Some Considerations Relating to Control Systems Employing the Invariance Principle

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Summary—**This paper presents a short survey of control systems employing Poncelet's principle, better known as the principle of invariance. Control systems based solely on the principle of invariance as well as those combining both feedback control and invariance principles are described and critically analyzed.**

General

Introduction

THE AIM of automatic control is to maintain the value of the controlled variable at a certain reference value. Realization of this aim in systems subjected to disturbances is possible by various methods, all of which, however, are characterized by the attempt to minimize the dependence of the controlled variable on the disturbances acting on the system.

The feedback control (Watt's) principle is based on comparison of the controlled variable of the system with a reference value and generation of an error signal which activates the system in a direction which tends to reduce the error.

A different approach, which appears to enable automatic control of a system, is control action actuated by the disturbance itself, a principle first proposed by Poncelet in 1829. Poncelet's principle, today generally referred to as the principle of invariance,[1] assumes that the disturbance which causes changes in the controlled variable can be used to generate an activating signal which will tend to cancel the effect of the same disturbance. Although Poncelet's attempt to apply his principle to the case of stabilizing the angular velocity of a steam engine failed, the principle itself found later applications in the field of industrial control.

The literature of automatic control deals mostly with control systems designed according to feedback control principle; recently, however, publications have begun to appear (mostly in Russian), on the invariance principle [1]–[21]. The present paper represents an attempt to carry out a short survey and a critical analysis of control systems using the invariance principle.

The Invariance Principle

Disturbances acting on control systems cause changes to occur in the controlled variable, which the system is supposed to maintain at a specific level determined by the command or reference. If the functional relation between the disturbance and the controlled variable is known, the effect of the disturbance can be cancelled without recourse to feedback control. To present the method we refer to a general linear system (plant) subjected to command (R) and disturbance (F) inputs, as represented in Fig. 1.

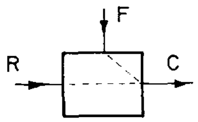

Fig. 1—System (plant) with command and disturbance inputs.

The differential equation relating the controlled variable $C(t)$ to the disturbance $F(t)$ alone, in the system of Fig. 1, is as follows:

$$a_m \frac{d^m C(t)}{dt^m} + \cdots + a_0 C(t) = b_n \frac{d^n F(t)}{dt^n} + \cdots + b_0 F(t), \quad (1)$$

which can be written in operational form[2]

$$A(D)C(t) = B(D)F(t)$$

$$D \triangleq \frac{d}{dt} \qquad D^k \triangleq \frac{d^k}{dt^k} \quad (2)$$

so that

$$C(t) = \frac{B(D)}{A(D)} F(t) \quad (3)$$

where

$$A(D) = a_m D^m + \cdots + a_0 \quad (4a)$$

$$B(D) = b_n D^n + \cdots + b_0. \quad (4b)$$

Thus, Poncelet's principle of control by disturbance refers to the possibility of altering the operator $B(D)$ which operates on the disturbance $F(t)$ in (2), so that the following relations are observed:

$$C(t) = 0 \text{ when } F(t) \neq 0. \quad (5)$$

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[1] In some publications "feedforward control" or "feedforward control for disturbance reduction" is used also.

[2] In the introduction and explanation of the invariance principle the operational form is used; in the next sections, however, the transfer form will be used for the convenience of control engineers.

It can be shown that application of Poncelet's principle, *i.e.*, generation of a signal that includes information on the disturbance, makes possible the required alteration of the operator $B(D)$ so that

$$B(D)F(t) = 0 \tag{6}$$

is realized.

The interesting point here is that the effect of the disturbance on the system is cancelled, or at least reduced, without recourse to comparison of the controlled variable with a reference value, as is necessary in control systems working on feedback control principle.

Investigation of the conditions, the realization of which brings about a situation where either the controlled variable is independent of the disturbance or this dependence is minimized, is the aim of the theory of invariance. Realization of the independence of the controlled variable upon the disturbance is designated as realization of its invariance to the disturbance.

Mathematically, the first case in which the invariance condition is realized is

$$B(D) = 0 \text{ when } F(t) \neq 0. \tag{7}$$

This is the case of absolute invariance, *i.e.*, the effect of the disturbance on the controlled variable is completely cancelled, and the cancellation is, moreover, independent of the type of disturbance. The transients in the controlled variable resulting from changes in the disturbance are eliminated as well. It has been proved that realization of absolute invariance, as well as other types of invariance, does not create a degenerate system [12], [19].

