

Lane Detection  
Using Images

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The slide has a blue background with a large, stylized white and blue geometric shape at the top. The title "Lane Detection Using Images" is centered in white. Below it, two bullet points introduce the speaker and the institution. At the bottom right is the logo for "NEWS<sup>Lab</sup>" which includes a stylized "O" icon and the text "嵌入式系統暨無線網路實驗室".

# Agenda

- ▶ Image Sensors Calibration
- ▶ Lane Detection for Self-Driving Cars

# Lane Detection Using Image Sensors

# Lane Detection

- ▶ Lane detection is a fundamental components for ADAS and self-driving cars. For example,
- ▶ **Lane departure warning (LDW)**: alerts the driver when the car changes lanes without turning on signals.
- ▶ **Lane Following**: follow the lane lines to drive the car.
- ▶ The core operation is to detect the lane line using onboard sensors.
- ▶ **Precision localization and HD map**: given the map and location of the vehicle, the vehicle can locate itself between lane lines.
- ▶ **Image sensors**: detect the lane lines according to the defined property of lane lines such as color and dimensions.
- ▶ **Lidar sensors**: detect the lane lines according to the intensity of the reflection.

# Finding Lane Lines from Images

1. Calibrate camera to remove distortion
2. Warping images
3. Finding lane lines
4. Curve fitting
5. Overlap Results on Image

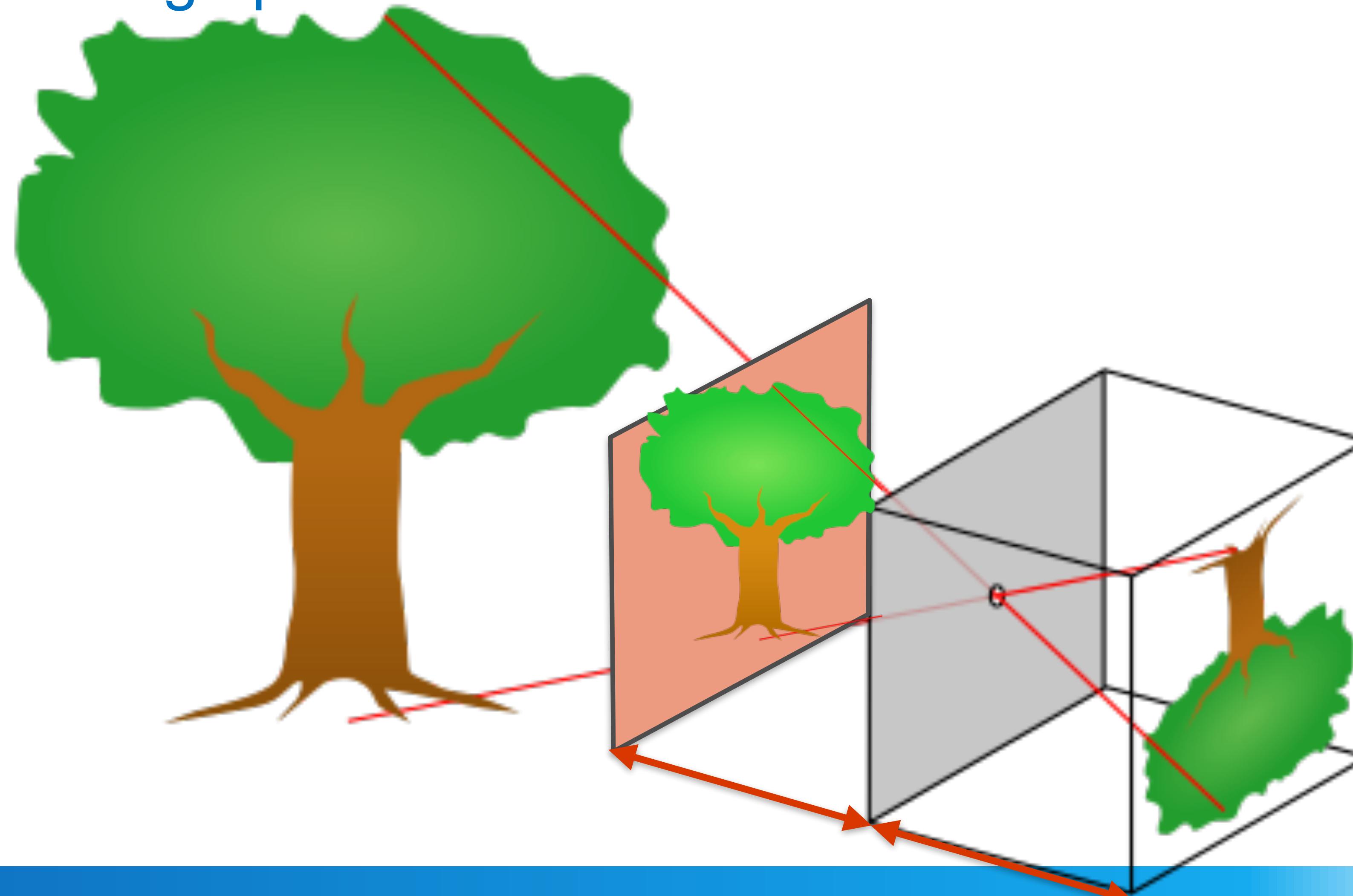
# Camera Calibration

# What is Camera Calibration?

- ▶ A camera projects 3D world-points onto the 2D image plane.
- ▶ **Calibration:** Finding the quantities internal to the camera that affect this imaging process
- ▶ Image center
- ▶ Focal length
- ▶ Lens distortion parameters

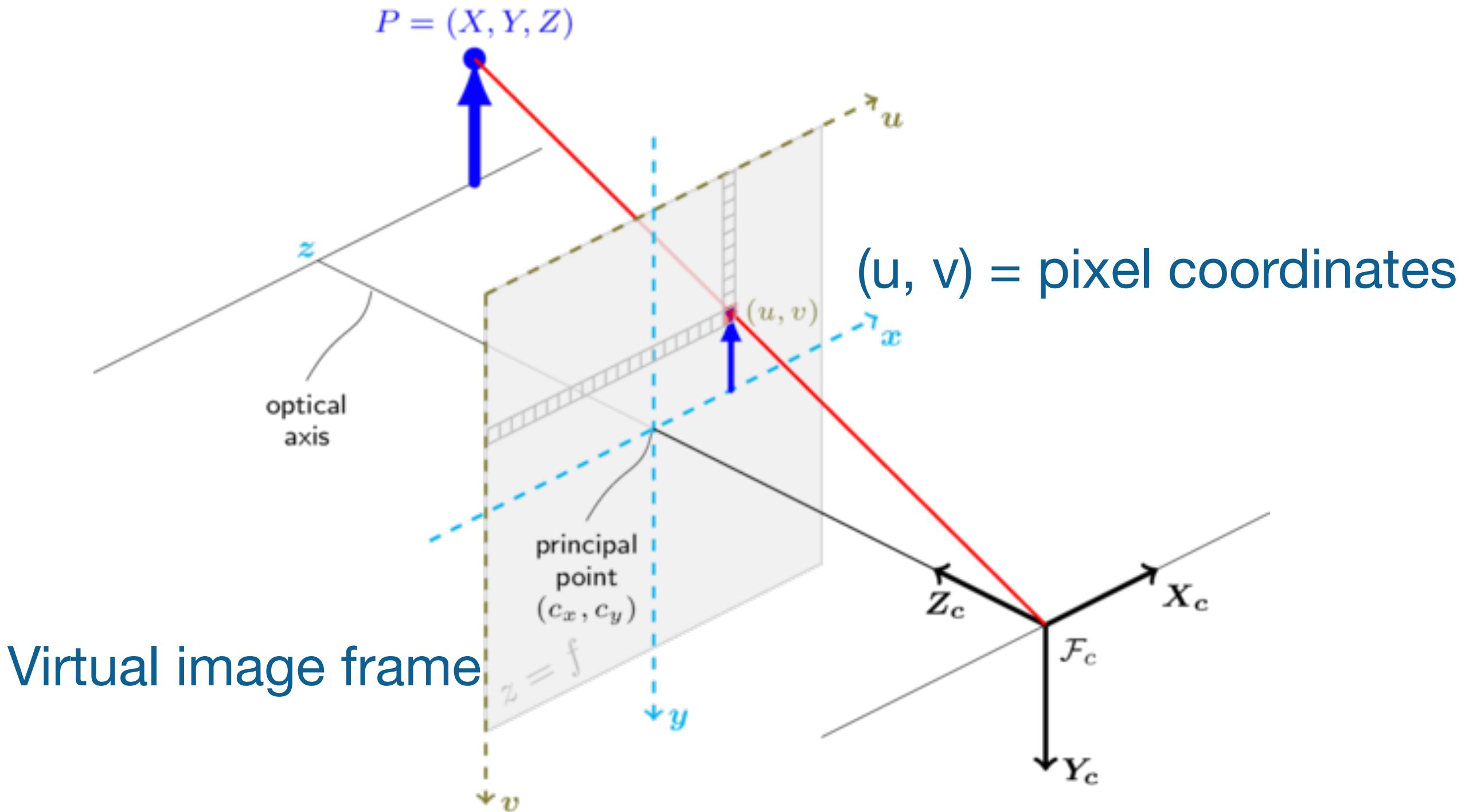
# Pin-Hole Camera Model

- ▶ A camera projects 3D world-points onto the 2D image plane.



# Pin-Hole Camera Model

$(x, y, z) = \text{world coordinates}$



Ideally, the light goes straight. However, in order to capture more light and increase field of views, lens are added to change the light path.

As a result, the points,  $(u, v)$ , projected on (virtual) camera frame need to be adjusted so that they are ***isomorphic*** to the points,  $(x, y, z)$ , in world coordinates.

The process to find the parameters is called ***calibration***.

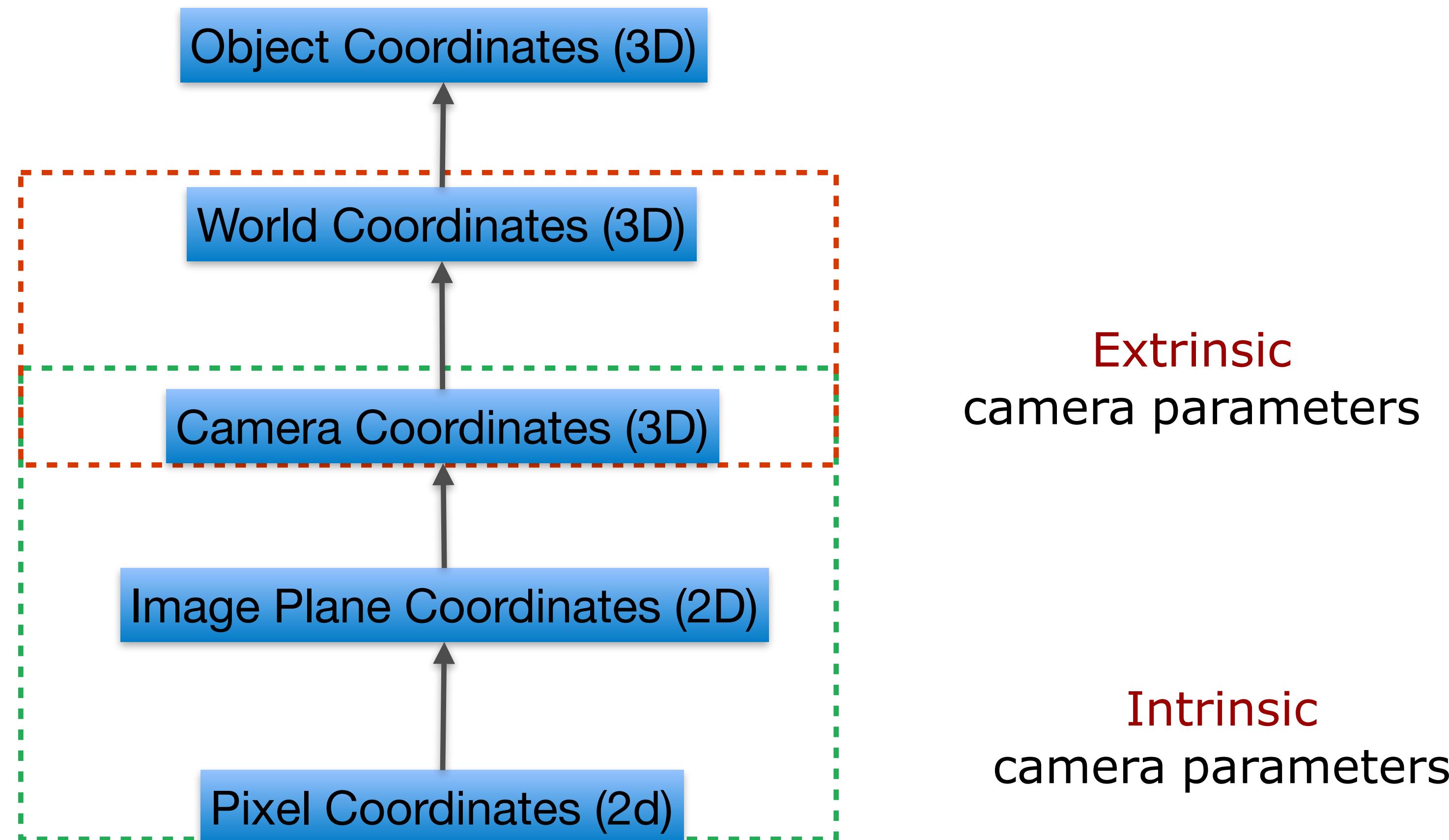
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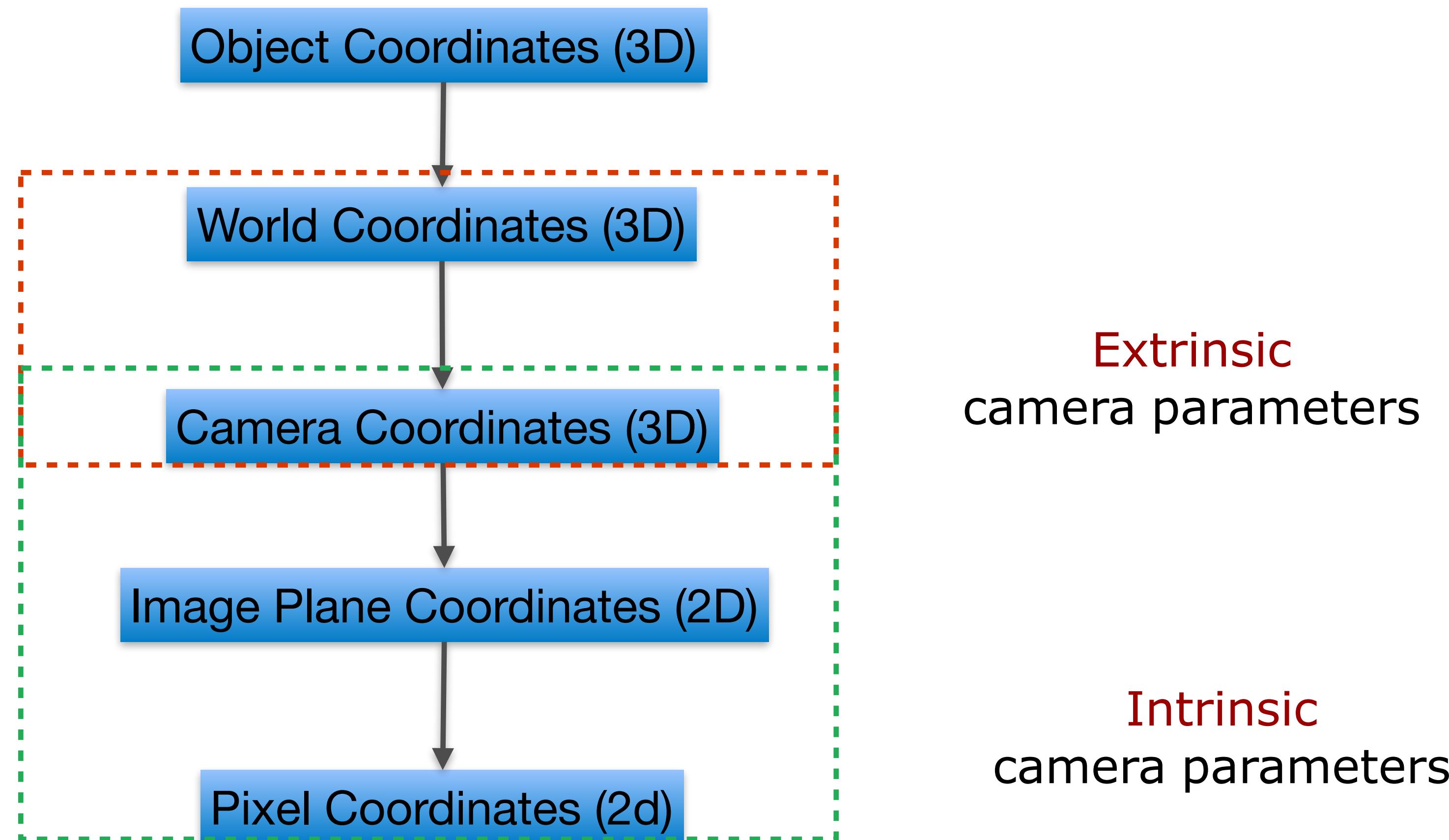
# Motivation

