5. $300 = 10X + 20Y$

I

若偏好為 $U = f(X,Y) = X^{\frac{2}{3}} Y^{\frac{1}{3}}$，則早餐消費決策為：Max $U = f(X,Y) = X^{\frac{2}{3}} Y^{\frac{1}{3}}$

subject to $300 = 10X + 20Y$

根據最適條件：$MRS_{xy} = \dfrac{\frac{2}{3} X^{-\frac{1}{3}} Y^{\frac{1}{3}}}{\frac{1}{3} X^{\frac{2}{3}} Y^{-\frac{2}{3}}} = \dfrac{P_x}{P_Y} = \dfrac{10}{20}$

$\Rightarrow Y = \frac{1}{4} X$，代回 $300 = 10X + 20Y$，可得 $X = 20, Y = 5$，∴ 每週會買 20杯奶茶及5個漢堡

II 若偏好為 $U = f(X,Y) = X + 3Y$，則消費決策為：Max $U = f(X,Y) = X + 3Y$

subject to $300 = 10X + 20Y$

依據最適消費條件：$MRS_{xy} = \frac{1}{3} < \dfrac{P_x}{P_Y} = \dfrac{10}{20} = \dfrac{1}{2}$

∴ $X = 0, Y = 15$，因此每週會買 0杯奶茶及15個漢堡

III 若偏好為 $U = f(X,Y) = \min(X,Y)$，則消費決策為：Max $U = f(X,Y) = \min(X,Y)$

subject to $300 = 10X + 20Y$

最適條件為 $X = Y$，代回 $300 = 10X + 20Y$

$\Rightarrow X = Y = 10$ ∴ 每週會買 10杯奶茶及10個漢堡

$\Rightarrow x = y = 10$ ∴ 每週會買10杯奶茶及10個漢堡包

6. ① $400x + 600y = 12000$ $U = x^{\frac{1}{2}} y^{\frac{1}{2}}$ $MRS_{xy} = \dfrac{\frac{1}{2} x^{-\frac{1}{2}} y^{\frac{1}{2}}}{\frac{1}{2} y^{-\frac{1}{2}} x^{\frac{1}{2}}} = \dfrac{400}{600}$

$\Rightarrow 3y = 2x$ $\Rightarrow x = 15,\ y = 10$

② $(s,t) \Rightarrow x + y = 23$

$\begin{cases} x + y = 23 \\ 400x + 600y = 12000 \end{cases}$

$\begin{cases} 4x + 4y = 92 \\ 4x + 6y = 120 \end{cases}$

$\dfrac{\quad\quad\quad\quad\quad}{-2y = -28}$

$y = 14$

$x = 9$

A: $y = 14,\ x = 9$