If the condition $B(D)=0$ is unobtainable, approximate invariance may be realized by minimizing the operator $B(D)$. ϵ-invariance, as defined in [4], can also be achieved.

Steady-state invariance cancels the influence of the disturbance on the steady-state value of the controlled variable and can be obtained by zeroing certain coefficients in the operator $B(D)$ [7]. Zeroing b_0 in (4b), for instance, permits cancellation of the system steady-state error for step disturbances. By zeroing additional coefficients, steady-state errors for other inputs may also be cancelled. Thus, an effect similar to the introduction of integration elements in feedback control systems is achieved.

The second case is

$$B(D)F(t) = 0$$

when

$$B(D) \neq 0, \qquad F(t) \neq 0 \tag{8}$$

which is the case of conditional invariance dependent upon the form of the disturbance. The system in this case remains absolutely invariant only for one form of disturbance. Conditional disturbance can be of great importance and value, provided the disturbance does, indeed, generally occur in a certain form, *e.g.*, a step, a ramp or a harmonic input with constant frequency.

Investigation of the conditions required of $B(D)$ for a given disturbance $F(t)$ so as to obtain conditional invariance has been carried out by Kulebakin [11], who proposed in his analysis an operational method of representing the disturbance functions which facilitates the determination of $B(D)$.

Control Systems Based on the Invariance Principle

Realization of Invariance by Means of the Disturbance Itself

Realization of invariance by means of an additional signal path activated by the disturbance itself is accomplished in the simplest manner by the configuration shown in Fig. 2. The disturbance signal F is added to the command signal R after passing through a suitable element, with transfer function $L_2(p)$, designed so that the controlled variable C remains invariant under the disturbance F. $G_1(p)$ denotes the transfer function of the command element and $L_1(p)$ is the disturbance input transfer function (after the disturbance input has been shifted in front of the plant input). $G_2(p)$ is the plant transfer function for command inputs. The relation between the controlled variable and the disturbance then becomes

$$\frac{C(p)}{F(p)} = G_2(p)[L_1(p) - L_2(p)G_1(p)]. \tag{9}$$

In order to achieve absolute invariance the transfer function $L_2(p)$ must be

$$L_2(p) = \frac{L_1(p)}{G_1(p)}. \tag{10}$$

If this condition is unrealizable, approximate invariance may be realized by minimizing the relation

$$L_1(p) - L_2(p)G_1(p) = \min. \tag{11}$$

By suitable design of $L_2(p)$, conditional or steady-state invariance can also be achieved. The problems of the stability and dynamic behavior of the system have no relevance in a system in which absolute invariance has been realized, since no transients appear at all with changes in the disturbance. Transients will appear in systems with only approximate or steady-state invariance, but the additional link from the disturbance does not endanger the stability of the controlled system, provided the plant elements $G_2(p)$, $L_1(p)$, command element $G_1(p)$, and $L_2(p)$ are all stable elements.

If the following notation is introduced:

$$G_1(p) = \frac{N_{G1}(p)}{D_{G1}(p)}, \qquad G_2(p) = \frac{N_{G2}(p)}{D_{G2}(p)},$$
$$L_1(p) = \frac{N_{L1}(p)}{D_{L1}(p)}, \qquad L_2(p) = \frac{N_{L2}(p)}{D_{L2}(p)}, \tag{12}$$

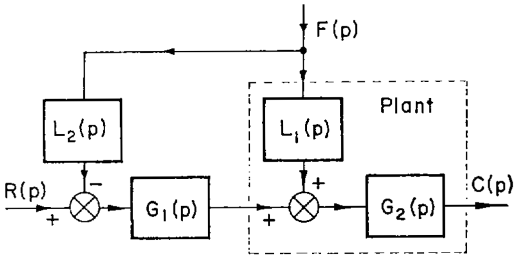

Fig. 2—System with realization of invariance by using additional path from the disturbance itself.

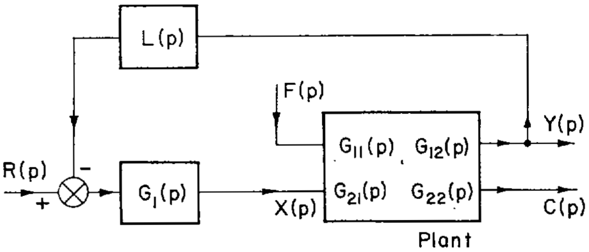

Fig. 3—Realization of invariance by using additional path from an internal variable, dependent upon the disturbance.

the characteristic equation of the system can be written

$$D_{G1}(p)D_{G2}(p)D_{L1}(p)D_{L2}(p) = 0. \tag{13}$$

The dynamic properties of the system in which approximate invariance or steady-state invariance is realized are determined by this equation. The fact that the additional path from the disturbance itself does not endanger the stability of the system constitutes an important advantage, which comes to the fore in systems which utilize both the invariance principle and the feedback control principle.

