**Chapter 05**

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## Applications of Higher Order Linear Differential Equations

The differential equation of the vibration of a mass on a spring:

Here, the forces are:

– The force of gravity, , as it acts downwards.

– The restoring force of the spring.

By Hooke’s Law,

where is the proportionality constant of the spring. This force tends to act upwards, thus .

In equilibrium position, i.e. , the restoring force magnitude is equal to the gravitation force, but in the opposite direction.

Thus,

Generally, .

– The resistive force of the medium, also known as the damping force. The resistive force is proportional to the magnitude of the velocity.

If this force acts upwards,

is called the damping constant.

– Any externally impressed force. At time , .

From ,

If , there is no damping. Such motion is called undamped motion, as opposed to damped motion.

If , there is no external force. Such motion is called free motion, as opposed to forced motion.

From free, undamped motion, .

A weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to rest in equilibrium position, thereby stretching the spring to . The weight is pulled down below its equilibrium position and released from rest at . Find the displacement of the weight as a function of time.

We know that, for free, undamped motion,

By Hooke’s law,

Auxiliary Equation:

A weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to rest in equilibrium position, thereby stretching the spring to . The weight is pulled down below its equilibrium position and released at with initial velocity directed downwards. Find the displacement of the weight as a function of time.

A weight is attached to the end of a spring. The spring is stretched . Find the equation of motion if the weight is released from rest at a point above equilibrium.

## 5.3 Example on Free and Damped Motion

Exercise 1:

A weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in equilibrium position, thereby stretching the spring . The weight is then pulled down below its equilibrium position and released at . No external forces are present, but the resistance of the medium in pounds is , where is the instantaneous velocity in . Determine the resulting motion of the weight on the spring.

As no external forces are present, .

Auxiliary equation:

## 5.6 Applications of Electric Circuit Problems

### Kirchhoff’s Voltage Law

Sum of voltage drops is equal to total electromotive force in a closed circuit.

Voltage drop across resistors:

Voltage drop across inductors:

Voltage drop across capacitors:

By the law,

and

Differentiating with respect to ,

Exercise 1

A circuit has a series electromotive force given by , a resistor of , an inductor of and a capacitor of . If the initial current and the initial charge on the capacitor are both , find the charge on the capacitor at any time .

We know,

Corresponding homogenous equation:

Auxiliary Equation:

U.C. Function:

U.C. Set:

Plugging this in we get,