Schaum's Outline - Complex Variables - Problem 1.132

Md. Mesbahose Salekeen

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Show that for any real numbers p and m,

$$e^{2mi\cot^{-1}p}\left\{\frac{pi+1}{pi-1}\right\}^m = 1$$

$$\begin{aligned}
L.H.S &= \left(e^{mi\cot^{-1}p}\right)^2\left\{\frac{pi+1}{pi-1}\right\}^m \\
&= \left(\cos(\cot^{-1}p) + i\sin(\cot^{-1}p)\right)^{2m}\left\{\frac{pi+1}{pi-1}\right\}^m \\
&= \left(\frac{p}{\sqrt{1+p^2}} + i\frac{1}{\sqrt{1+p^2}}\right)^{2m}\left\{\frac{pi+1}{pi-1}\right\}^m \\
&= \left\{\frac{p^2-1+2ip}{1+p^2}\right\}^m\left\{\frac{pi+1}{pi-1}\right\}^m \\
&= \left\{\frac{p^2-1+2ip}{(1+ip)(1-ip)}\right\}^m\left\{\frac{pi+1}{pi-1}\right\}^m \\
&= \frac{(p^2-1+2ip)^m}{(pi+1)^m(1-ip)^m}\left\{\frac{pi+1}{pi-1}\right\}^m \\
&= \frac{(p^2-1+2ip)^m}{(-1)^m(pi-1)^{2m}} \\
&= \frac{(p+i)^{2m}}{(-1)^m(pi-1)^{2m}} \\
&= \frac{(p+i)^{2m}}{(-1)^m(pi-1)^{2m}} \\
&= \frac{i^{2m}(\frac{p}{i}+1)^{2m}}{(-1)^m(pi-1)^{2m}} \\
&= \frac{(-pi+1)^{2m}}{(pi-1)^{2m}} \\
&= 1
\end{aligned}$$

[Showed]