

Problem 1

Start with the vector $u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Multiply again and again by the same “Markov matrix”, $A = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$. The next three vectors are u_1 , u_2 and u_3 . Find out these vectors. What property do you notice for all four vectors u_0 , u_1 , u_2 and u_3 ?

Problem 2

For the vectors $v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $w = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ test the Schwarz inequality on $v \cdot w$ and the triangle inequality on $|v + w|$. Find $\cos \theta$ for the angle between v and w .

Problem 3

Find a unit vector a in the direction of $w = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. Find a unit vector v that is perpendicular to a . How many possibilities of v ?

Problem 4

Describe geometrically (line, plane or all of \mathbb{R}^3) all linear combinations of

$$a) \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \begin{bmatrix} 5 \\ 8 \\ 11 \end{bmatrix} \quad b) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \quad c) \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

Problem 5

In the xy plane, mark all nine of these linear combinations-

$$c \begin{bmatrix} 3 \\ 2 \end{bmatrix} + d \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

with $c=0,1,2$ and $d=0,1,2$

Problem 6

Find two different combinations of the three vectors $u=(1,3)$, $v=(2,7)$ and $w=(1,5)$ that produce $b=(0,1)$. If I take any three vectors u,v,w in the plane, will there always be two different combinations that produce $b=(0,1)$?

Problem 7

Write down three equations for c,d,e so that $cu+dv+ew=b$.

$$u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$$

Can you find c,d,e for this b ?

Problem 8

Draw three dimensional vectors u,v,w so that their combinations $cu+dv+ew$ fill only a line. Draw vectors u,v,w so that their combinations $cu+dv+ew$ fill a plane.

Problem 9

Four corners of this cube are $(0,0,0)$, $(0,0,1)$, $(0,1,0)$ and $(0,0,1)$. Find the other four corners of this cube. Find the co-ordinates of the center of this cube. What are the center points of the six faces of this cube?

