Association Analysis (2)
Generating Rules

Rule Generation

- An association rule can be extracted by partitioning a frequent itemset Y into two nonempty subsets, X and $Y - X$, such that

  $X \rightarrow Y - X$

  satisfies the confidence threshold.

- Each frequent k-itemset, Y, can produce up to $2^k - 2$ association rules
  - ignoring rules that have empty antecedents or consequents.

Example

Let $Y = \{1, 2, 3\}$ be a frequent itemset.

Six candidate association rules can be generated from Y:

$\{1, 2\} \rightarrow \{3\}$,
$\{1, 3\} \rightarrow \{2\}$,
$\{2, 3\} \rightarrow \{1\}$,
$\{1\} \rightarrow \{2, 3\}$,
$\{2\} \rightarrow \{1, 3\}$,
$\{3\} \rightarrow \{1, 2\}$.

Computing the confidence of an association rule does not require additional scans of the transactions.

Consider $\{1, 2\} \rightarrow \{3\}$. The confidence is

$$\sigma(\{1, 2, 3\}) / \sigma(\{1, 2\})$$

Because $\{1, 2, 3\}$ is frequent, the anti-monotone property of support ensures that $\{1, 2\}$ must be frequent, too, and we know the supports of frequent itemsets.

Confidence-Based Prunning I

Theorem.

If a rule $X \rightarrow Y - X$ does not satisfy the confidence threshold,

then any rule $X' \rightarrow Y - X'$, where X' is a subset of X, cannot satisfy the confidence threshold as well.

Proof.

Consider the following two rules: $X' \rightarrow Y - X'$ and $X \rightarrow Y - X$, where $X' \subseteq X$.

The confidence of the rules are $\sigma(Y) / \sigma(X')$ and $\sigma(Y) / \sigma(X)$, respectively.

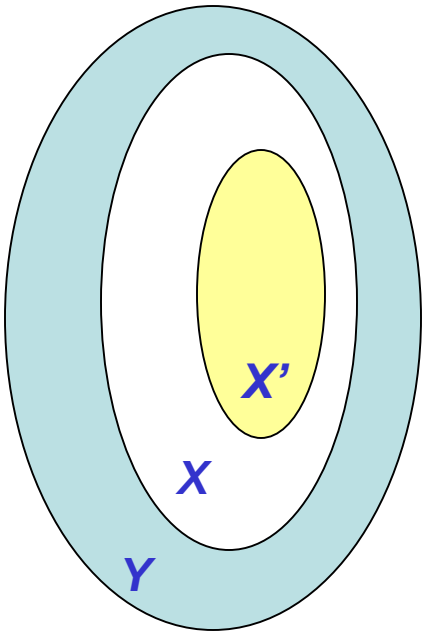
Since X' is a subset of X, $\sigma(X') \geq \sigma(X)$.

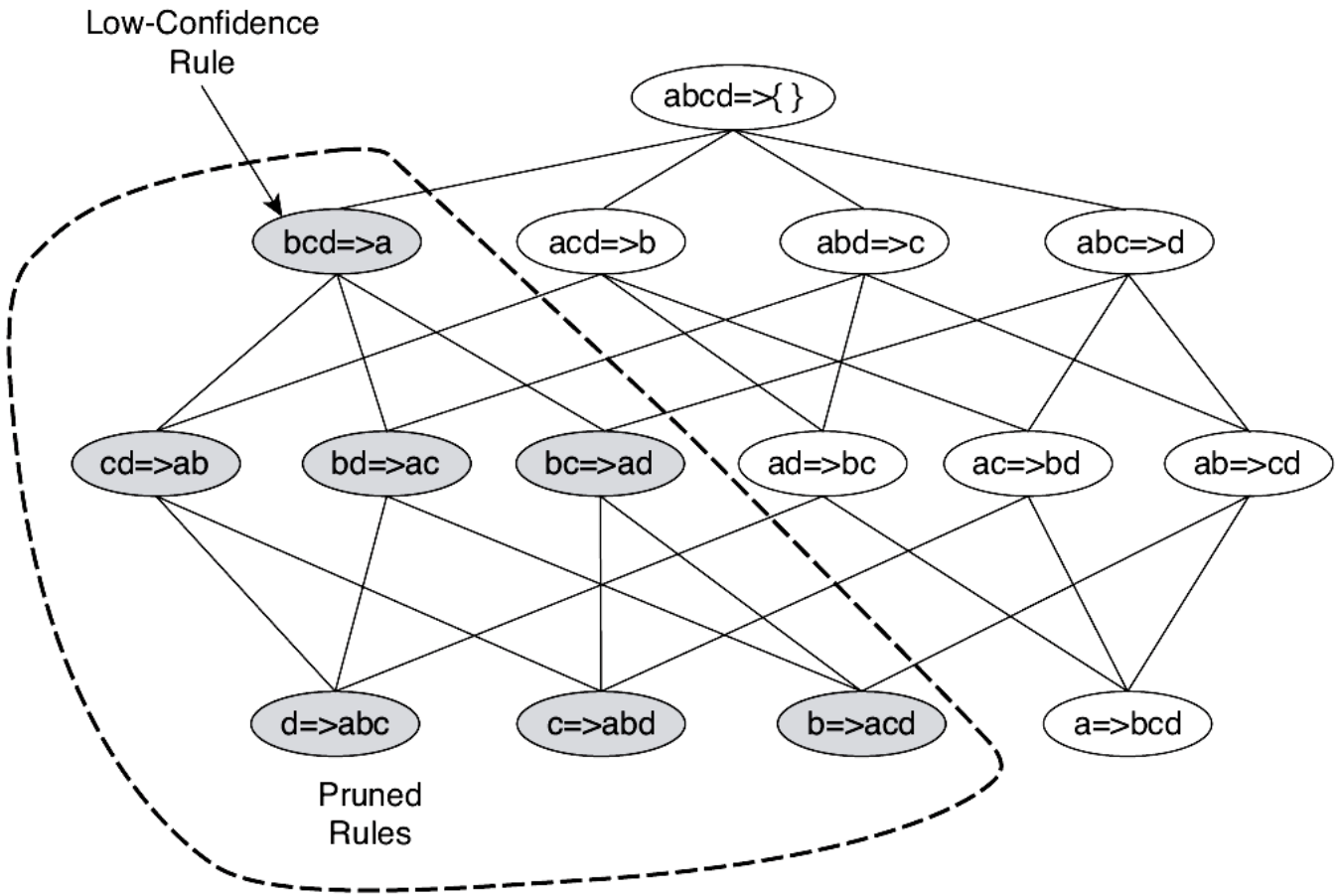
Therefore, the former rule cannot have a higher confidence than the latter rule.

Confidence-Based Prunning II

- Observe that:

  $X' \subseteq X$ implies that $Y - X' \supseteq Y - X$





Confidence-Based Prunning III

- Initially, all the highconfidence rules that have only one item in the rule consequent are extracted.
- These rules are then used to generate new candidate rules.
- For example, if
  - {acd} → {b} and {abd} → {c} are highconfidence rules, then the candidate rule {ad} → {bc} is generated by merging the consequents of both rules.

Confidence-Based Prunning IV

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

{Bread,Milk}→{Diaper} (confidence = 3/3) threshold=50%

{Bread,Diaper}→{Milk} (confidence = 3/3)

{Diaper,Milk}→{Bread} (confidence = 3/3)

Confidence-Based Prunning V

Merge:

{Bread,Milk}→{Diaper}

{Bread,Diaper}→{Milk}

{Bread}→{Diaper,Milk} (confidence = 3/4)

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