**Tutorial Class 02 – Discrete Random Variables**

1.

a)

This is a Bernoulli trial, since a series of the same experiment with two outcomes repeatedly takes place.

Let be a random variable where

.

Thus, .

For this, we can tell that . Notice that has been used, since either probability could cause the required wins. Thus, we can define two more random variables here, and , where and . As such,

For games to take place,

b)

c)

Let be the event that wins and be the event that the series ends in games. Thus,

2. a)

Say the brothers reach a decision in tosses. Thus, . Thus, each round of tossing is a Bernoulli experiment. Since we keep going until the first success (the first success in this case being when all three tosses do not produce the same result), , where is the probability of a ‘success’.

Let and . Both of these are binomial random variables with three possible values. Since is the probability that an odd result occurs in a round of tosses,

Notice that we did not bother with here. This is because when we calculate , we are simultaneously calculating as well, since these are complements. Similarly, when considering , is simultaneously considered.

Since we have a value of now,

and thus,

The probability that we do not get a decision in the first tosses is just , so

If we had found this value by summing up the PMFs, we would reach the same value.

b)

Since we have to find ,

Notice that the power has changed, since previously, we had been considering rounds instead of rounds.

3.

Let and .

Let . Since is being considered for the next customers, we can consider this to be Bernoulli experiments, with a success being when a customer makes a purchase with a credit card. Thus,

4.

Essentially, the parity checker will be unable to detect the error if an even number of bits are changed.

Let . .

Since there are two possibilities that are opposites of each other for every bit, , considering a bit being changed to be a ‘success’.

The probability that an error will be undetected is

Notice that we used a trick here to only take the even numbers.

b)

c)

For every character, there are three cases now, either there is a detected error, , there is an undetected error, , or there is no error at all.

Let . Thus, , since an undetected error can be considered a success and no errors or a detected error can both be considered failures.

I have doubts about this last part, but what I understood is that we already took into account the probability of the error actually occurring in the first place when we calculated .

5.

Let .

6.

Let , and .

and , so

Say . Thus, .

Say the player picks the number . Thus, . Thus,

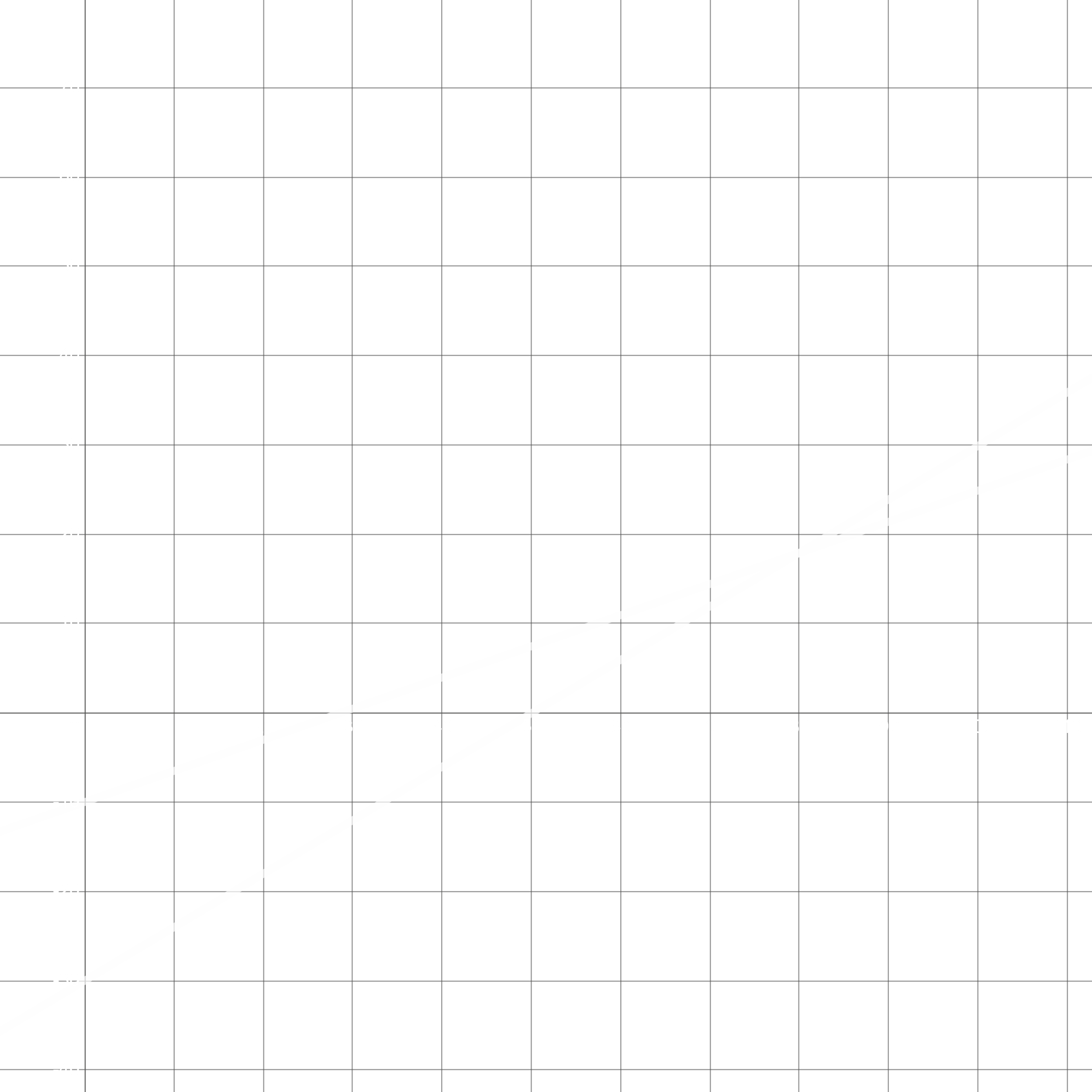
7.

The price we get for a circuit can be or . The probability of failure for an ordinary device is and that for an ultra-ordinary device is . The price for an ordinary device is and the price for an ultra-ordinary device is .

Let and . has two values, and . can be either or depending on whether or not the circuit works. Similarly, or .

For an ordinary circuit, and for an ultra-ordinary circuit .

From this, it becomes clear that we cannot find an answer just from the equations for expected value, since we have no clue what the value of is.



From , we get . Thus, below thus value, the expected value for is higher, and above this value the expected value for is higher.