Math 4441: Probability and Stochastic Processes
Tutorial Class 2: Discrete Random Variables

Following problems will be discussed in the tutorial class.

1. Suppose that two players (A and B) play a series of games that ends when one of them has won 5 games. Suppose that each game played is, independently, won by player A with probability 0.58.
   a) Find the probability that the series ends in seven games.
   b) Find the probability that the series ends in less than seven games.
   c) If the series ends in seven games, what is the probability that player A wins the series.

2. A father asks his sons to cut their backyard lawn. Since he does not specify which of the three sons is to do the job, each boy tosses a coin to determine the odd person, who must then cut the lawn. In the case that all three get heads or tails, they continue tossing until they reach a decision. Let p be the probability of heads and $q = 1 - p$, the probability of tails.
   a) Find the probability that they reach a decision in less than n tosses.
   b) If $p = 0.5$, what is the minimum number of tosses required to reach a decision with probability 0.95.

3. Past experience shows that 30% of the customers entering a clothing store will make a purchase. Of the customers who make a purchase, 85% use credit cards. Let X be the number of customers who make a purchase and use a credit card, among the next 6 customers to be entered the store. Find the probability mass function and expected value of X.

4. The simplest error detection mechanism used in data communication is *parity checking*. Usually messages sent consist of characters, each character consisting of a number of bits (a *bit* is the smallest unit of information and is either 1 or 0). Assume that the number of bits in a character is 7. In parity checking, a 1 or 0 is appended (the total number of bits in a character finally becomes 8) to the end of each character at the transmitter to make the total number of 1's even (and the parity checking mechanism is known as even parity).

   The receiver checks the number of 1's in every character received, and if the number of 1's is odd it signals an error. Suppose that each bit in a character is received correctly with probability 0.999, independently of other bits of the characters.

   a) Find the probability that character is received in error, but the error is not detected by the parity check mechanism.
   b) Find the probability that the parity check mechanism detects the error, if one or more bits are incorrectly received.
   c) Suppose that a message consisting of six characters is transmitted. Find the probability that the message is erroneously received (at least one character is erroneously received), but none of the errors is detected by the parity check mechanism.

5. Customer arrives at a bookstore at a Poisson rate of six per hour. Given that the store opens at 9:30 am, what is the probability that exactly one customer arrives by 10:00 am and at least one customer by 10:30 am.

6. A game is often played in carnivals and gambling houses is called chuck-a-luck, where a player bets on any number 1 through 6. Then three fair dice are tossed. If one, two, or all three land the same number as the player's, then he or she receives one, two, or three times the original stake plus his or her original bet, respectively. Otherwise, the player loses his or her stake. Let X bet the net gain of the player per unit stake. First find the probability mass function of X; then determine the expected amount that the player will lose per unit of stake.

7. A particular circuit works if all 10 of its component devices work. Each circuit is tested before leaving the factory. Each working circuit can be sold for k dollars, but each nonworking circuit is worthless and must be thrown away. Each circuit can be built with either ordinary devices or ultrareliable devices. An ordinary device has a failure probability of $q = 0.1$ while an ultrareliable device has a failure probability of $\frac{q}{2}$, independent of any other device. However, each ordinary device costs \$1 while an ultrareliable device costs \$3. Should you build your circuit with ordinary devices or ultrareliable devices in order to maximize your expected profit $E[R]$? Keep in mind that your answer will depend on k.