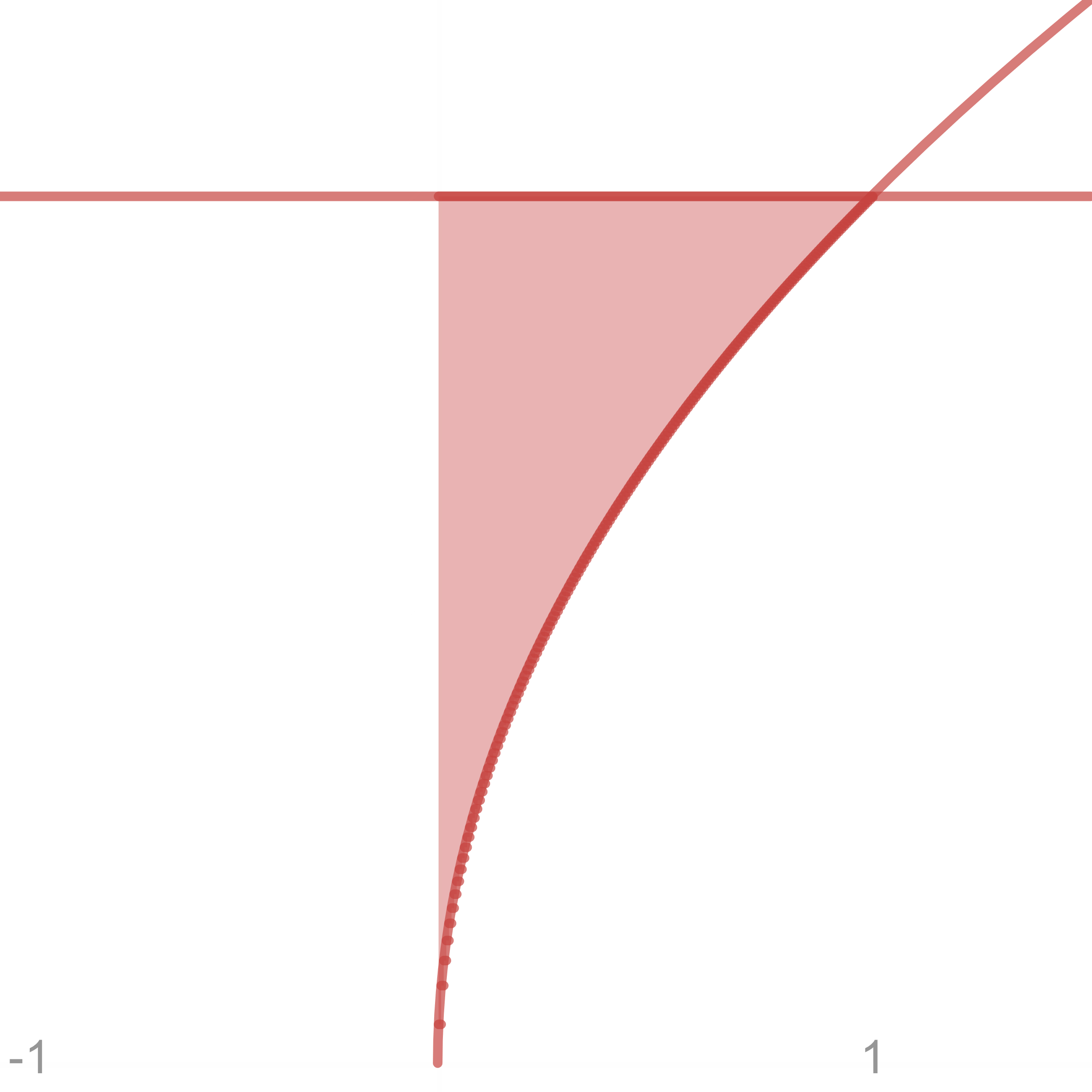
Exercise 23

The volume of the region bounded by , and and rotated about the -axis is

Exercise 39 – 44

Volume by Washers

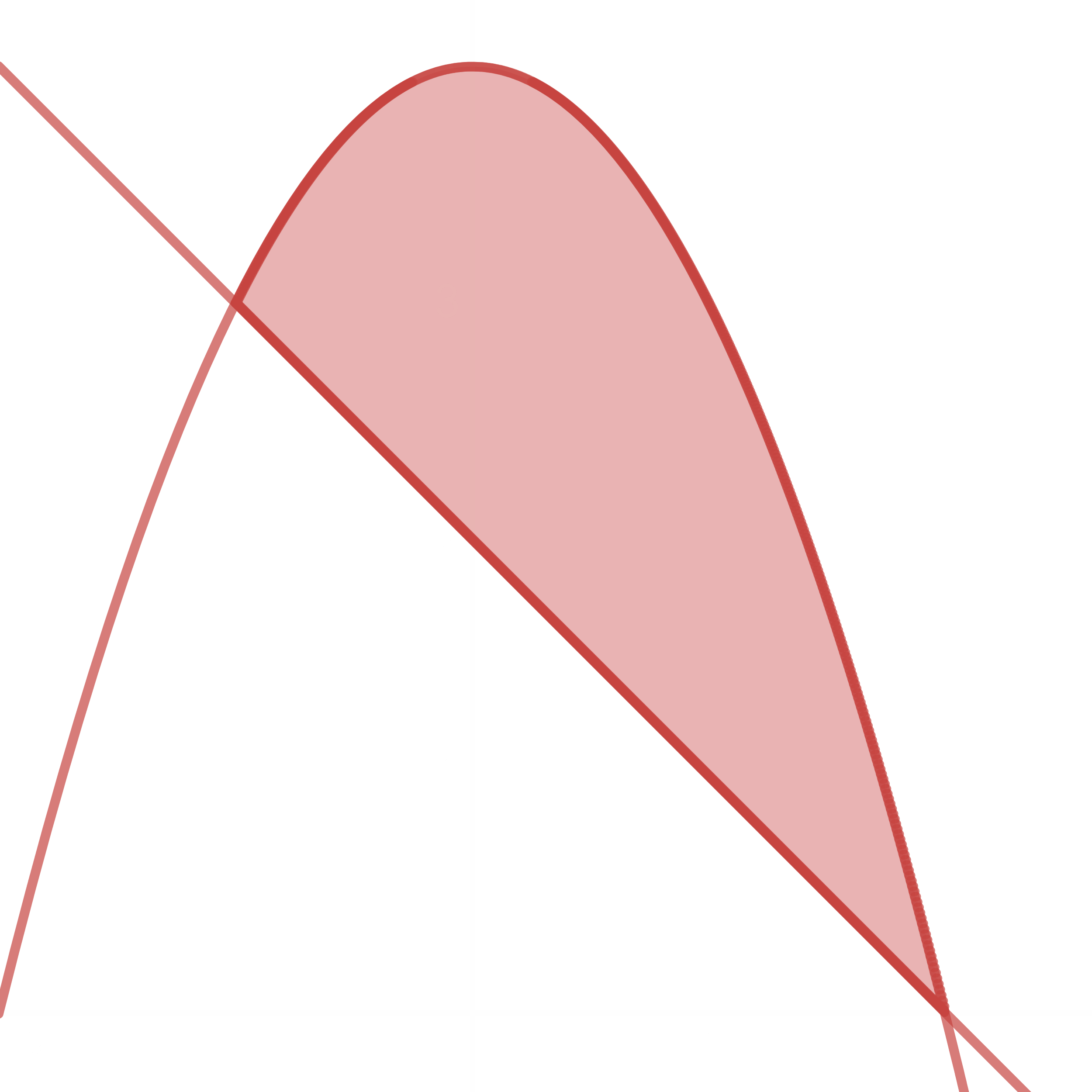
Exercise 40



The region is bounded above by the line , below by the curve , on the left by and on the right by .

The volume of the region rotated about the -axis is

Exercise 42



The region is bounded above by the curve , below by the line , on the left by and on the right by .

The volume of the region rotated about the -axis is

## 6.3 Arc Length

Arc length of a curve from the point to is given by

Similarly, for the curve ,

Example 1 – 4

Example 3

Find the length of the curve , .

By definition we know,

Arc Length,

Exercise 1 – 10

Exercise 2

from to

Arc Length,

Let

Exercise 19 – 20

Exercise 19

a) Find the curve through the point whose length integral is

b) How many curves are there?

One

## 6.4 Areas of Surfaces of Revolution

The area of the surface generated by revolving the graph of on about the -axis is given by

For ,

Exercise 13 – 23

Exercise 14

-axis

We know the area of the surface generated by revolving the graph of on about the -axis is

Let

Exercise 9 – 12

Exercise 10

Find the lateral surface area of the core generated by revolving the line segment , , about the -axis. Check your answer with the geometric formula:

Given,

We know, the area of the surface generated by revolving the graph of on about the -axis is

Slant Height

Base Circumference

(Proven)