- ▶ Causes of errors for calibrations:
  - ▶ Camera production errors
  - ▶ Cheap lenses
- ▶ Precise calibration is required for
  - ▶ 3D interpretation of images
  - ▶ Reconstruction of world models: e.g., lane lines
  - ▶ Robot interaction with the world (Hand-eye coordination)

# From 2D to 3D pixel frame



# From 3D to 2D pixel frame



# Projective Geometry

- Extension of Euclidean coordinates towards points at infinity

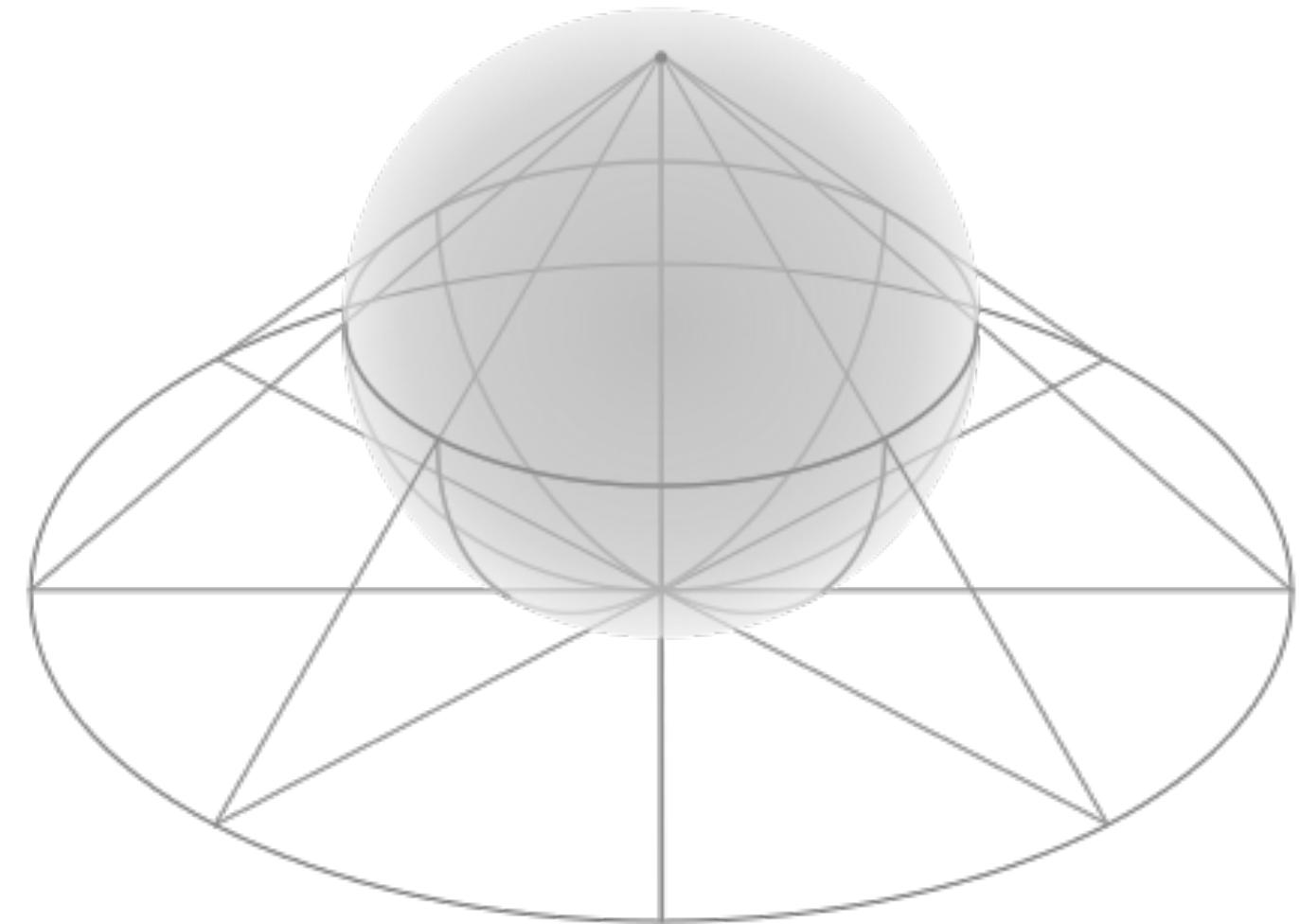
$$\mathbb{R}^n \rightarrow \mathbb{P}^n : (x_1, x_2, \dots, x_n) \rightarrow (\lambda x_1, \lambda x_2, \dots, \lambda x_n, \lambda) \in \mathbb{R}^{n+1} \setminus 0_{n+1}$$

- Here, equivalence is defined up to scale:

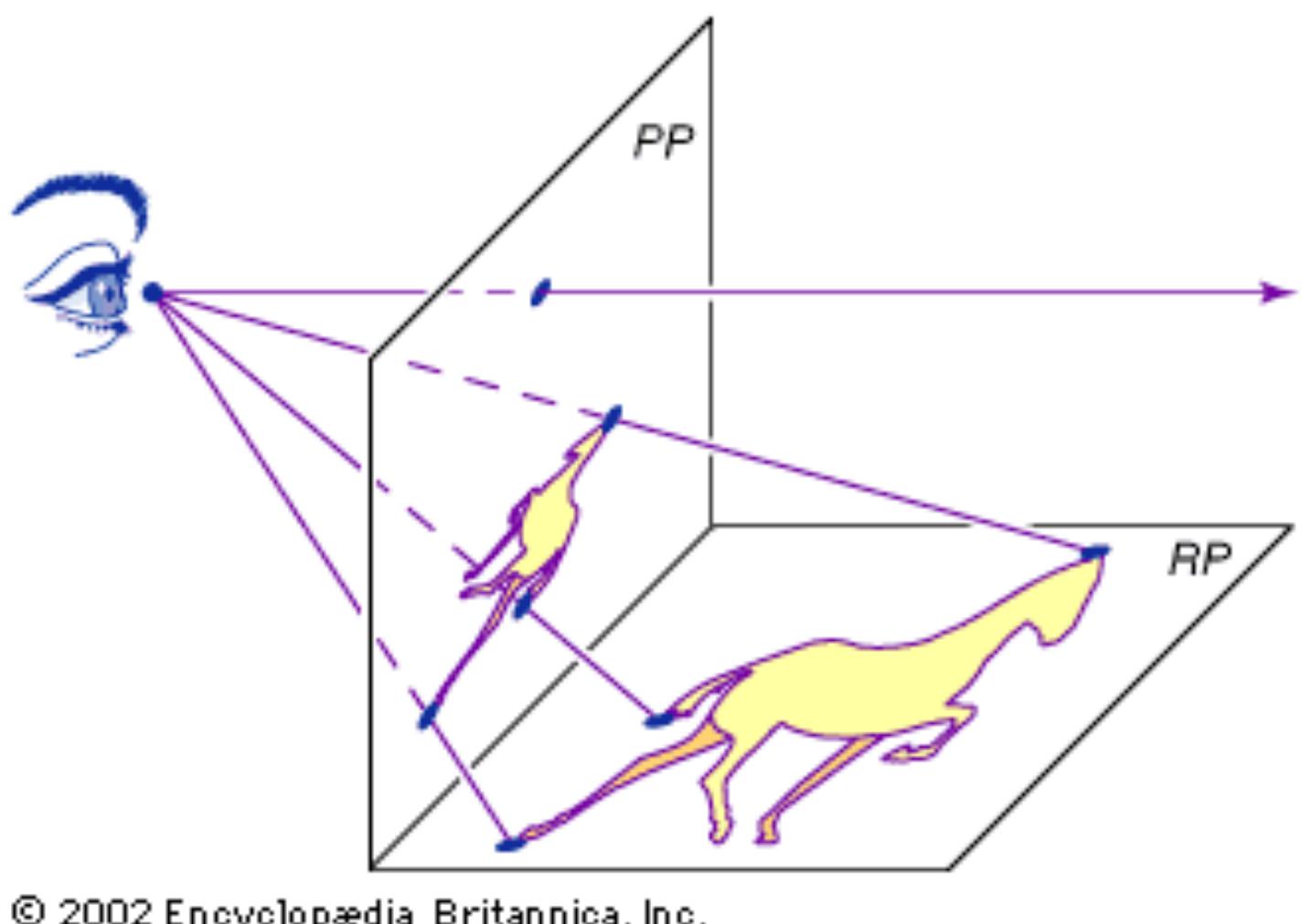
$$\hat{x} \sim \hat{y} \Leftrightarrow \exists \lambda \in \mathbb{R} \setminus \{0\} : \hat{x} = \lambda \hat{y}$$

- Special case: Projective Plane  $\mathbb{P}^2$

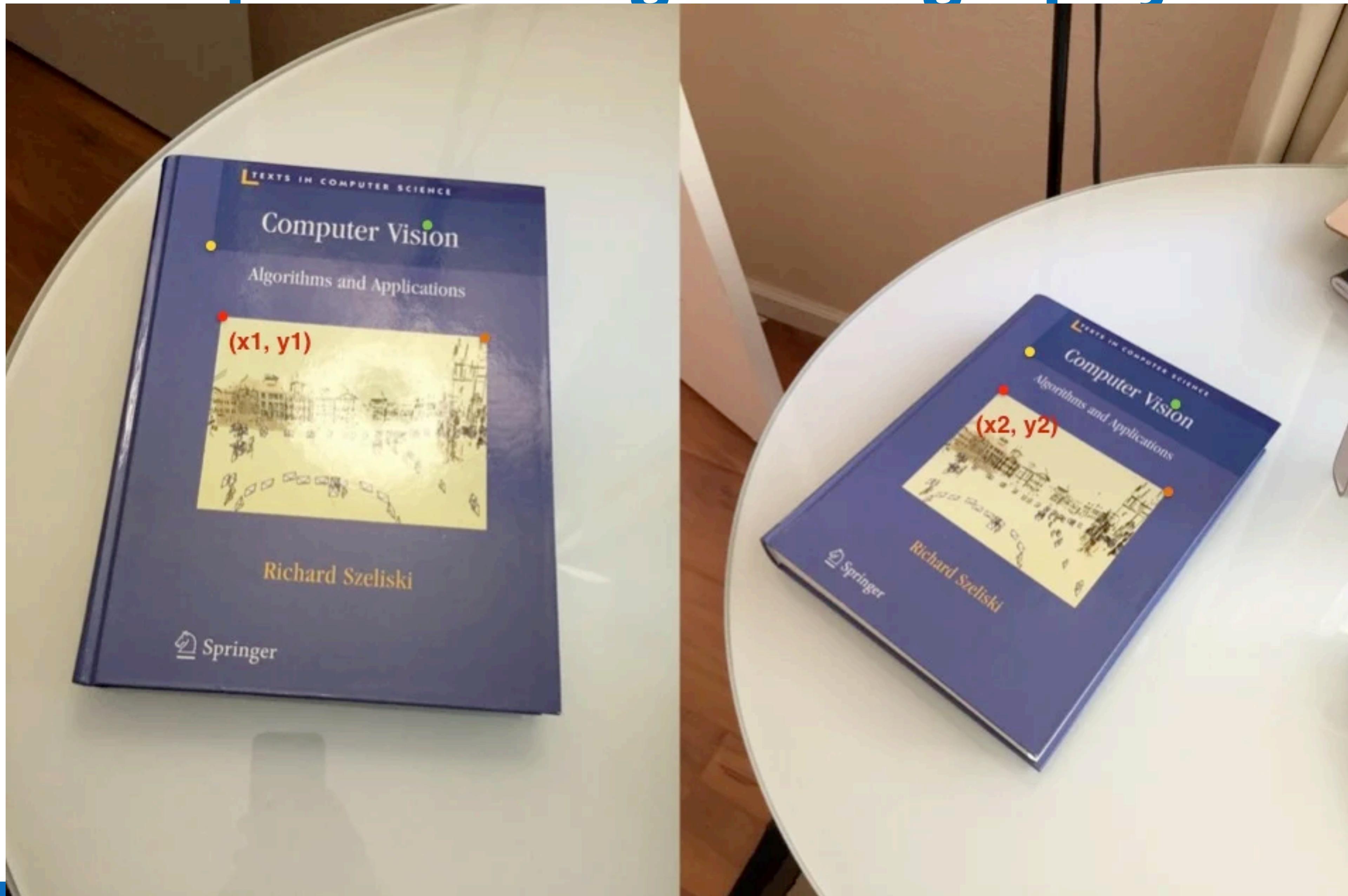
- A linear transformation within is called a homography  $\mathbb{P}^2$



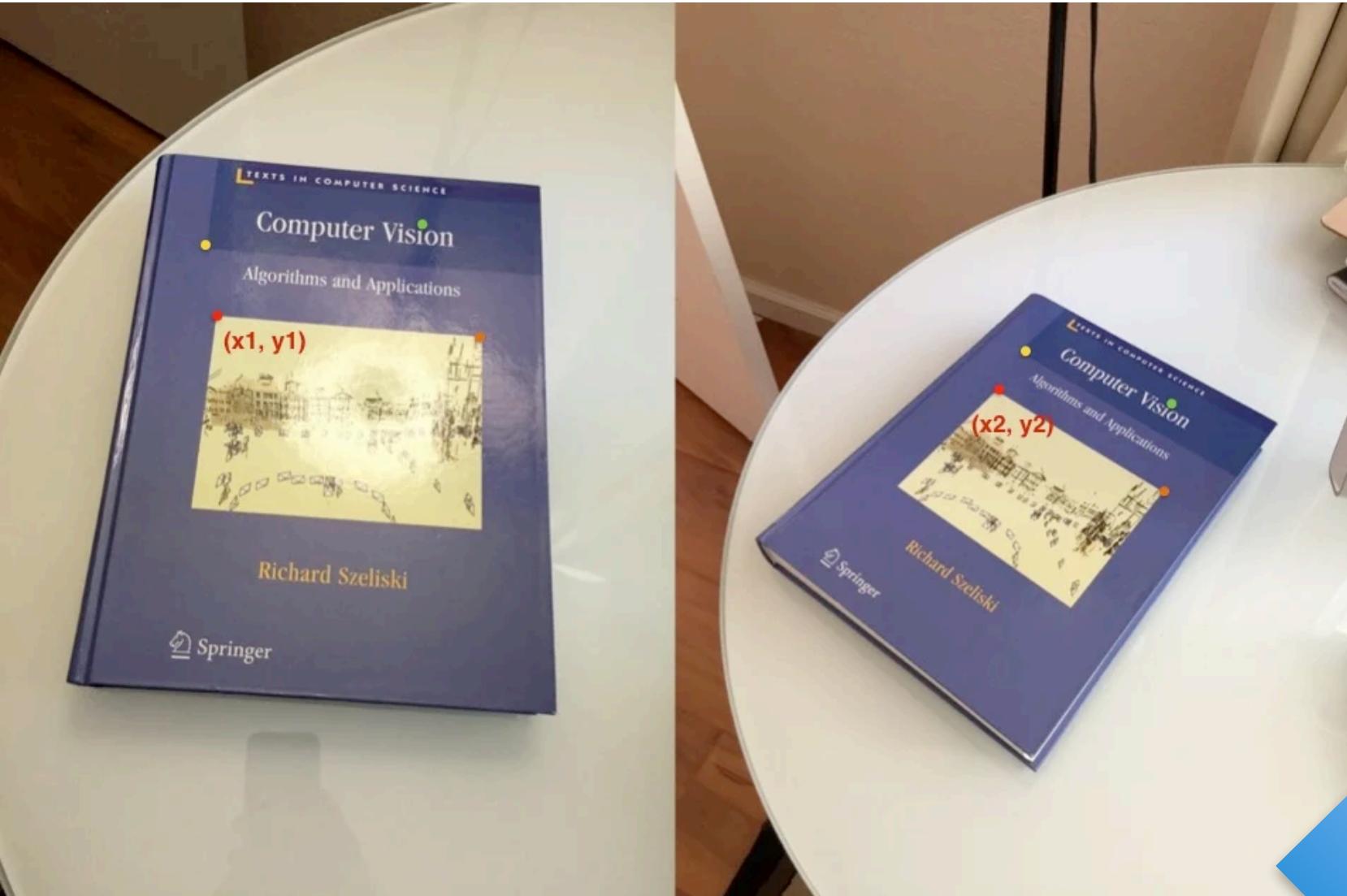
Projecting a sphere to a plane.



