**Differentials**

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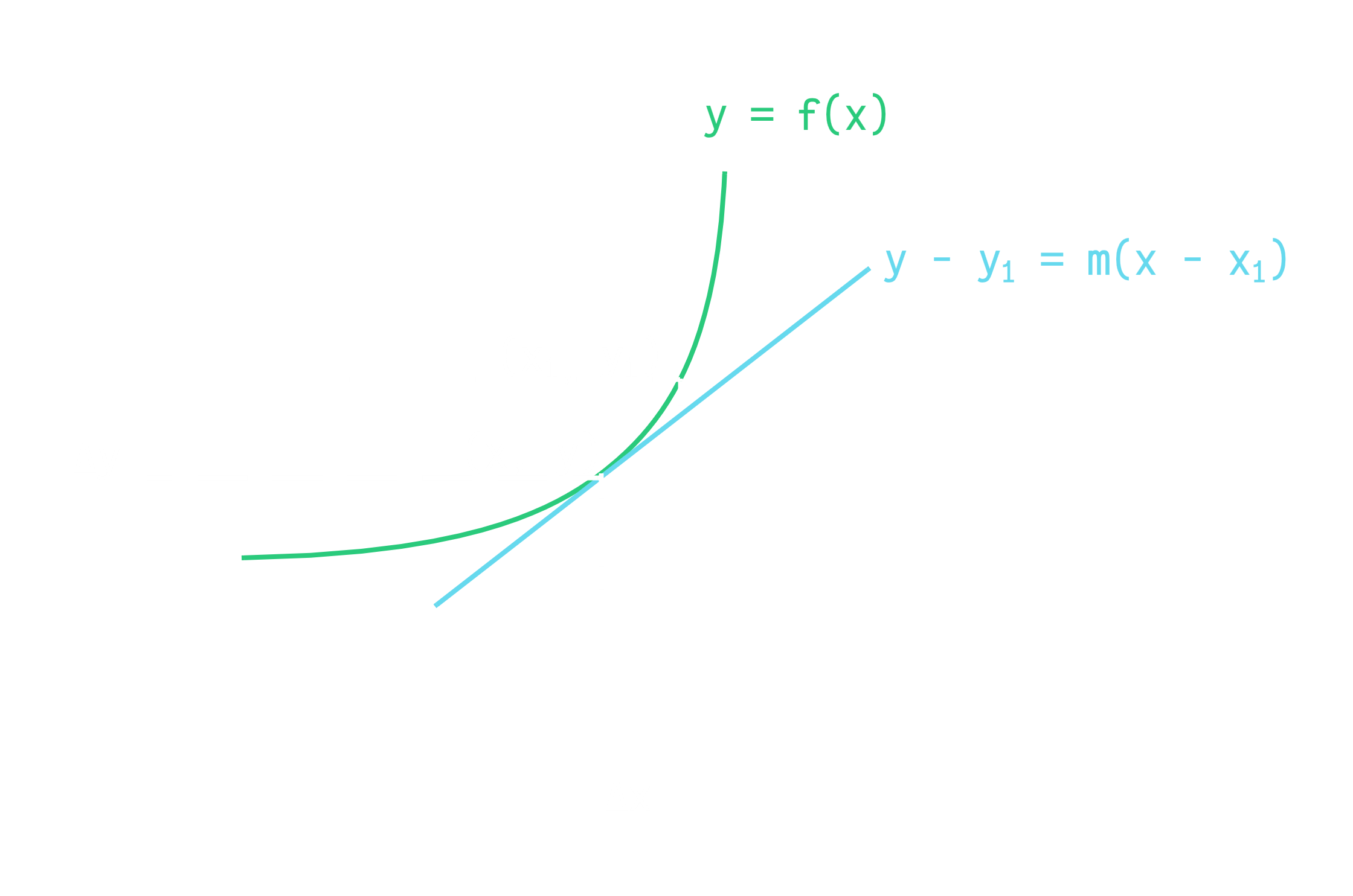
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For the curve , the **tangent** at the point is given by

where is a point that is very close to the point on the same curve.

**and**  are also denoted as  **and**  and is also denoted as . However, there is a small issue here. Consider the graph below:



At the point , there is a **tiny difference** between the **curve** and the **tangent**. If we consider the **vertical** difference, the height different between and on the **tangent** is , while the height difference between the same points on the **curve** is . is the **increment** and is the **differential**. We assume that .

From here,

We switched from to , to indicate that this is the **linear approximation** of , not the exact value of at .

Say . The square root of is a little difficult to find, so lets say we decide . Thus,

This gives us . The exact value of is .

## Functions of Two Variables

If we start considering functions of **two variables**, we get similar terminology. For , and are the increments of and respectively and the increment of is given by:

Since we are using **linear approximation**, we can say that and . Based on this,

We can of course extend this to accommodate more independent variables.

## Differentiability

For a **differentiable** function, , we can say that the differential is **approximately** the same as .

A similar approximation can be made for functions of **two variables**, and if the approximation can be made, the function is said to be differentiable at the point .

Here, both and as .

The function is further said to be **differentiable in a region** if it is differentiable at **every point** in the region.

Say we have a function .

Thus, is differentiable.

If is a function of and and and are **continuous** in an open region , this is a **sufficient condition** to say that is **differentiable** in . Similarly, if is **differentiable** at a point , it is also **continuous** at that point. However, note that itself being continuous does not guarantee that it is differentiable. Both and must be continuous for to be differentiable.

For very small values of and , we can ignore and . This makes . Thus,

The approximation of is again, a **linear approximation**.