**Laurent’s Theorem and Cauchy’s Residue Theorem**

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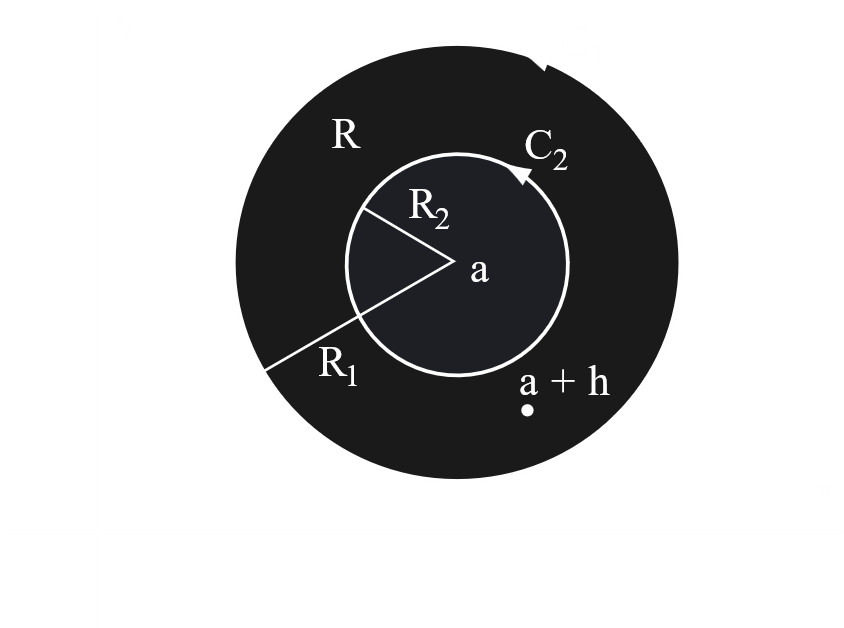
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## Laurent’s Theorem

If we are required to **expand** about a point where is **not analytic**, we user **Laurent’s Theorem**.

Say we have **two circles**, and , both with their centre at , one with radius and the other with radius .



The region **bounded** by the two circles is called an **annular region**. Suppose is analytic in this region, but not inside .

For a scenario like this,

where

Here, the part is called the **analytic** part while the part is called the **principle** part. If the principle part is **zero**, Laurent’s Series reduces to **Taylor’s Series**.

## Residue in Complex Number

If we wish to **integrate** a Laurent’s series term by term, a lot of the integrals will become **zero**. The terms that do not become zero are called **residues**.

The **first coefficient** of the **principle part**, i.e. , is the residue of at .

### Finding Residues

1. The residue at a **simple pole** is given by
2. The residue at a **pole or order**  is given by

Example

Determine the poles of over and the residue at each pole.

The poles are , and .

Since is outside the region (the circle of radius ), we can exclude it.

### Residue Theorem

If a function is **analytic** in a **closed curve** except at finite poles within , then

Example

over