# Example of Using Homography

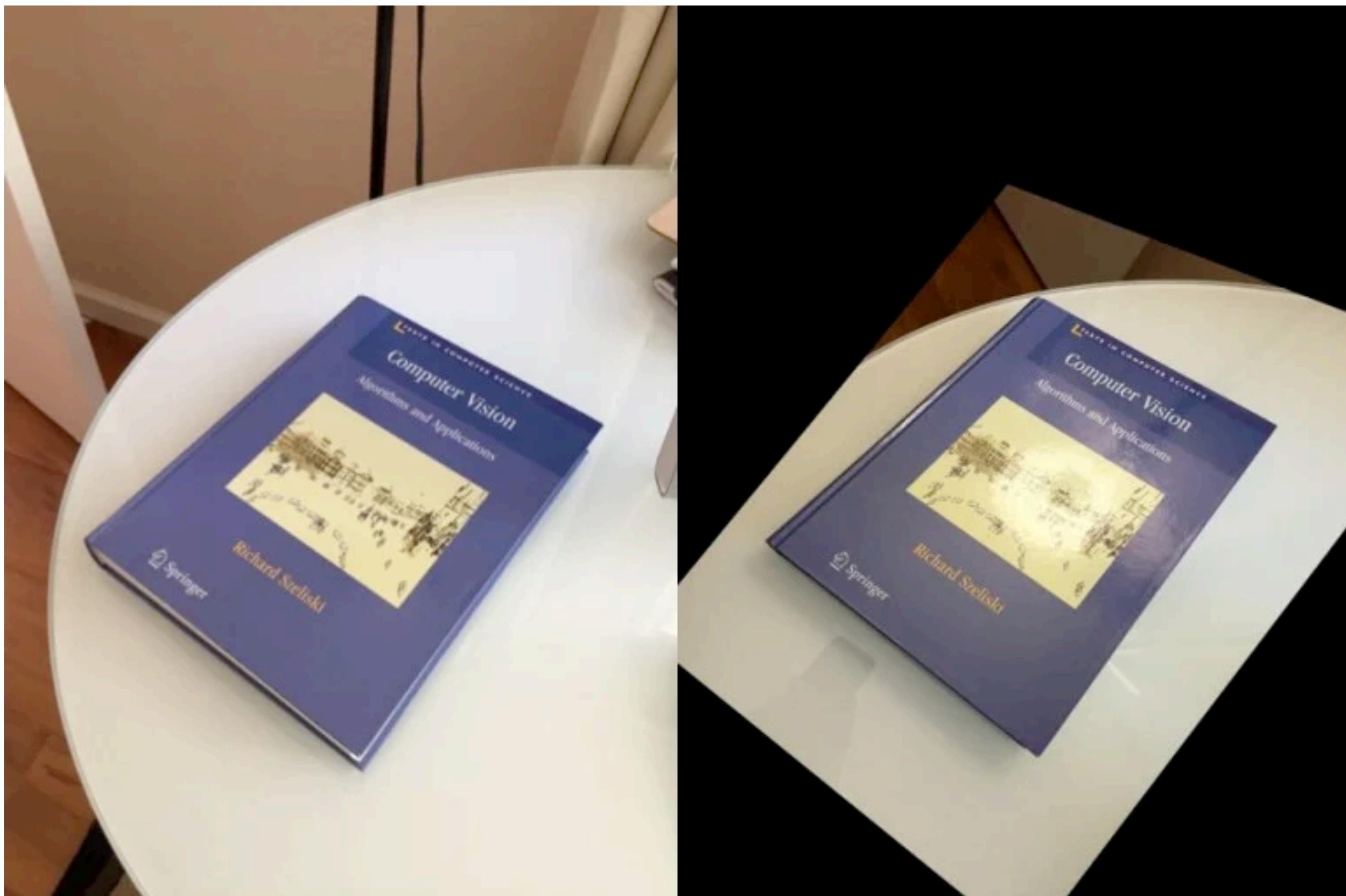


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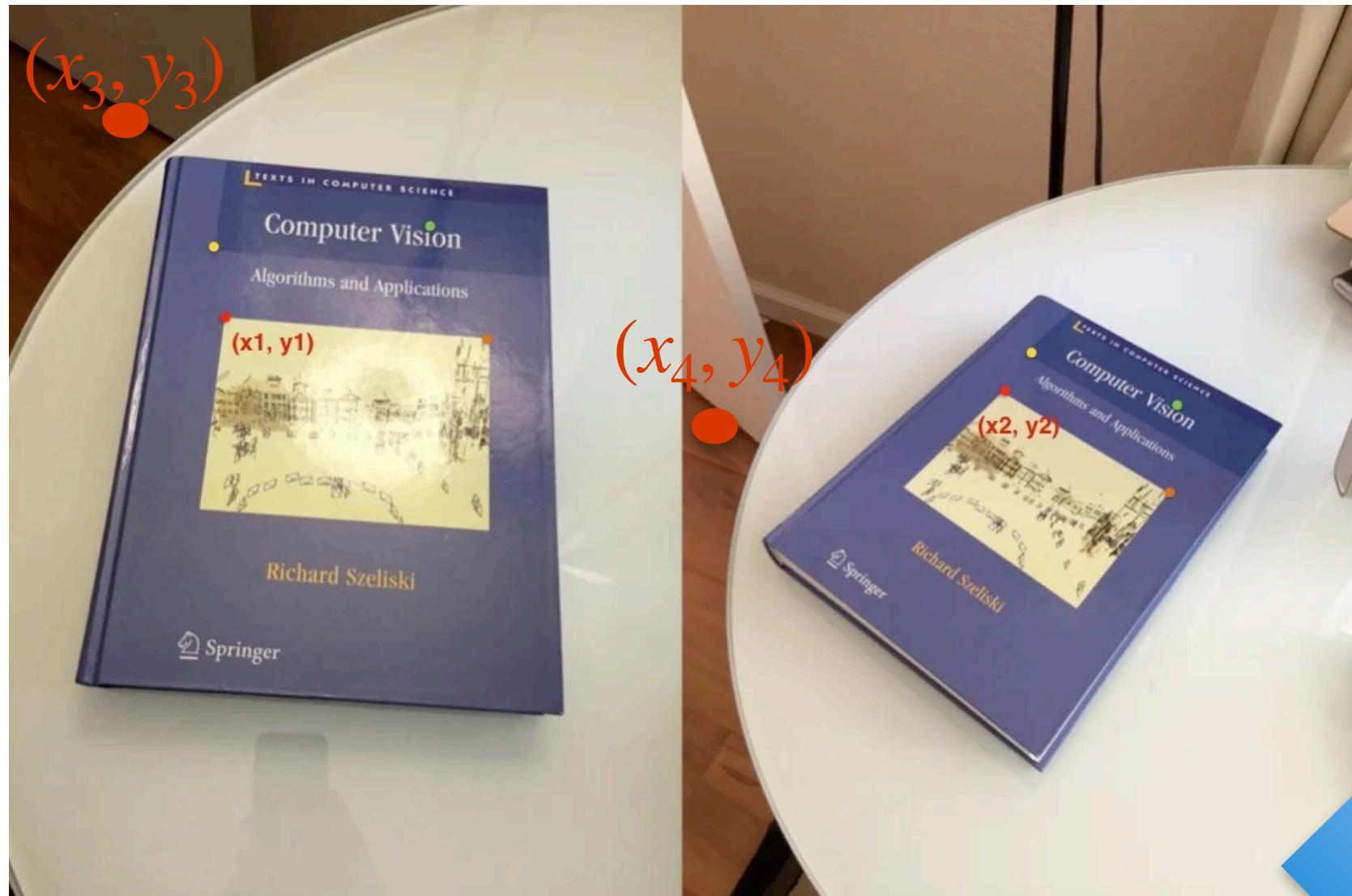


$(x_1, y_1)$  and  $(x_2, y_2)$  are the points located at the same place in world coordinates



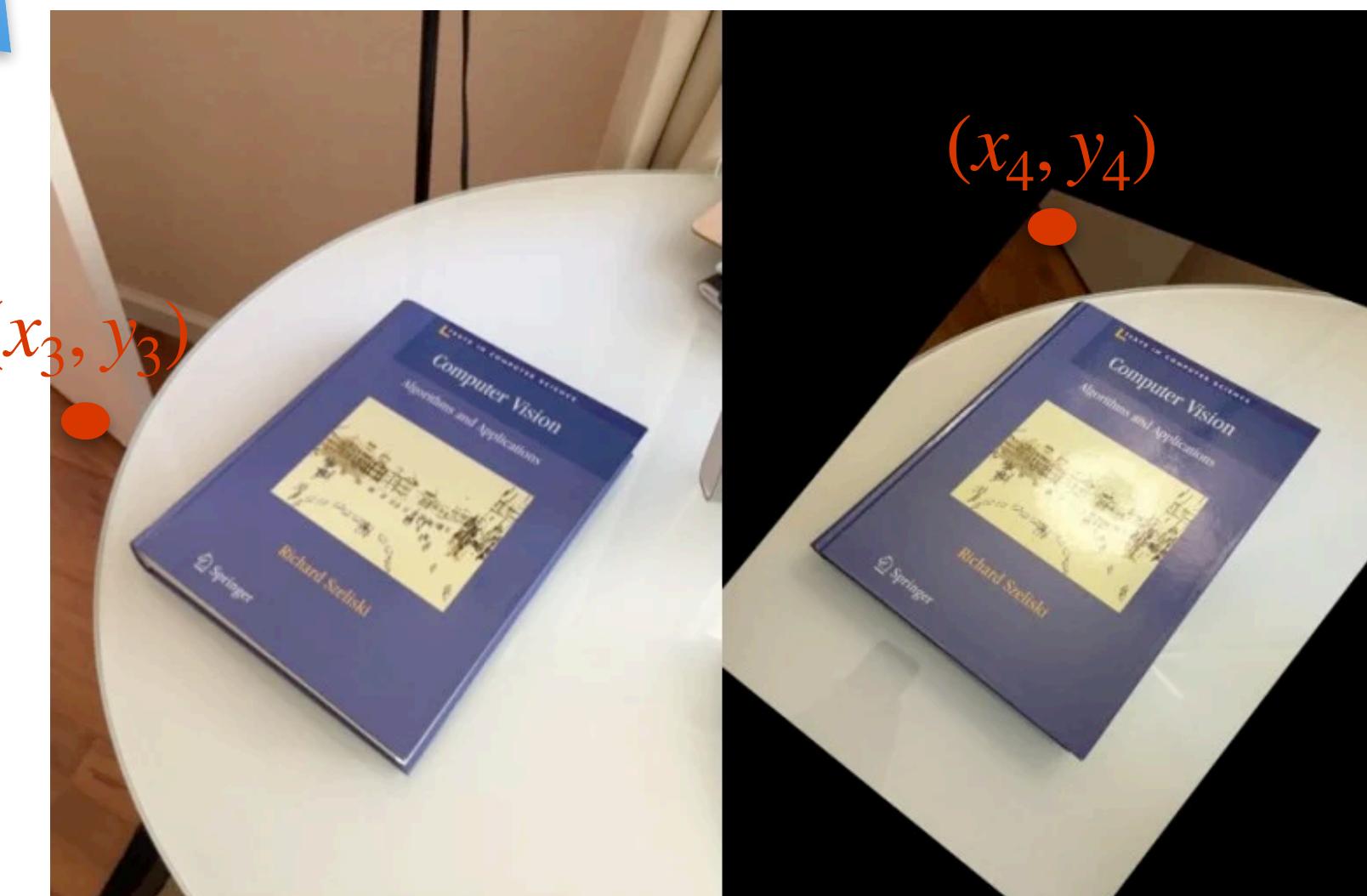


# Example of Using Homography



$(x_1, y_1)$  and  $(x_2, y_2)$  are the points located at the same place in world coordinates

$(x_3, y_3)$  and  $(x_4, y_4)$  are **NOT** the points located at the same place in world coordinates.



# Homography

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim \underbrace{\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}}_{\text{Homography } H} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- ▶  $H$  has 9-1(scale invariance)=8 DoF
- ▶ One pair of points gives us 2 equations
- ▶ Therefore, we need at least 4 point correspondences for calculating a homography matrix.