Realization of invariance by means of an additional path for the disturbance, as in Fig. 2, does not reduce the sensitivity of the system to variations in the parameters of the elements. System sensitivity is not affected by execution of the invariance and remains as in the open-loop state. This constitutes a major disadvantage of control systems working only with the invariance principle.

Two cases are discernible in the effect of parameter variations on the maintenance of the invariance.

1) Parameter changes occur only in $G_2(p)$, in which case the system remains invariant with regard to the disturbance and the output varies solely in compliance with the command signal.
2) Parameter changes occur in one of the transfer functions $G_1(p)$, $L_1(p)$, $L_2(p)$, in which case the invariance conditions are affected. If the changes occur in $G_1(p)$, the system response to a command signal will also change.

Use of the Internal Variable for Realization of Invariance

In plants with a structure as that in Fig. 3, there exists a further possibility for realization of the invariance principle. In this case an additional path from the internal variable, which is dependent upon the disturbance, is introduced. In Fig. 3, in order to show the internal variable dependent upon the disturbance, the plant is represented as an element with two inputs and two outputs. All considerations discussed in the previous section with regard to realization of the various types of invariance also hold true in this case. Naturally, the invariance conditions obtained for the additional loop in the present case differ from those of the previous section as a result of the more complicated relationships.

The over-all transfer function between the controlled variable and the disturbance, according to Fig. 3, is

$$\frac{C(p)}{F(p)} = \frac{G_{21}(p) + G_1(p)L(p)[G_{12}(p)G_{21}(p) - G_{11}(p)G_{22}(p)]}{1 + G_{12}(p)G_1(p)L(p)} \tag{14}$$

and so, for realization of absolute invariance, the transfer function of the additional path $L(p)$ must be

$$L(p) = \frac{G_{21}(p)}{G_1(p)[G_{11}(p)G_{22}(p) - G_{12}(p)G_{21}(p)]}. \tag{15}$$

In the case $G_{12}(p)=0$ (internal variable independent of the command signal), $L(p)$ becomes

$$L(p) = \frac{G_{21}(p)}{G_1(p)G_{11}(p)G_{22}(p)}. \tag{16}$$

By suitable design of $L(p)$ other types of invariance can also be achieved.

The novelty in the realization of the invariance in this case lies also in the stability considerations. Whereas in the previous case (using an additional path from the disturbance) the various invariance conditions did not affect the stability of the system (a stable element being used in the additional path), in the present case the stability is directly affected, as can be seen by comparing the characteristic equations of the system before and after introduction of the additional path from the internal variable.

The sensitivity of these systems to parameter variations in the various elements is here dependent upon the location of the variation, as is evident from (14). From the sensitivity viewpoint, these systems behave like open-loop systems in reaction to parameter variations in the transfer functions $G_{21}(p)$, $G_{12}(p)$. Their sensitivity is, however, reduced as in closed-loop systems with regard to variations in $G_{11}(p)$, $G_{22}(p)$, $G_1(p)$, and $L(p)$. As regards the effect of parameter variations on the invariance condition, it should be noted that here the invariance condition will be upset by parameter changes anywhere in the system.

Realization of the principle of invariance depends on the structure of the plant. It is not possible in all plants to realize this principle by means of a simple additional path from the internal variable. For example, let us

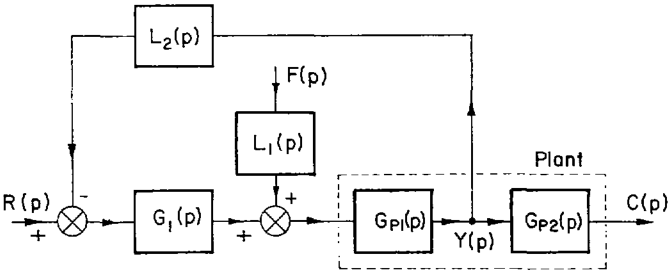

Fig. 4—Plant configuration which makes impossible the achievement of absolute invariance by using additional path from an internal variable.

consider a plant with a structure like that in Fig. 4. The internal variable $Y(p)$ is to be used when the relation between the controlled variable and the disturbance is

$$\frac{C(p)}{F(p)} = \frac{G_{p1}(p)G_{p2}(p)L_1(p)}{1 + L_2(p)G_1(p)G_{p1}(p)} \tag{17}$$

and no finite function $L_2(p)$ can be found to realize absolute invariance. The same negative results would be obtained by using the controlled variable in an additional path. Thus absolute invariance cannot be achieved by control systems working only on feedback control principle. A general mathematical approach to the question of the realizability of the principle of invariance is given by Petrov [12], [19].

