Assignment 04
Divide and Conquer

Please submit your solutions in PDF format. The PDF must be typed, NOT handwritten. Solution for each problem must start on a new page. The solutions should be concise; complicated and half-witted solutions might receive low marks, even when they are correct. Solutions should be submitted on the course website.

Problem 1: Collaborators [2 points]

List the name of the collaborators for this assignment. If you did not collaborate with anyone, write "None" (without the quotes).

Problem 2: Go Off on a Tangent [5 points]

Assume that you are running Convex Hull Algorithm (as taught in class) on a set of points S. For $CH(A)$ $(a_1, a_2, \ldots, a_p)$, let a_1 be the point with maximum x. For $CH(B)$ $(b_1, b_2, \ldots, b_q)$, let b_1 be the point with minimum x. Let L as the vertical line that separates A and B. We define $y(i, j)$ as the y-coordinate of the intersection between L and the line segment (a_i, b_j).

Argue that (a_i, b_j) is the uppertangent if and only if it maximizes $y(i, j)$.

Problem 3: GiveIn: Shakes, Fries, Burgers [33 points]

Consider that your friend wants to setup a burger joint chain, *GiveIn*. *GiveIn* restaurants will be opened on different locations of a city. The city can be considered as an undirected graph $G = (V, E)$, where each potential location is denoted by the set of vertices. The adjacent locations are connected via the edges. Now, to avoid competition between two *GiveIn* restaurants, they won't be opened on adjacent vertices. Each vertex u has an integer $p_u (\geq 0)$ associated with it, denoting the profit of opening restaurant in that location. Your goal is to design an algorithm to find out a set of vertices O that maximizes the total profit $\sum_{u \in O} p_u$. Consider that G is acyclic.

(a) [5 points] Consider the following greedy approach of opening *GiveIn*:

- Set $O = \emptyset$
- Sort the vertices in the descending order of their profit.
- Repeat the following steps until V is empty.
  - Pick the first vertex u (that has the highest profit) from V and add it to O.
  - Remove u and all of its neighbors from V.

Draw an example graph where the algorithm will not work.

(b) [10 points] Provide an efficient algorithm to solve the problem.

(c) [8 + 4 points] Assume that all the potential locations are equally good. So the goal is to find out the largest set of vertices to open the restaurant. Provide a simple greedy algorithm to solve this problem and argue its correctness.

(d) [6 points] Assume that the graph is not necessarily acyclic. Provide an algorithm to solve the problem.