# Pinhole Camera Model

- ▶ Perspective transformation using homogeneous coordinates:

$$b = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

↓                              ↓

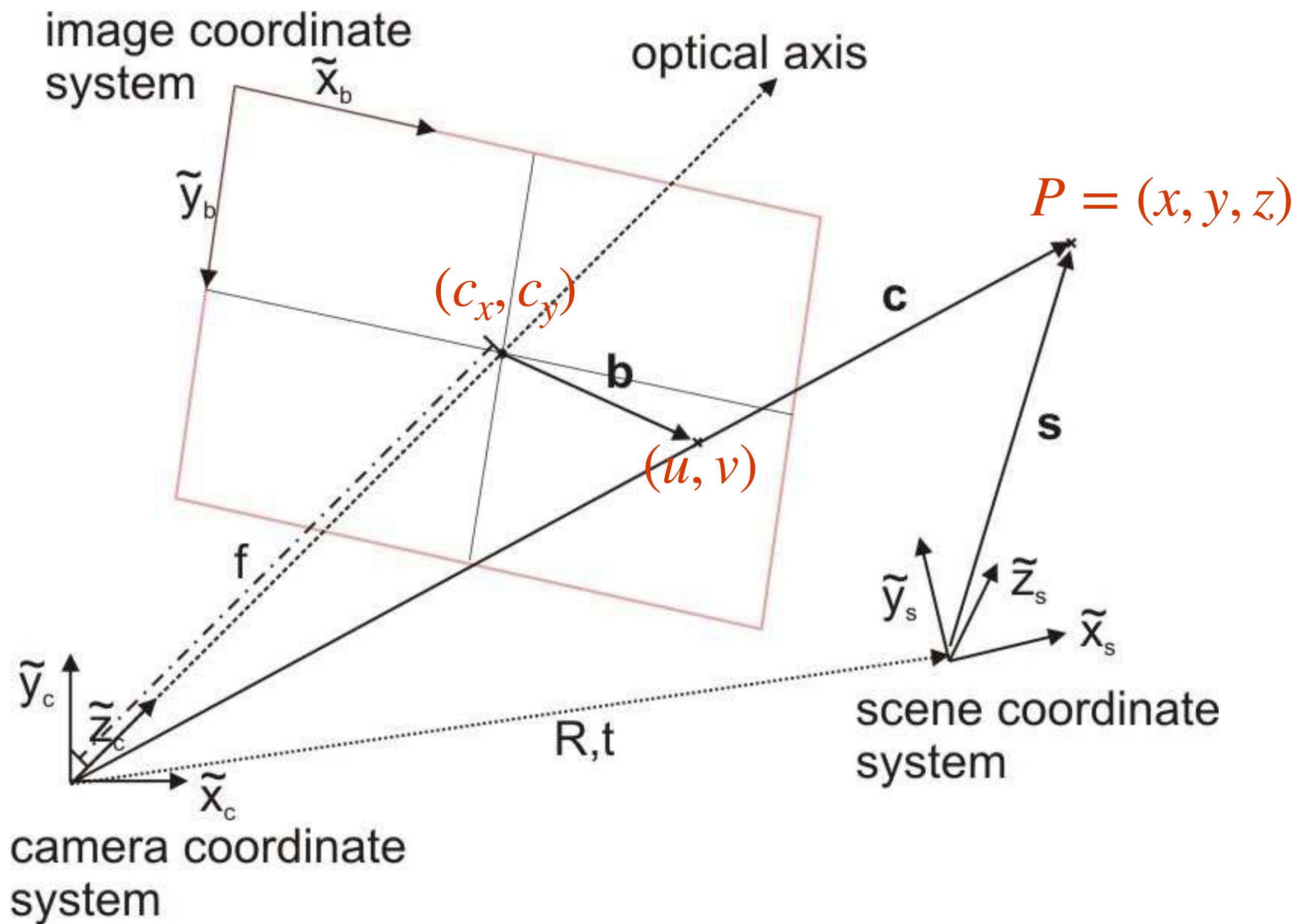
Intrinsic  
camera parameters

Extrinsic  
camera parameters

**Intrinsic parameters:** parameters to project points on pixel frame onto reference frame.

**Extrinsic parameters:** parameters to project points on camera reference frame onto world reference frame.

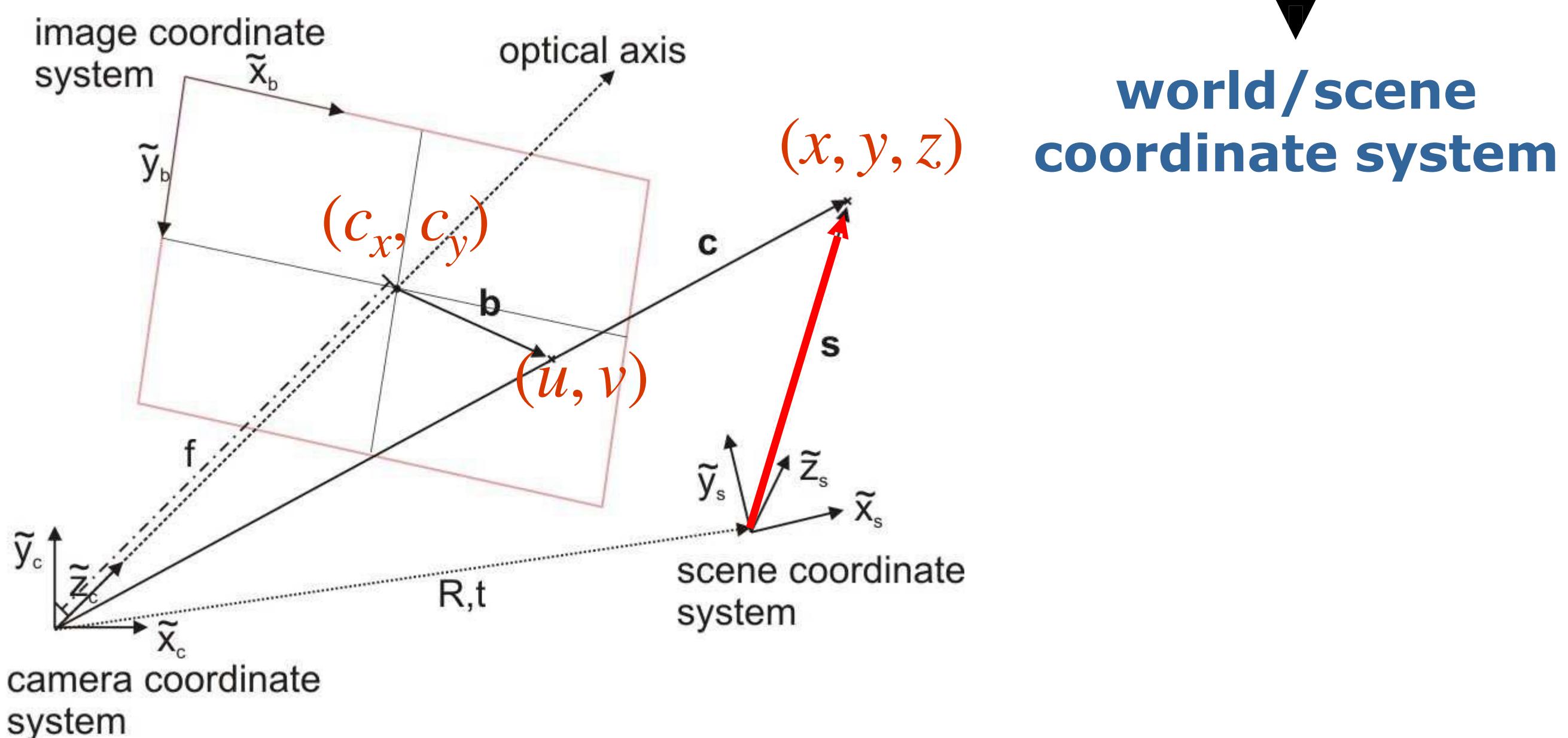
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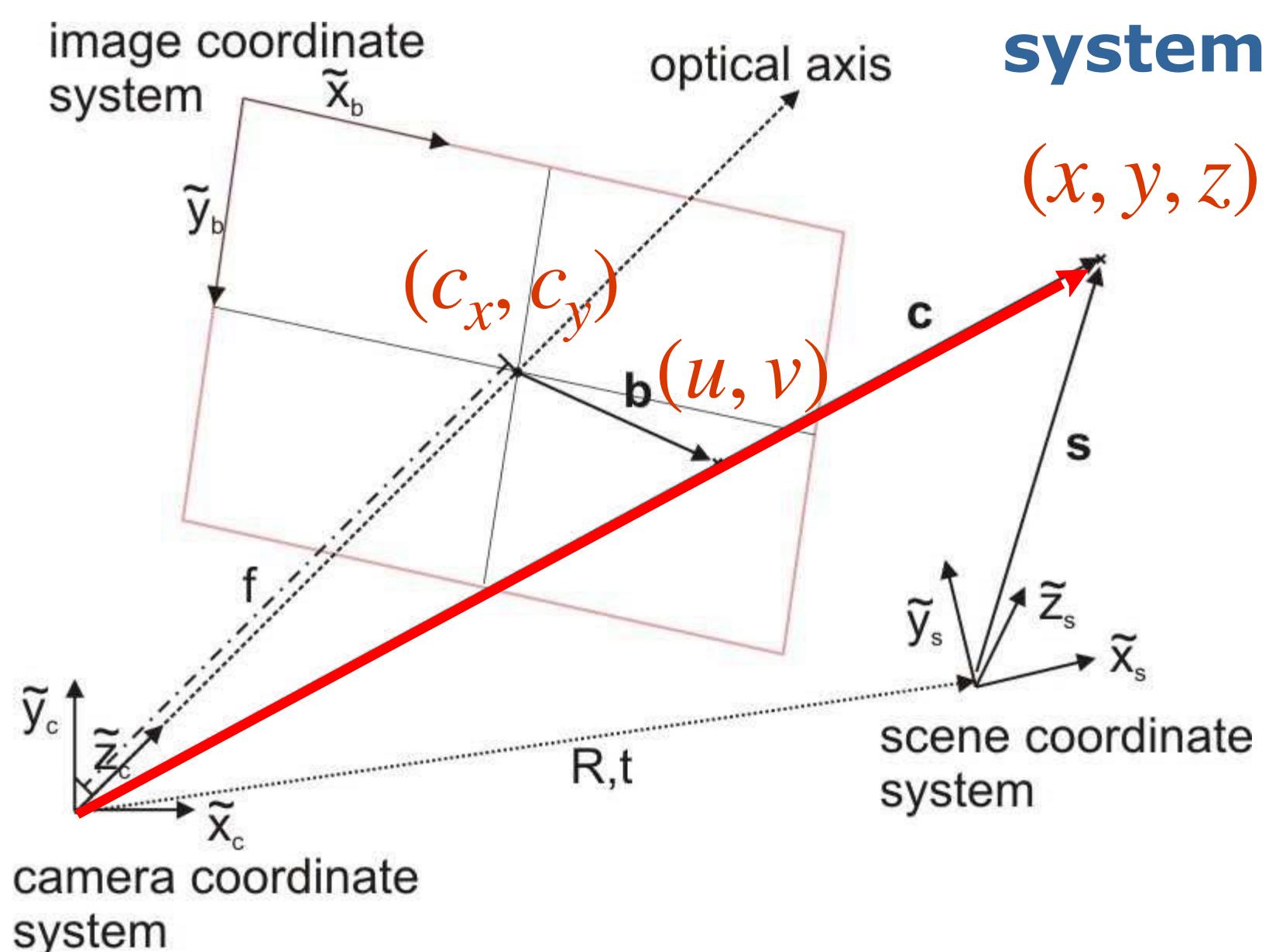


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**camera coordinate system**



# Pinhole Camera Model

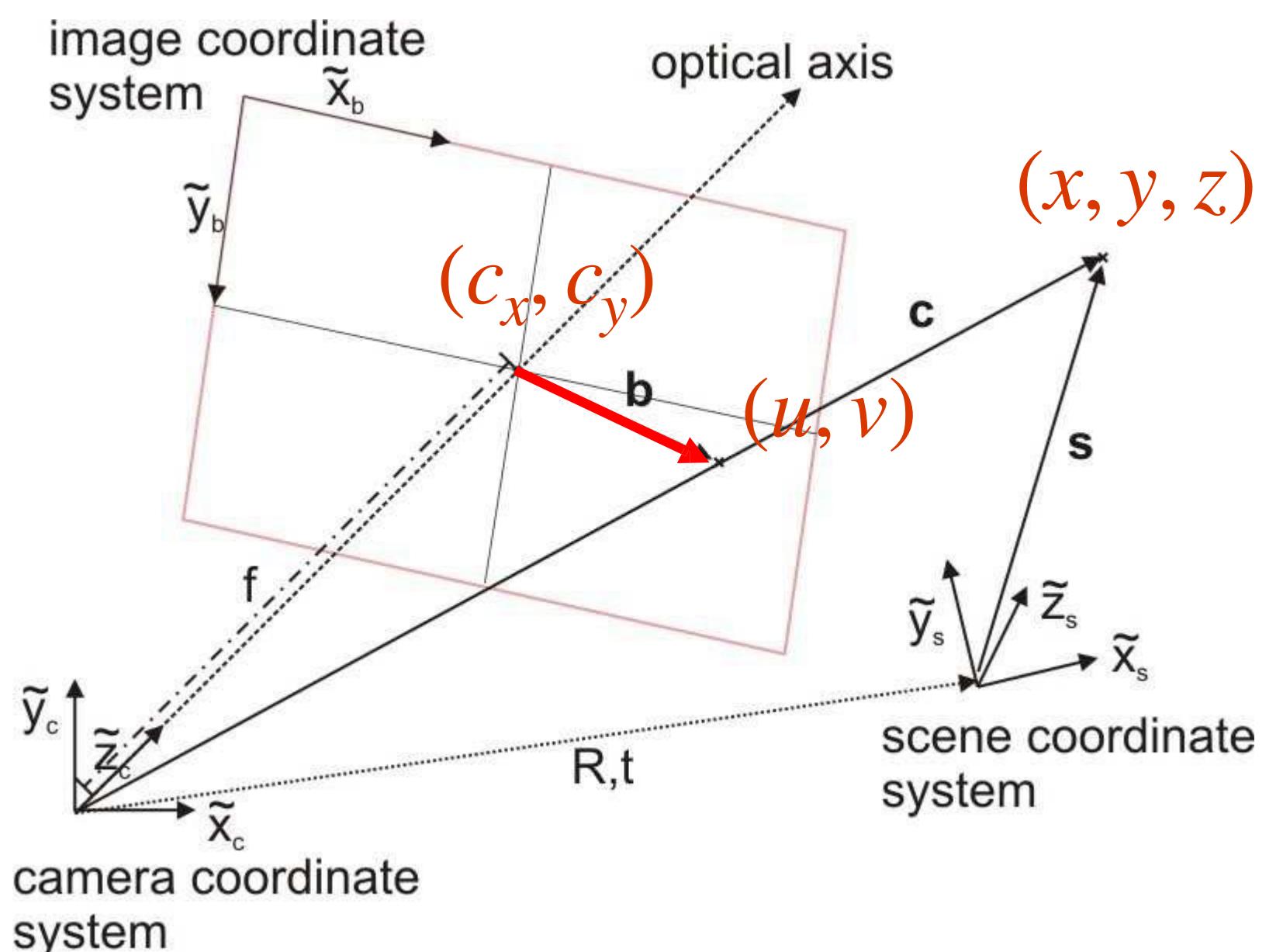
▶ Perspective transformation using

homogeneous coordinates:

