**Multiple Server Queuing Systems**

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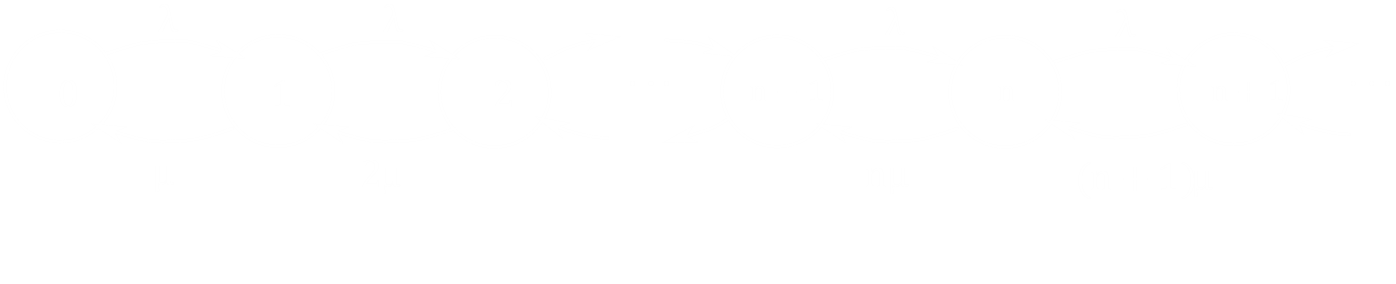
## Infinite Server Queueing System

Imagine that we have the same system as an **SSQS**, but this time, we have an **infinite number of queues**, an system. Of course, such a system would be purely hypothetical, but let’s analyse it anyways.

In an infinite server queueing system, we would not need a **queue** at all. Since we have an **infinite** number of **servers**, every customer would be assigned a server immediately after arriving.

### State Transitions

The state transition diagram for such a system would look like this:



Each state tells us the number of customers in the system. Thus, since customers are **arriving** at a rate of , the system moves from each state to the next at the rate of . However, the **departure rate** is interesting.

The rate at which the system goes from one state to the previous state is proportional to the **number of customers** in the system. This is why the system goes from state to state at the rate , but goes from state to state at the rate .

Think about why this is. In an SSQS system, we had a **single server**. This meant that one customer could not be served until the previous one had been served. This kept the departure rate **constant**. In this system however, we can server 2 (or more) customers **parallelly**. This means that the service rate must increase by a factor equal to the number of services being performed parallelly. Thus, the service rate becomes , being the number of customers in the system.

### Steady State Probabilities

For an SSQS system, we saw that the steady state probabilities were:

Since the departure rates have changed, for this system, the equations become:

However, since all are equal and all are equal,

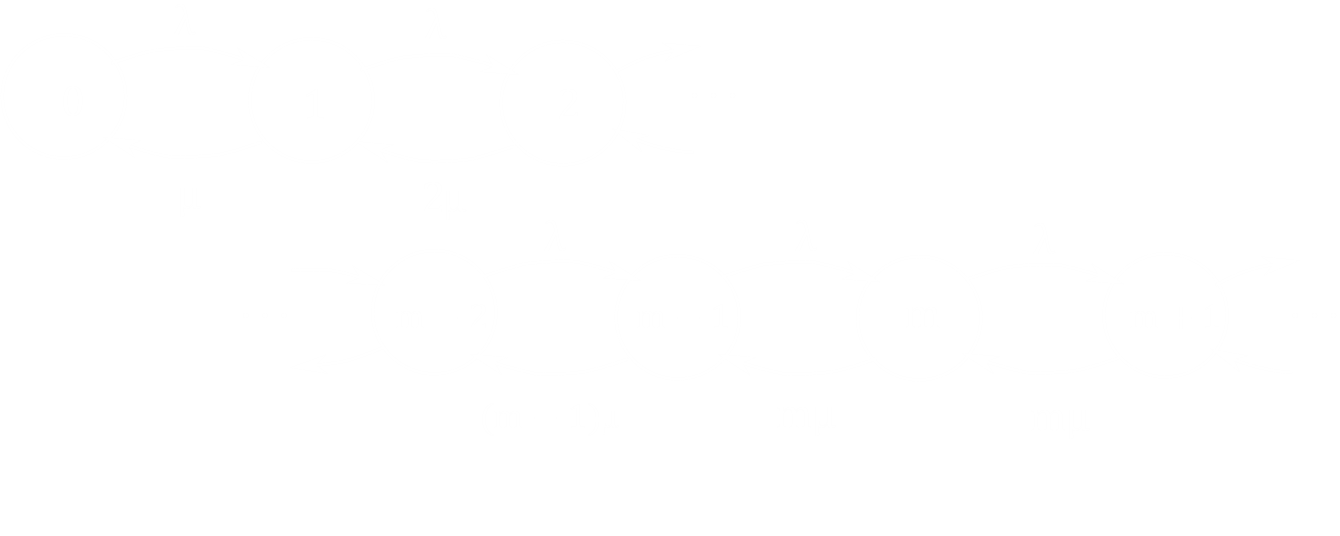
This is just a **Poisson distribution**. The **average** value for a Poisson distribution is given by:

From here, applying **Little’s Formula**,

## Server Queueing System

Now consider a system with servers. In this system, the **maximum** departure rate is . If we get more than customers, the extra customers join the **queue**.

### State Transition Diagram



The diagram above shows the same situation. The departure rates get larger as we go into higher states, but the maximum rate of is not crossed.

### Steady State Probabilities

The steady state probabilities are:

Here, .