Present-Day Use of Control Systems Working by Poncelet's Principle Only, and Future Possibilities

Control systems designed according to Poncelet's principle only, which achieve absolute invariance relative to a certain disturbance, constitute a theoretically ideal solution of the automatic stabilization problem through their ability not only to stabilize the controlled variable in the steady state, but also to prevent the appearance of transients in it (which conventional feedback control systems are, of course, incapable of doing). All the same, systems of this kind are rarely encountered in practice, and then only with certain specific plants where steady-state invariance alone is realized. The main factors that disqualify the use of these systems for real control applications are the following:

1) The practical difficulty encountered in realization of absolute invariance, which generally demands the employment of zeros in the additional path from the disturbance, or the internal variable relative to the disturbance. The realization of steady-state invariance is much easier, and such systems are consequently more common (*e.g.*, compoundation in electric machines).
2) Their great sensitivity to parameter changes (from this point of view they act like open-loop systems). Achievement and maintenance of absolute invariance demands highly accurate components and extremely precise adjustment of the system, every aberration causing the invariance to be upset. The inability of the system to adapt itself to such changes, due to its inherent lack of self-adaptive properties, limits design to approximate invariance only, by means of which only relatively crude control is possible.
3) The lack of accurate inexpensive instrumentation for measurement of the disturbance in uncomplicated industrial control systems (*i.e.*, the measurement of mechanical moments, loads, powers, etc.). In certain industrial control systems all the conditions necessary for realization of the required regulation by means of the invariance principle exist [*i.e.*, parameter variations in the system are insignificant, and only a single main disturbance appears in the system (usually a load change)] but lack of suitable cheap instrumentation for accurate measurement of the main disturbance prevents this.
4) The appearance of additional disturbances, apart from the main one for which the system is designed. Theoretically one may design the system so as to be invariant with regard to each disturbance, but this is unfeasible in practice.

The above-mentioned disadvantages disqualify control systems using only the invariance principle for most industrial applications,[3] but all the same the definite advantages of these systems indicate that they have a future in combination with conventional feedback control systems.

Combined Systems

Introduction

In combined systems, in addition to the closed loop from the controlled variable, an additional path from the main disturbance, or from an internal variable proportional to it, is provided (Figs. 5 and 6). The combined systems [11], [13] do indeed possess the advantages of both methods, so that results are obtained which neither principle on its own could supply; the complete cancellation of the influence of the main disturbance, coupled with the reduced sensitivity to parameter variations, at least theoretically enables extremely high quality systems to be designed. It appears, therefore, that the future of the invariance principle lies in its application to combined systems of the above sort. Such systems, even when mere approximate invariance or steady-state invariance is accomplished, promise results unobtainable in the conventional monolithic designs.

Combined Systems with an Additional Path from the Disturbance Itself

As shown in Fig. 5 these systems consist of closed-loop control plus an additional path from the disturb-

[3] This does not refer to the use of the invariance principle for parametric stabilization, frequently employed in compensation (*e.g.*, compensation of temperature-induced changes in measuring instruments, etc.).

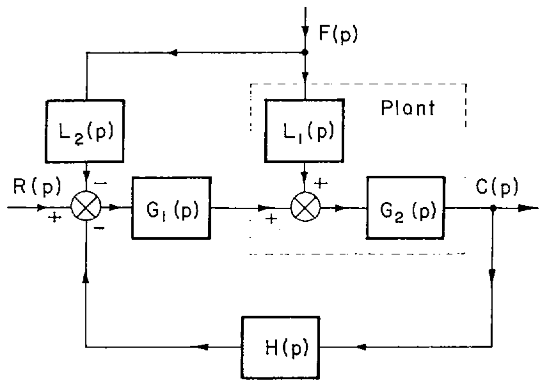

Fig. 5—Combined system with additional path from the disturbance itself.