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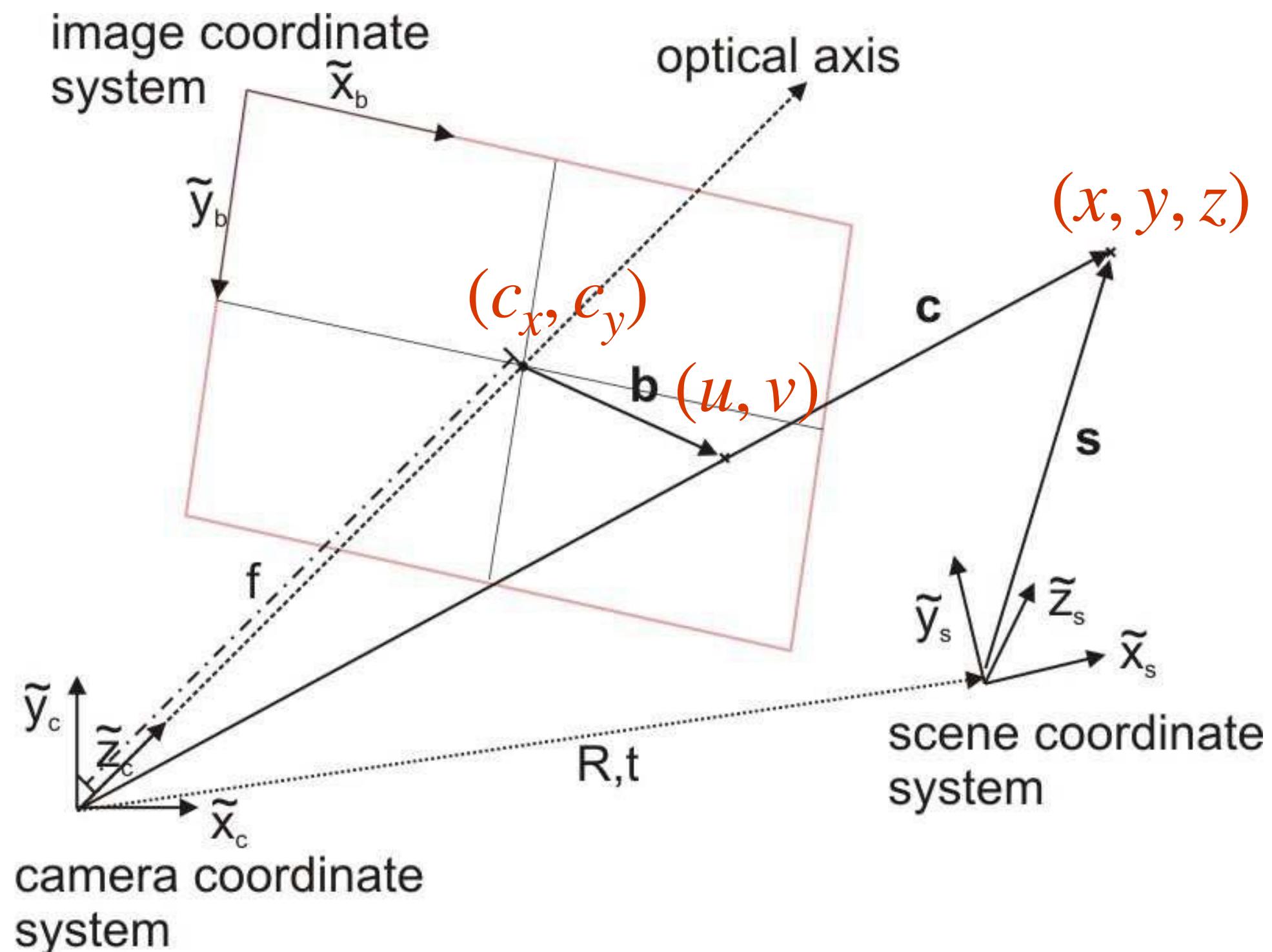


image coordinate system



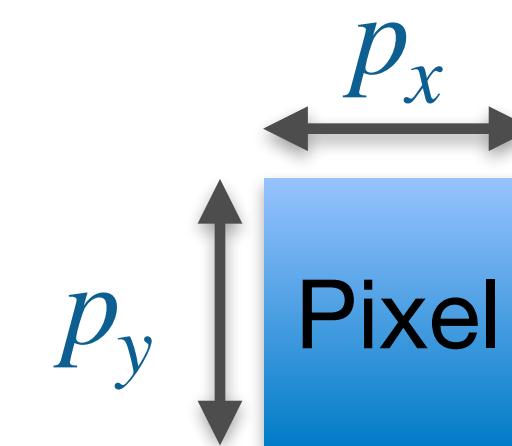
# Pinhole Camera Model

- ▶ Interpretation of intrinsic camera parameters:



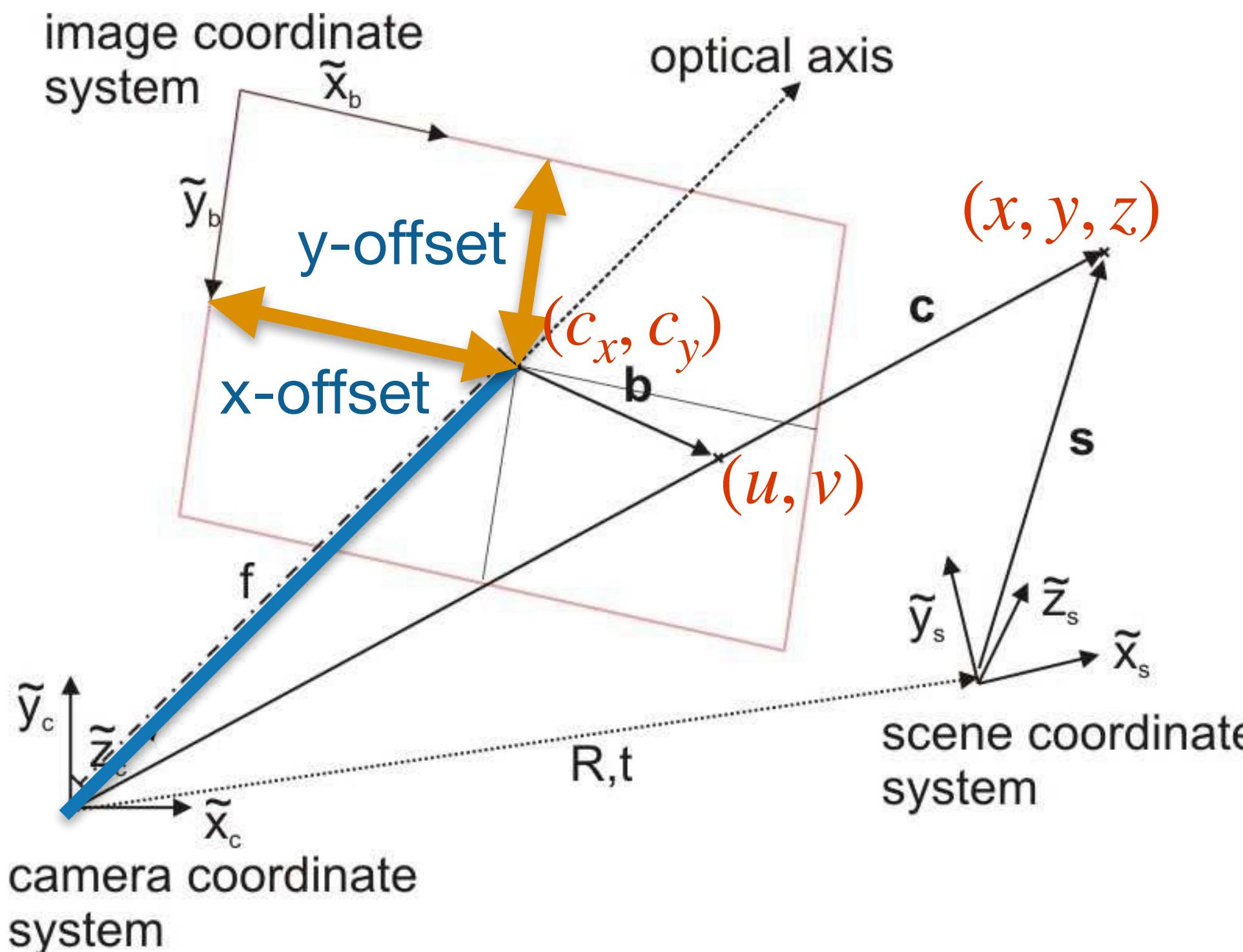
$$K = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

$(f_x, f_y)$ : focal length in pixels.  
 $f_x = \frac{F}{p_x}$  and  $f_y = \frac{F}{p_y}$   
F: focal length in world units  
 $(p_x, p_y)$ : size of pixels in world units



# Pinhole Camera Model

- ▶ Interpretation of intrinsic camera parameters:



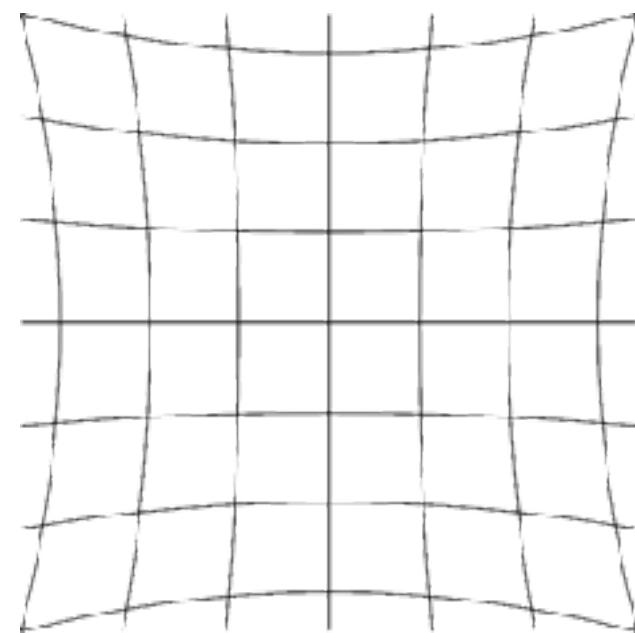
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focal length      offset

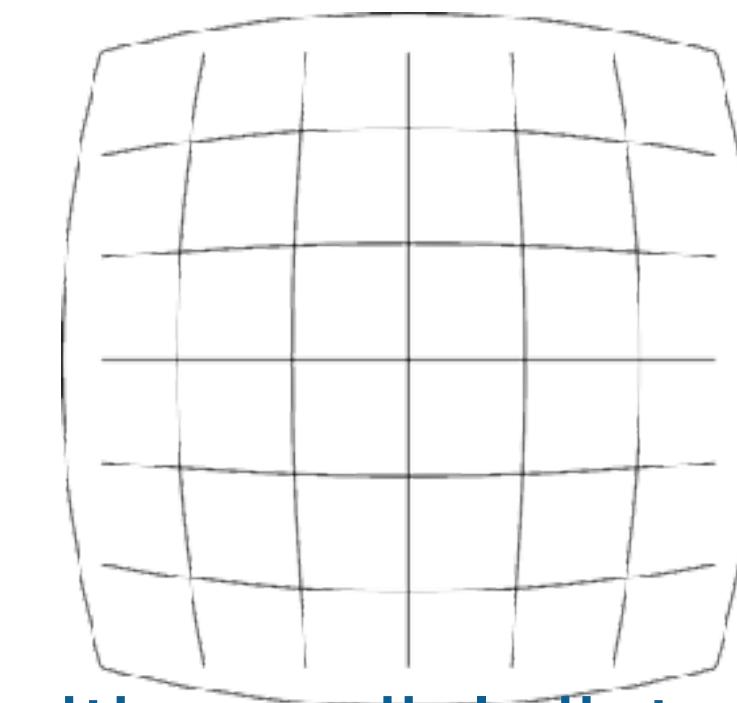
# Lens distortion

Non-linear effects:

- ▶ Radial distortion
- ▶ Tangential distortion
- ▶ Compute corrected image point:



Negative radial distortion



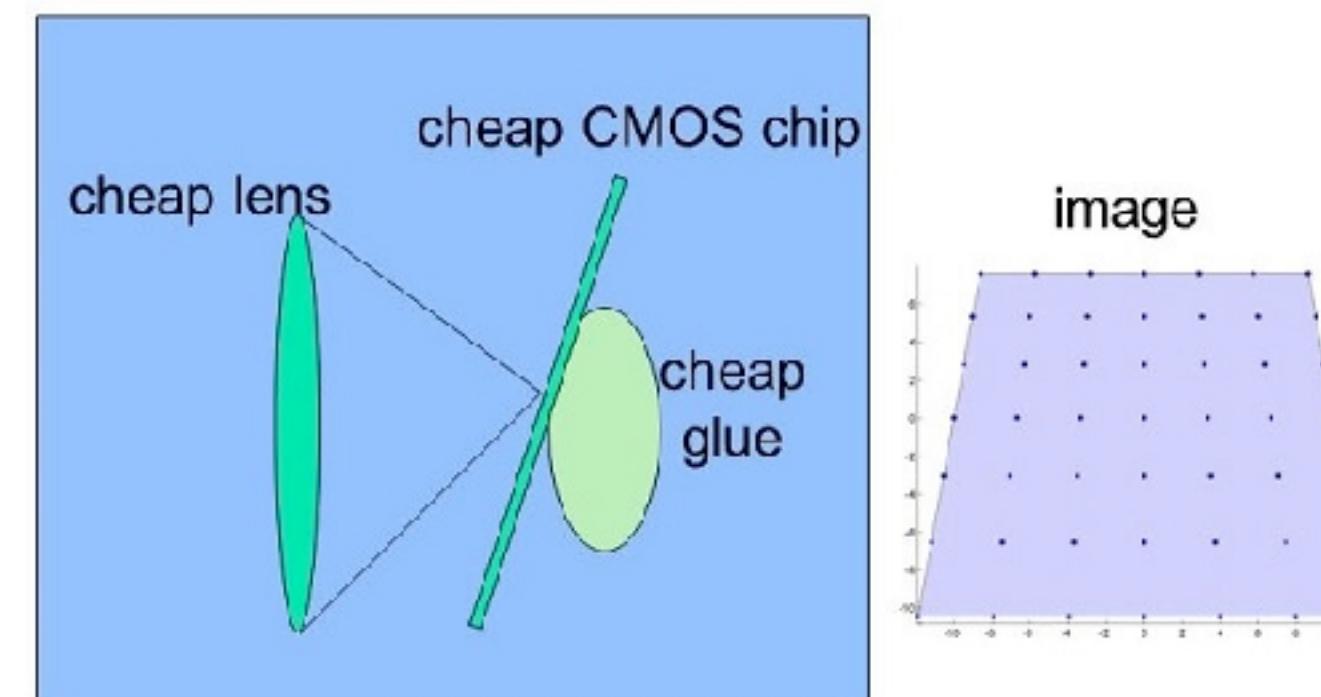
Positive radial distortion

$$(1) \begin{aligned} x' &= x/z \\ y' &= y/z \end{aligned}$$

$$(2) \begin{aligned} x'' &= x'(1 + k_1 r^2 + k_2 r^4) + 2p_1 x'y' + p_2(r^2 + 2x'^2) \\ y'' &= y'(1 + k_1 r^2 + k_2 r^4) + p_1(r^2 + 2y'^2) + 2p_2 x'y' \end{aligned}$$

where  $r^2 = x'^2 + y'^2$ ,  $k_1, k_2$ : radial distortion coefficients,  
and  $p_1, p_2$ : tangential distortion coefficients.

$$(3) \begin{aligned} u &= f_x \cdot x'' + c_x \\ v &= f_y \cdot y'' + c_y \end{aligned}$$



Tangential distortion

# Camera Calibration

- ▶ Calculate intrinsic parameters and lens distortion from a series of images
  - 2D camera calibration
  - 3D camera calibration
  - Self calibration