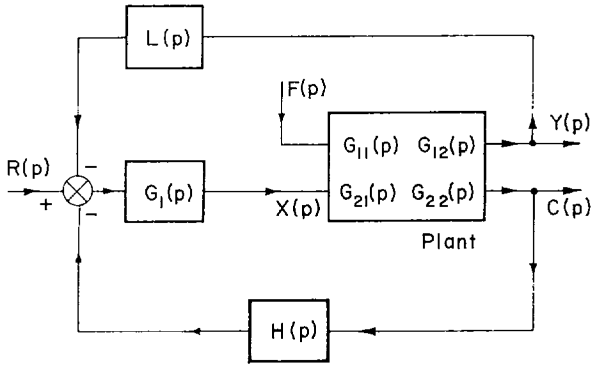

Fig. 6—Combined system with additional path from an internal variable.

ance. By adjustment of $L_2(p)$ in compliance with (10) (the invariance conditions are not affected by the feedfack) all types of invariance are obtainable here too: absolute, conditional, approximate and steady-state.

Realization of absolute invariance may here, as well, engender practical difficulties (the need for active networks), but the use of approximate and steady-state invariance also gives good results, due to the action of the feedback loop compensating for the disadvantages inherent in the invariance principle design alone. In fact, the use of combined systems with approximate invariance is essential in the control of plants with large time constants relative to the disturbance and command, where exclusive use of feedback control principle produces a sluggish system with large deviations from the required level during the transient phenomena [21].

Similarly, steady-state invariance constitutes a highly efficient stabilizing means in regulators, where reduction of the steady-state error by feedback control principle methods is hindered by the lack of integration elements and by necessary limitations on the system gain.

In combined systems with an additional path from the disturbance itself, stability is not endangered by the additional path (providing, of course, that the element $L_2(p)$ is stable); this is a great advantage of the system. The sensitivity of such systems is identical to that of an equivalent pure feedback control system. Reduction of sensitivity is obtained not only to elements in the main loop as $G_1(p)$, $G_2(p)$, but also to $L_1(p)$ and $L_2(p)$. This, in its turn, helps to prevent the disabling effect of variation of parameters in the additional loop on the invariance conditions. This is clearly seen from the equation relating the controlled variable with the disturbance (Fig. 5),

$$\frac{C(p)}{F(p)} = \frac{G_2(p)[L_1(p) - L_2(p)G_1(p)]}{1 + G_1(p)G_2(p)H(p)}. \tag{18}$$

The effect of the signal relative to the deviation from the invariance condition on the controlled variable is reduced by the negative feedback. In a pure invariance system, however, the controlled variable would be directly affected by this signal.

Together with the outstanding advantages of combined systems with an additional path from the disturbance itself, there are certain unavoidable difficulties connected with the problem of obtaining accurate and appropriate instrumentation for measuring disturbances. For this reason, combined systems which use some internal variable dependent on the disturbance, rather than the disturbance itself, in order to effect the control are more common. These systems are the subject of the following section. However, in all cases where the requisite instrumentation is available the systems described in the present section may be unhesitatingly adopted as promising excellent results.

Combined Systems with an Additional Path from an Internal Variable Dependent on the Disturbance

Combining an additional path from the internal variable dependent on the disturbance with a conventional feedback loop (Fig. 6) also produces a system possessing the advantages of both methods. As mentioned above, this type of system is the more common in industrial applications since in most plants it is easier to measure some variable related to the disturbance than to measure the disturbance itself. In these systems, too, all types of invariance can be realized, although in practice the most common cases are with approximate and steady-state invariance. All that has been said as to the advantages of combined systems with an additional path from the disturbance itself applies to these systems as well. The principal difference between the two varieties is that in the latter type the stability and quality of the system are affected by the presence of the additional path from the internal variable related to the disturbance. The dynamic properties of the system are directly influenced by the additional path, which on the one hand presents dangers to stability but on the other enables the dynamic behavior of the system to be tailored to required specifications by means of appropriate design of the transfer function of the additional path from the variable related to the disturbance. The equa-

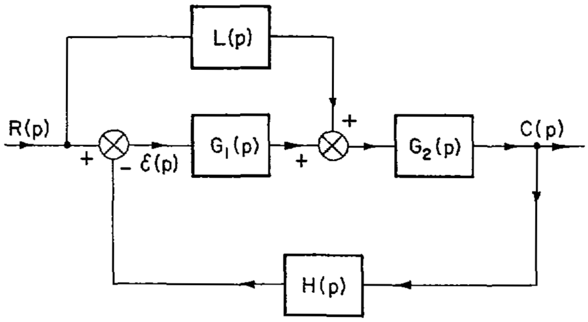

Fig. 7—Combined follow-up system.

tion relating the controlled variable to the disturbance is

$$\frac{C(p)}{F(p)} = \frac{G_{21}(p) - G_1(p)L(p)[G_{12}(p)G_{21}(p) - G_{11}(p)G_{22}(p)]}{1 + G_1(p)[G_{12}(p)L(p) + G_{22}(p)H(p)]}, \quad (19)$$

from which the conclusions regarding stability and sensitivity can be drawn.