# Camera Calibration

▶ Calculate intrinsic parameters and lens distortion from a series of images

- 2D camera calibration
- 3D camera calibration

need external pattern

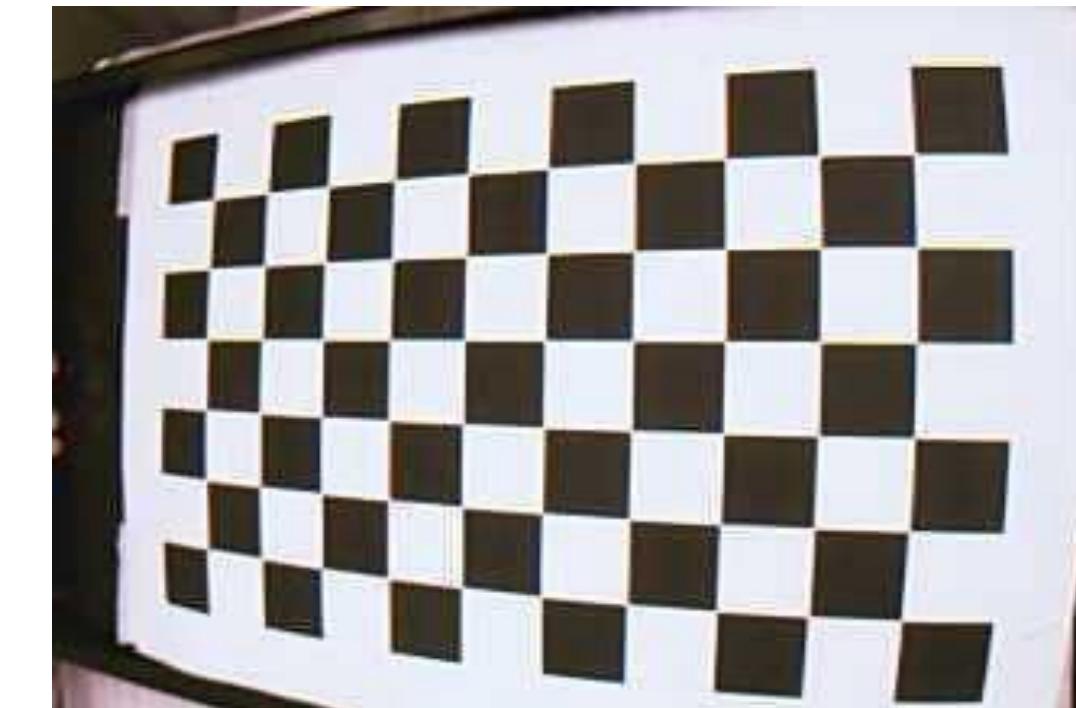
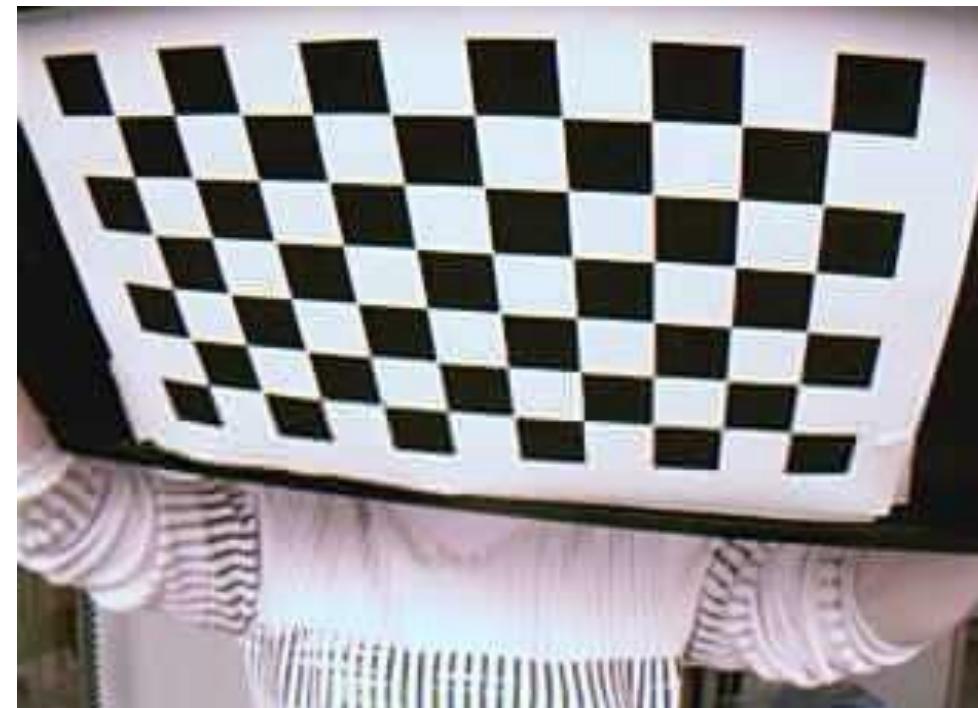
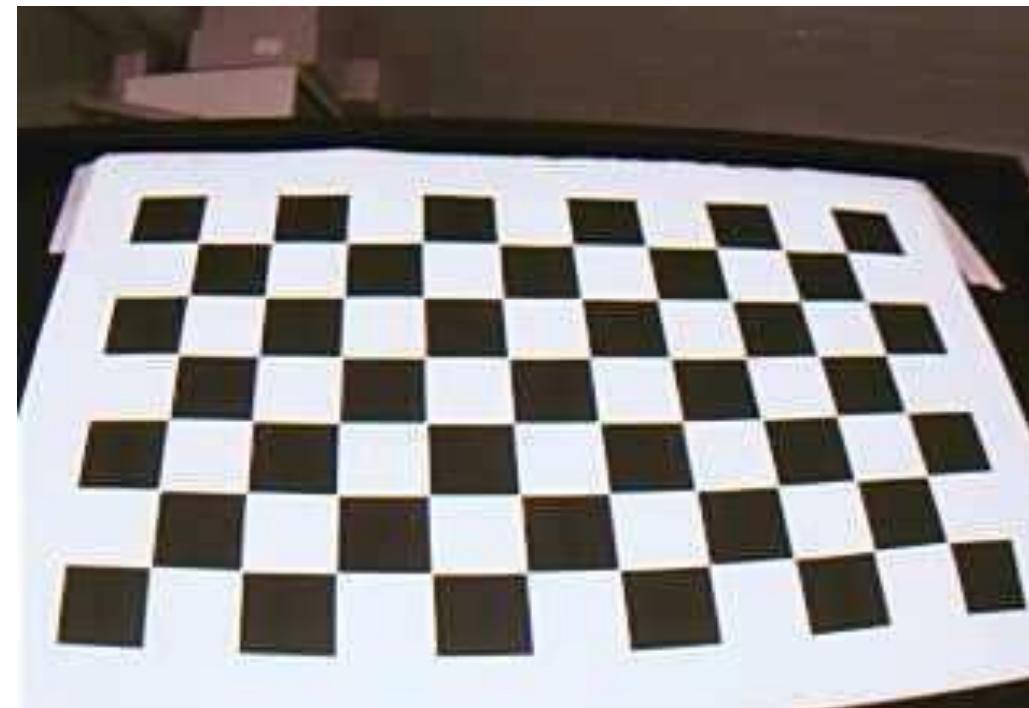
- Self calibration

# Camera Calibration

- ▶ Calculate intrinsic parameters and lens distortion from a series of images
  - 2D camera calibration
  - 3D camera calibration
  - self calibration

# 2D Camera Calibration

- ▶ Use a 2D pattern (e.g., a checkerboard)



- ▶ Trick: set the world coordinate system to the corner of the checkerboard and the dimension of the checker board in world units are known.

# 2D Camera Calibration

- ▶ Use a 2D pattern (e.g., a checkerboard)



- ▶ Trick: set the world coordinate system to the corner of the checkerboard and the dimension of the checkerboard in world units are known.
- ▶ Now: All points on the checkerboard lie in one plane!

# 2D Camera Calibration

$$b = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$

- ▶ Since all points lie in a plane, their  $z$  components are 0 in world coordinates

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- ▶ Thus, we can delete the 3rd column of the Extrinsic parameter matrix.

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# 2D Camera Calibration

$$H = (h_1, h_2, h_3) = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix}}_{(r_1, r_2, t)}$$



$$(h_1, h_2, h_3) = K(r_1, r_2, t)$$

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$$(h_1, h_2, h_3) = K(r_1, r_2, t)$$


$$r_1 = K^{-1}h_1, r_2 = K^{-1}h_2$$

# 2D Camera Calibration

Note that  $(r_1, r_2, r_3)$  form an orthonormal basis, thus:  $r_1^T r_2 = 0, \|r_1\| = \|r_2\| = 1$ .

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$$r_1 = K^{-1}h_1, r_2 = K^{-1}h_2$$

$$\xrightarrow{r_1^T r_2 = 0} h_1^T K^{-T} K^{-1} h_2 = 0$$

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$$h_1^T K^{-T} K^{-1} h_2 = 0$$

$$\|r_1\| = \|r_2\| = 1 \quad \rightarrow \quad h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$$

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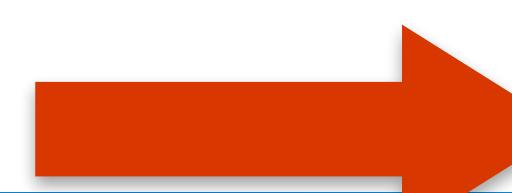
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$$h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0$$

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# 2D Camera Calibration

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$$h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0 \quad (2)$$

- $B := K^{-T} K^{-1}$  is symmetric and positive definite.

# 2D Camera Calibration

$$h_1^T K^{-T} K^{-1} h_2 = 0 \quad (1)$$

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- $B := K^{-T} K^{-1}$  is symmetric and positive definite.

Thus:  $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$

Note:  $K$  can be calculated from  $B$  using Cholesky factorization.

# 2D Camera Calibration

$$h_1^T K^{-T} K^{-1} h_2 = 0 \quad (1)$$

$$h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0 \quad (2)$$

- $B := K^{-T} K^{-1}$  is symmetric and positive definite.

Thus:  $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$

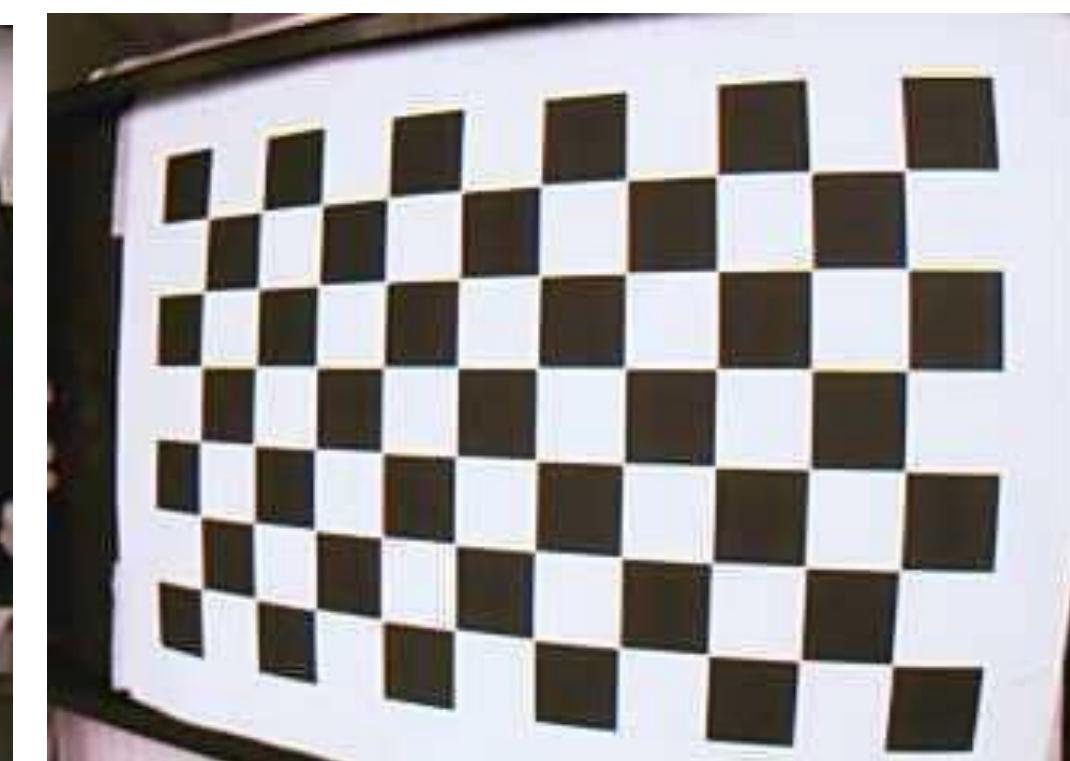
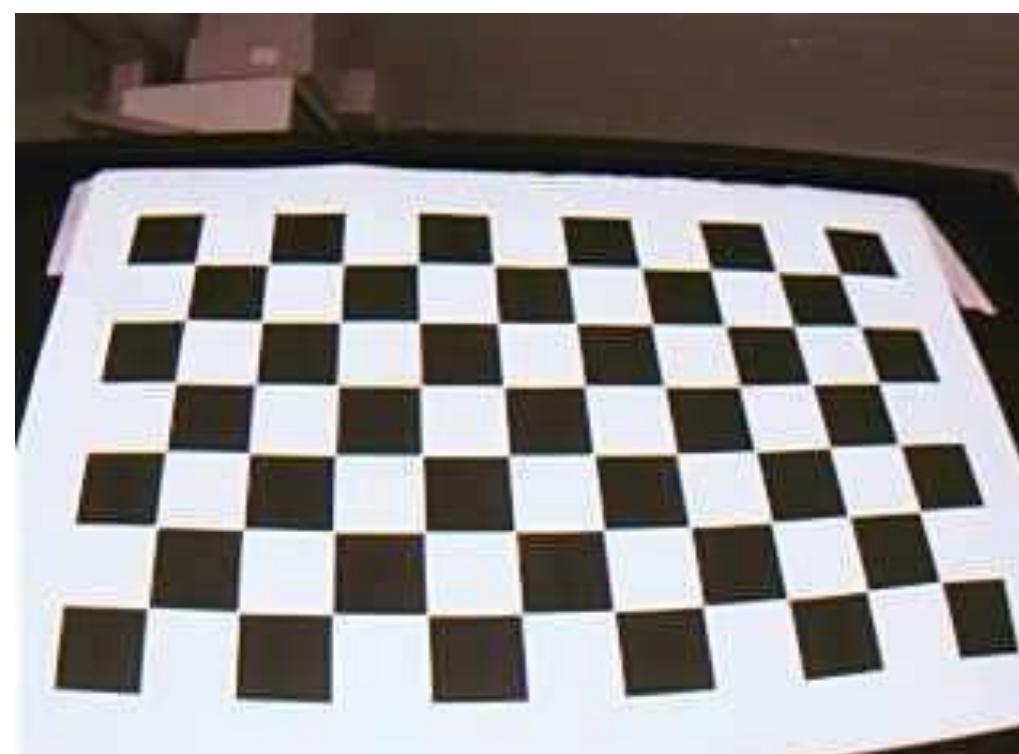
Note:  $K$  can be calculated from  $B$  using Cholesky factorization.

- define:  $b = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33}) \quad (3)$
- Reordering of (1)-(3) leads to the system of the final equations :

$$Vb = 0$$

# Direct Linear Transformation

- ▶ Each plane gives us two equations
- ▶ Since  $B$  has 6 degrees of freedom, we need at least 3 different views of a plane.



- ▶ We need at least 4 points per plane.