The main use of the additional path from the variable dependent upon the disturbance is in systems with plants sluggish relative to disturbance and command inputs. Here the additional path is exploited to speed up the system response to a disturbance, a refinement which is lacking in a pure feedback control system. This is, in effect, equivalent to realization of merely approximate invariance.

Conditional Invariance in Combined Systems

By realizing conditional invariance in combined systems, harmonic disturbances with constant frequency can be completely eliminated [11]. This fact can be of considerable value since such disturbances can never be completely cancelled in linear systems by the use of feedback principle alone but, generally speaking, the practical realization of conditional invariance is not easier than that of absolute invariance.

Combined Follow-Up Systems

The invariance principle is best applied to stabilizing systems (regulators) which generally undergo a single main disturbance. But at least theoretically the principle of invariance can be applied to follow-up systems in order to obtain an improvement in their steady-state and dynamic characteristics. The approach necessary in applying invariance to follow-up systems is somewhat different. What was formerly regarded as the disturbance is now the command, *i.e.*, the reference input. The value which has now to be kept invariant under disturbance is no longer the controlled variable, but the actuating error. Here, too, invariance is realized by the introduction of an additional path (feedforward) leading from the "disturbance" [now the command (Fig. 7)].

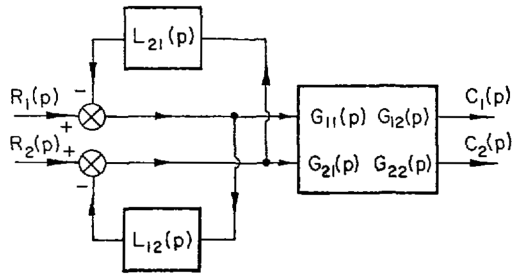

Fig. 8—Cancellation of cross links in a plant by using the invariance principle.

The relationship between the actuating error and the "disturbance" is here

$$\frac{\epsilon(p)}{R(p)} = \frac{1 - G_2(p)H(p)L(p)}{1 + G_1(p)G_2(p)H(p)} \quad (20)$$

and absolute invariance is obtained when

$$L(p) = \frac{1}{G_2(p)H(p)}. \quad (21)$$

In practice it is likely to be difficult to realize an $L(p)$ as given by (21) owing to the necessity for zeros of first and second order. However, additional (feedforward) path from the input can be of great value in the effort to minimize the tracking error (achieving approximate invariance only).

Use of Invariance Principle in Multivariable Systems

In systems with multiple inputs and outputs the invariance principle has been in use for a long time for the purpose of cancelling cross links in the plant (mathematically, this means diagonalization of the transfer matrix of the plant). What is actually done is to achieve invariance of a specific output to specific inputs. Fig. 8 shows a block diagram of a system in which appropriate design of $L_{12}(p)$ and $L_{21}(p)$ accomplishes cancellation of the cross links.

Naturally the invariance principle can be utilized in multivariable systems for the cancellation of the disturbance effect as well (either with regard to a single disturbance or to several); again the main use is in plants with large time constants relative to the disturbance and command, where the advantages of introducing an additional path from the disturbance or from an internal variable, functionally related to the disturbance, are clear. The design of these relationships is carried out in accordance with the principles outlined in connection with unidimensional systems.

Conclusions

Control systems depending exclusively on the invariance principle suffer from high sensitivity to parameter variations. Their practical realization is difficult, mainly due to the lack of sufficiently accurate and inexpensive instrumentation for measurement of the main disturbances, but also owing to the necessity of active elements (zeros) in the additional path. For these reasons these systems are rare in industrial applications and no more than a small number of systems which employ steady-state invariance only are to be found.

The future of the principle of invariance in industrial applications lies therefore in combined control systems where it is used with the feedback principle in order to exploit the advantages of both methods while obviating the most outstanding disadvantages. In most cases the combined system furnishes a superior performance to a system designed to employ feeback principle alone. In particular, the introduction of an additional "invariance path" into closed-loop systems with sluggish plants having large time constants relative to the disturbance and command inputs appears to be of considerable advantage.

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