# Direct Linear Transformation

- ▶ Real measurements are corrupted with noise
- ▶ Find a solution that minimizes the least-squares error

$$b = \arg \min_b Vb$$

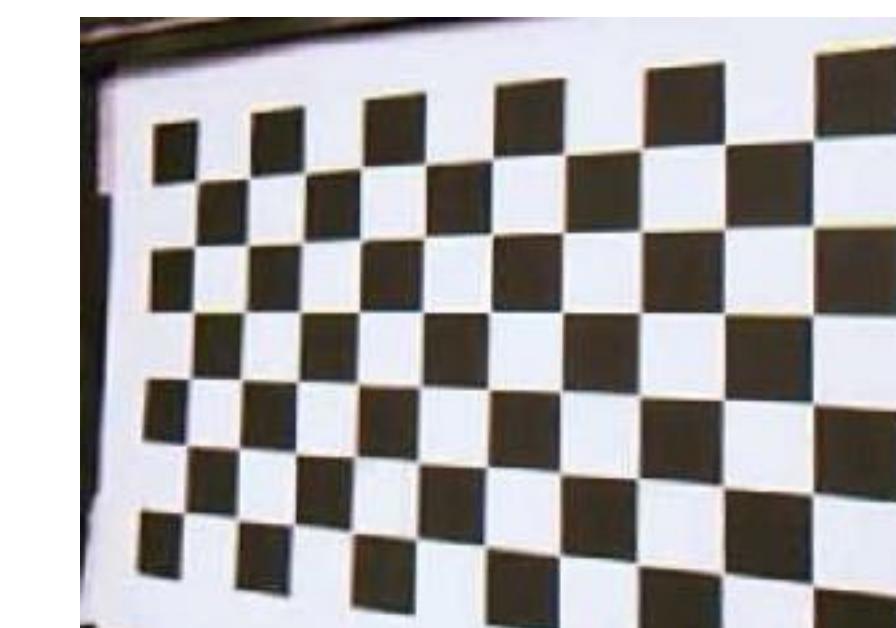
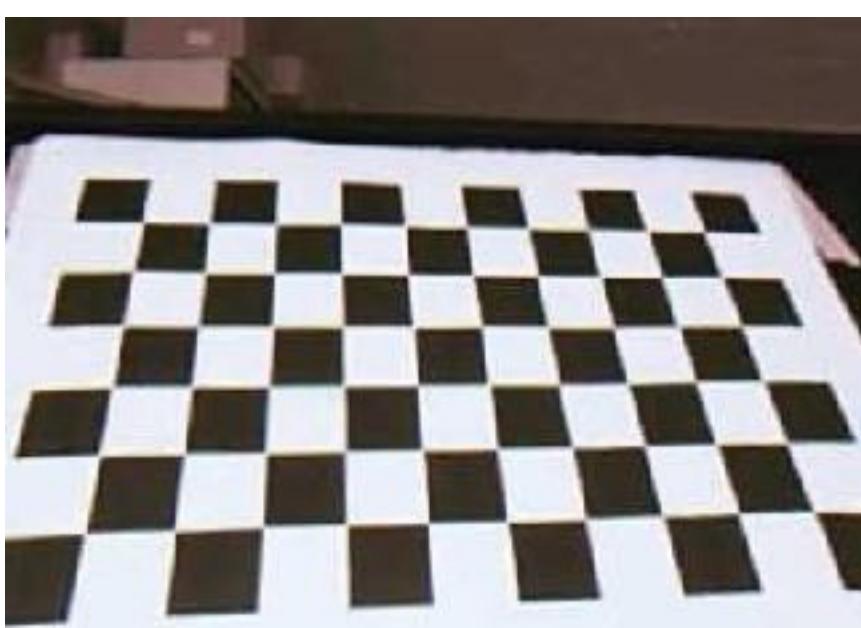
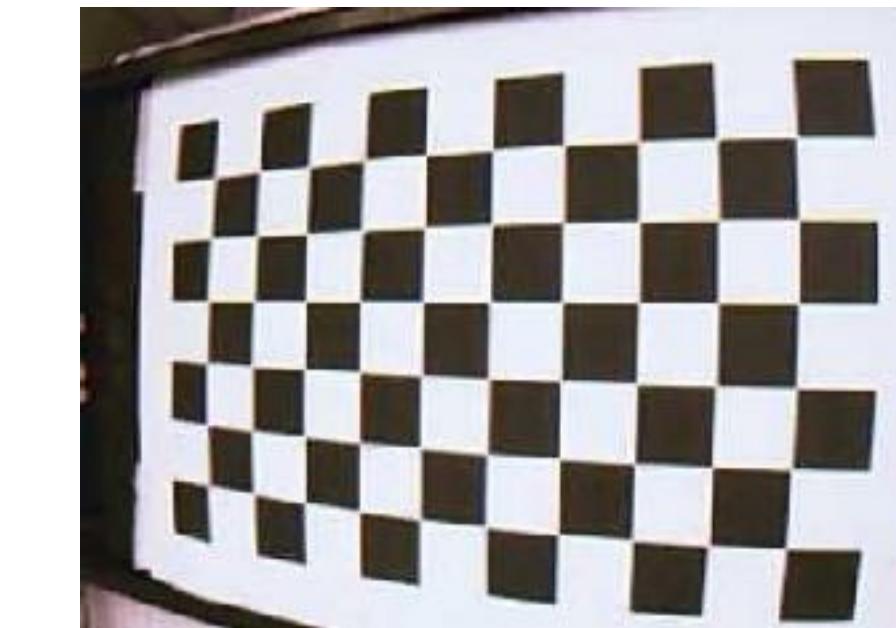
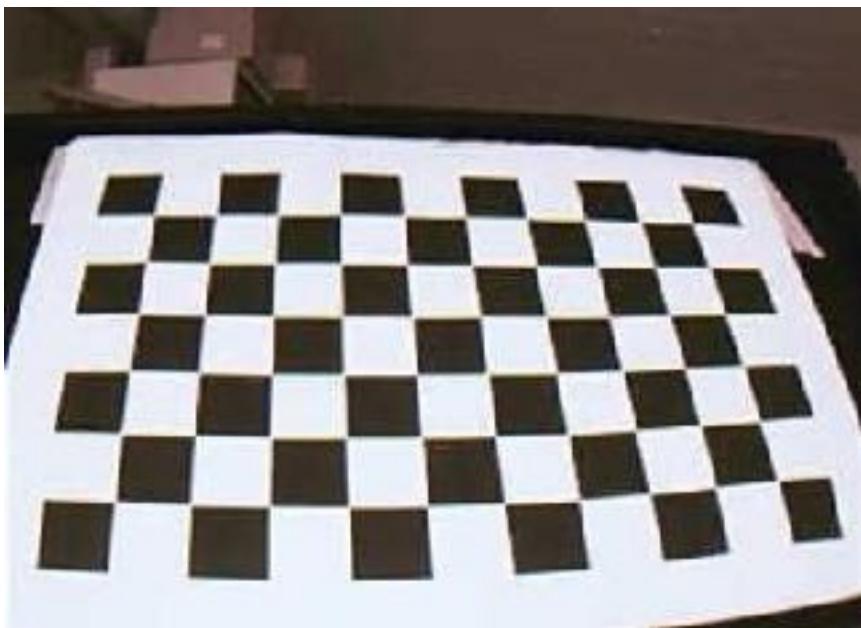
# Non-Linear Optimization

- ▶ Lens distortion can be calculated by minimizing a non-linear function

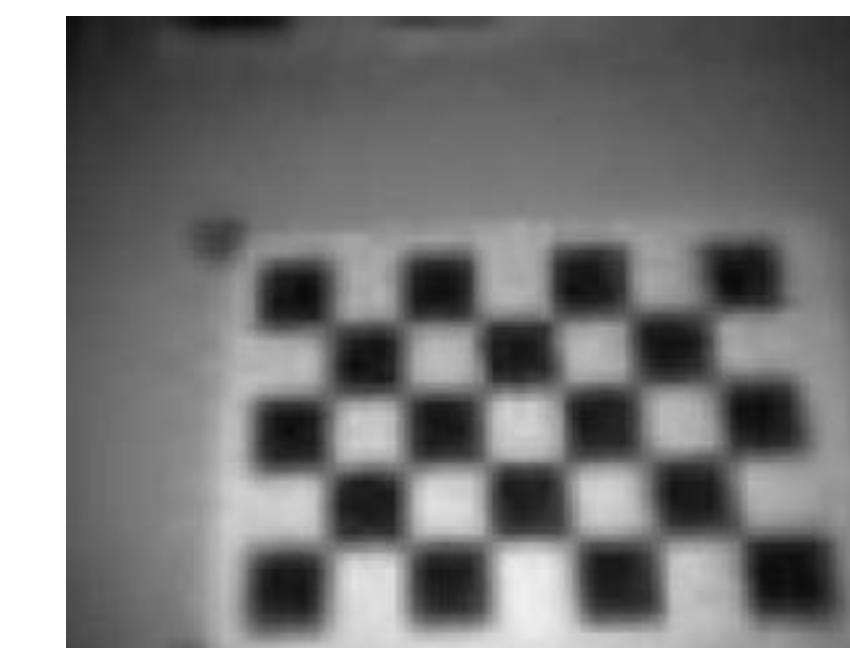
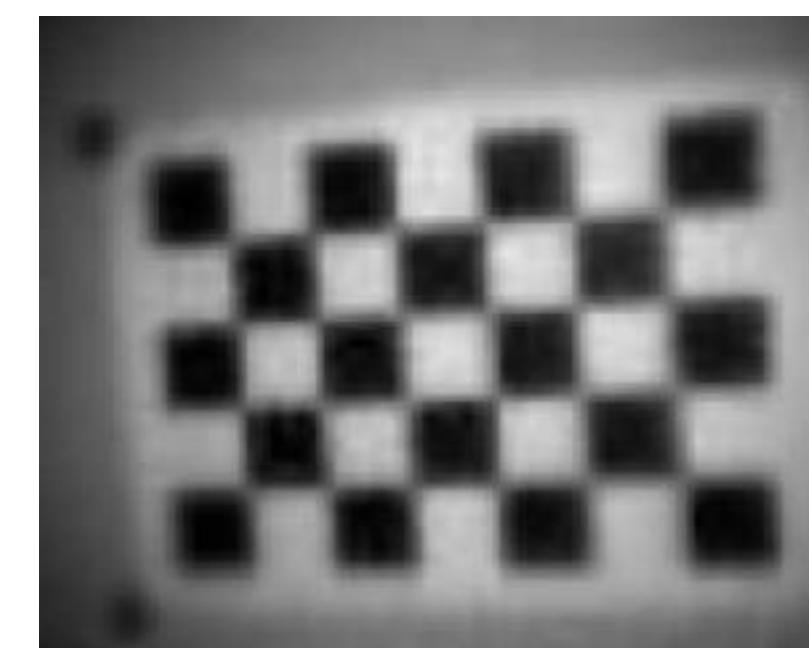
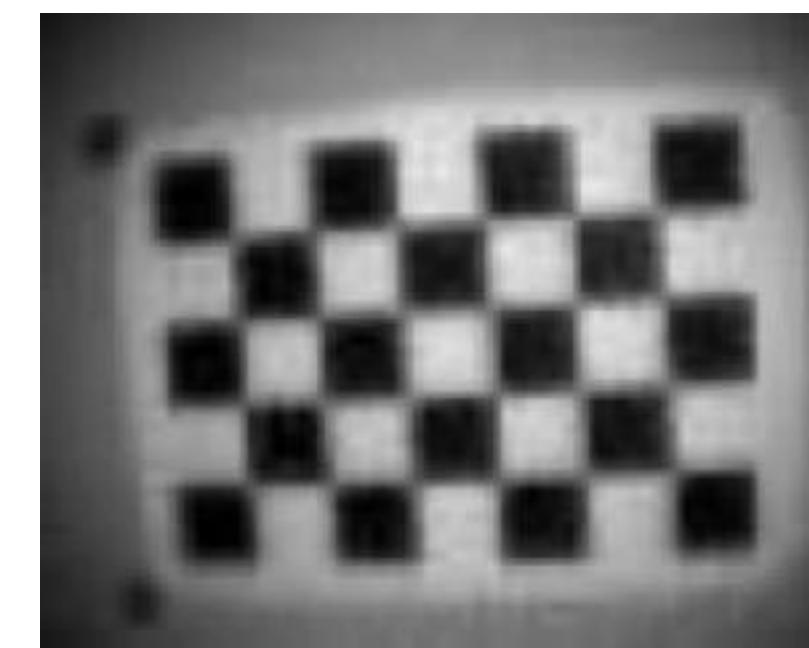
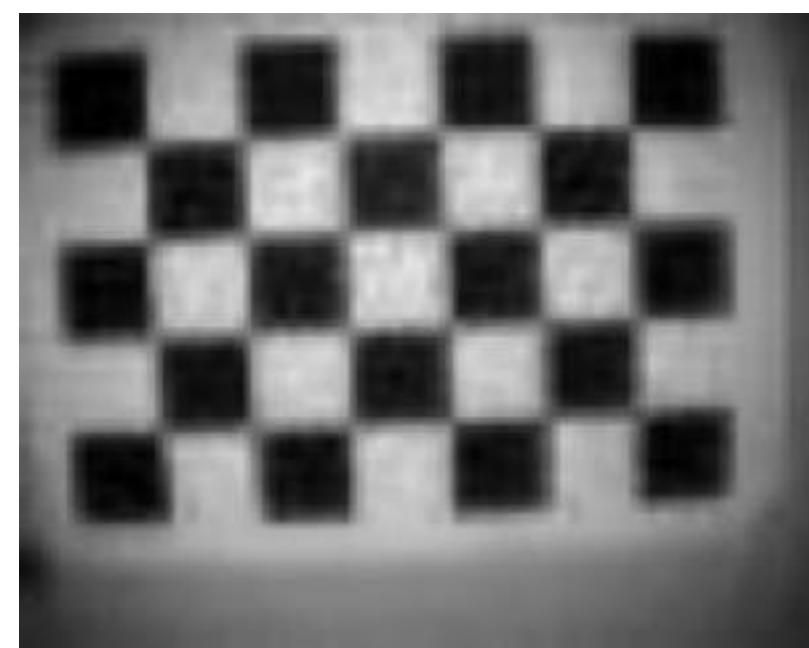
$$\min_{K, k, R_i, t_i} \sum_i \sum_j \|x_{i,j} - \hat{x}(K, k, t_i; X_{ij})\|^2$$

- ▶ Estimation of  $k$  using non-linear optimization techniques (e.g., Levenberg-Marquard)
- ▶ The parameters obtained by the linear function are used as starting values.

# Results: Webcam



# Results: ToF-Camera



# Summary

- ▶ Pinhole Camera Model
- ▶ Non-linear model for lens distortion
- ▶ Approach to 2D Calibration that
  - ▶ accurately determines the model parameters and is easy to realize

# Finding lane lines for self-driving cars

Reference from Udacity

# Finding lane lines is perception and decision

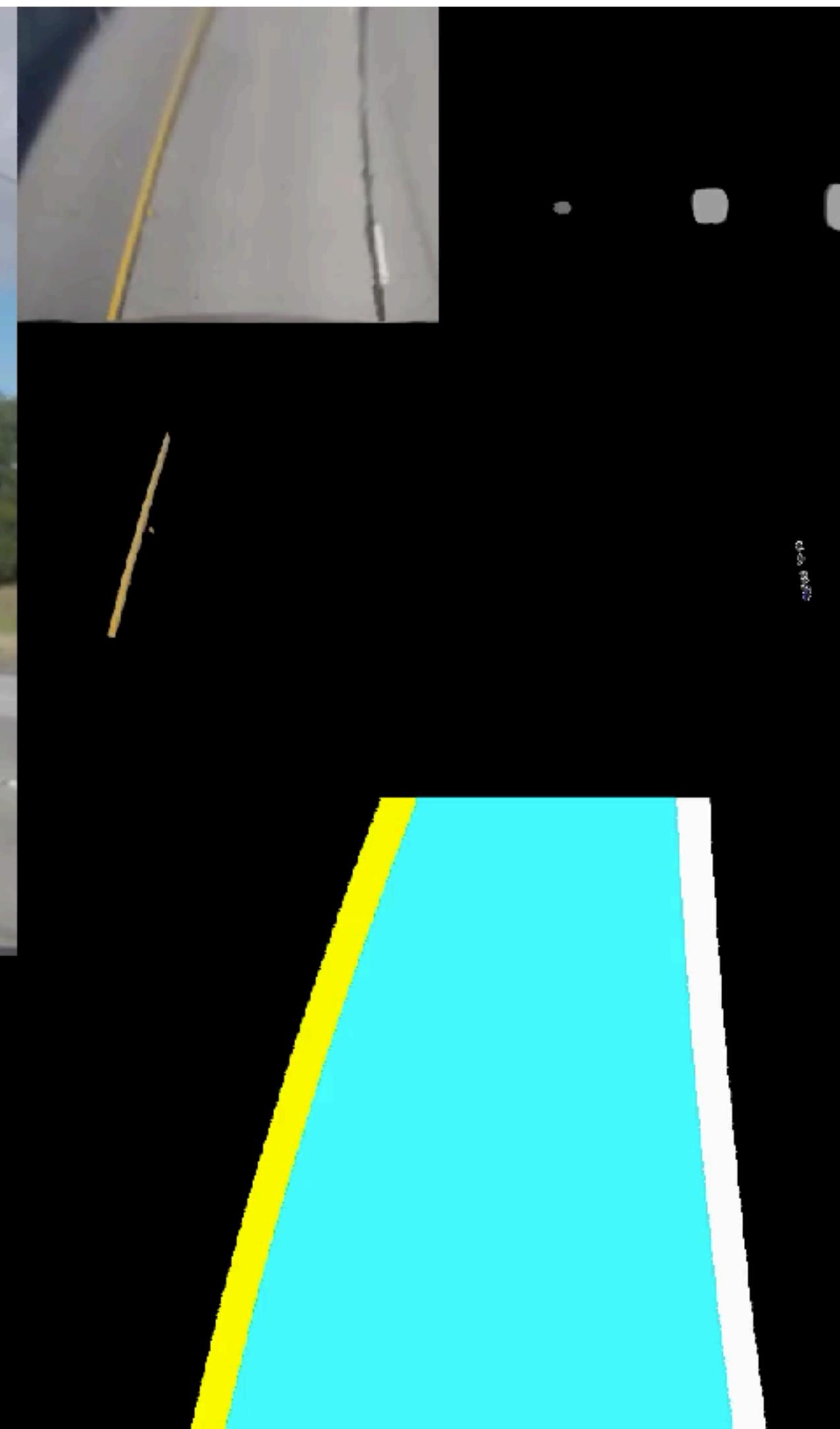


Radius of curvature: 0501m  
0.11m Right of center



Curvature: Right = -13777.09, Left = -7113.53  
Lane deviation: 36.47 cm.



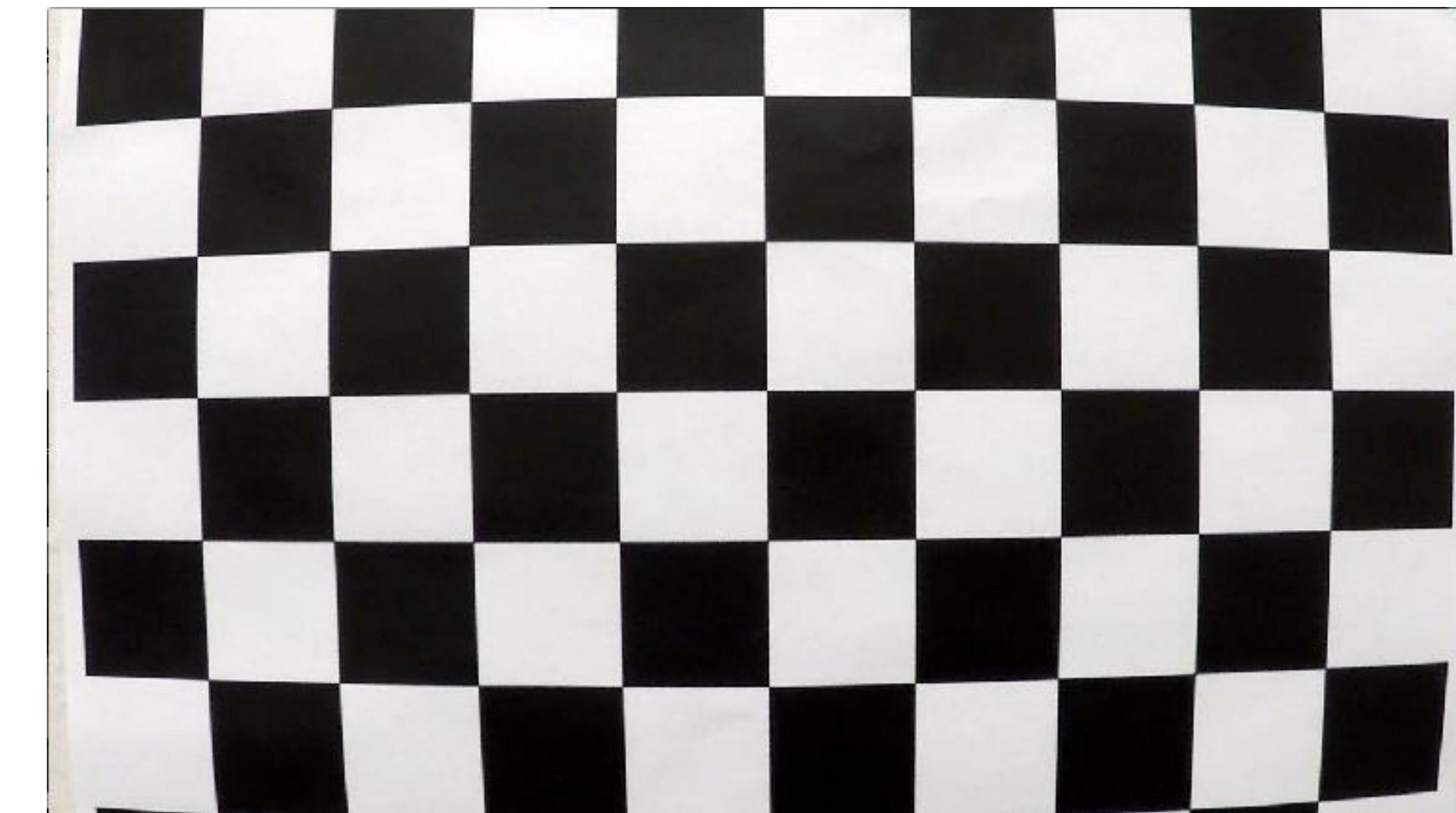
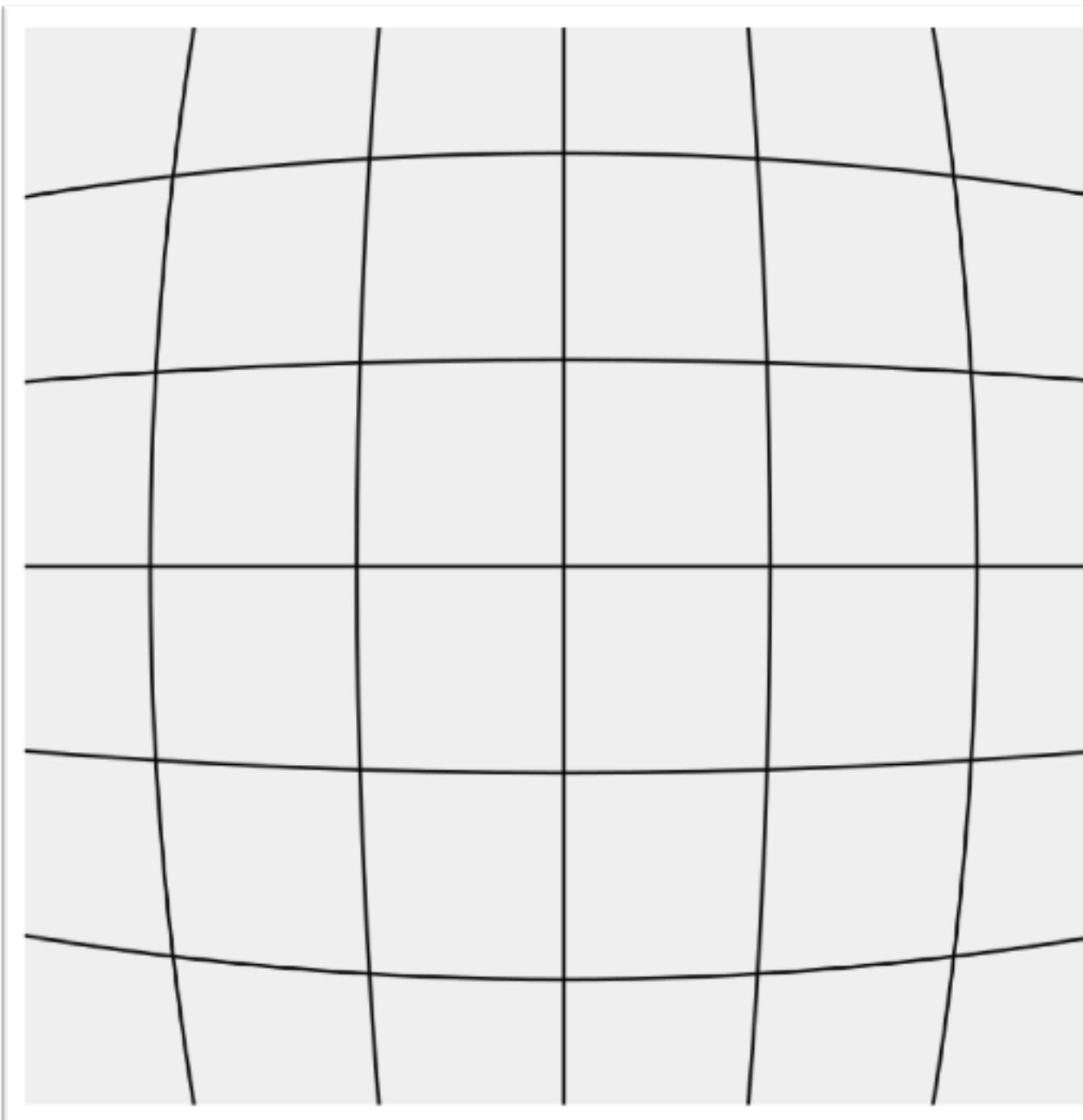


# Finding Lane Lines from Images

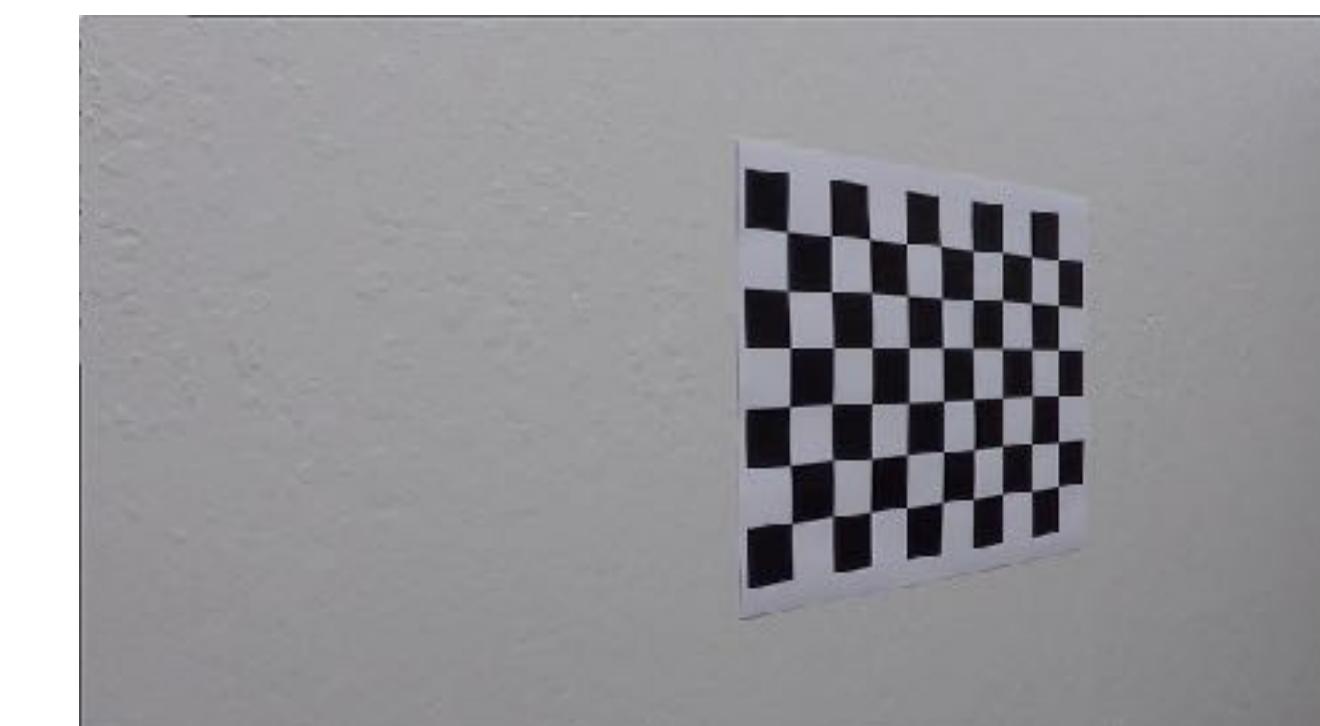
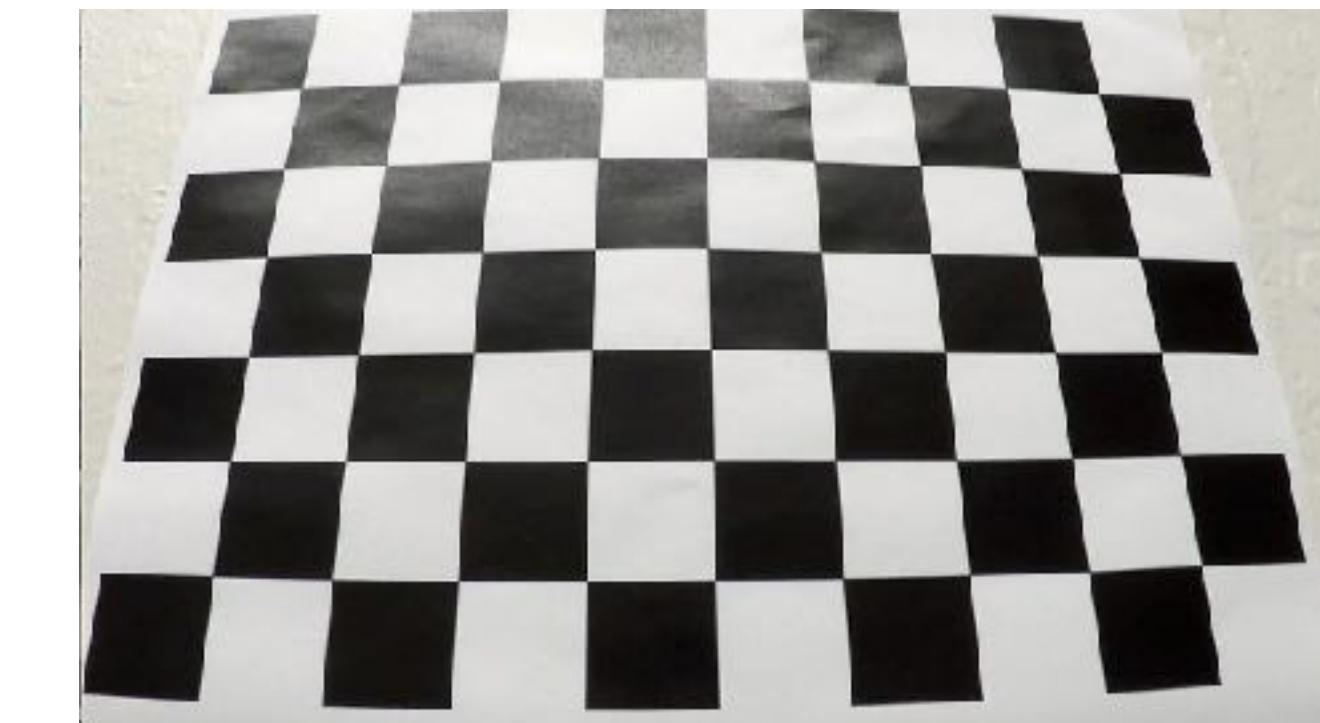
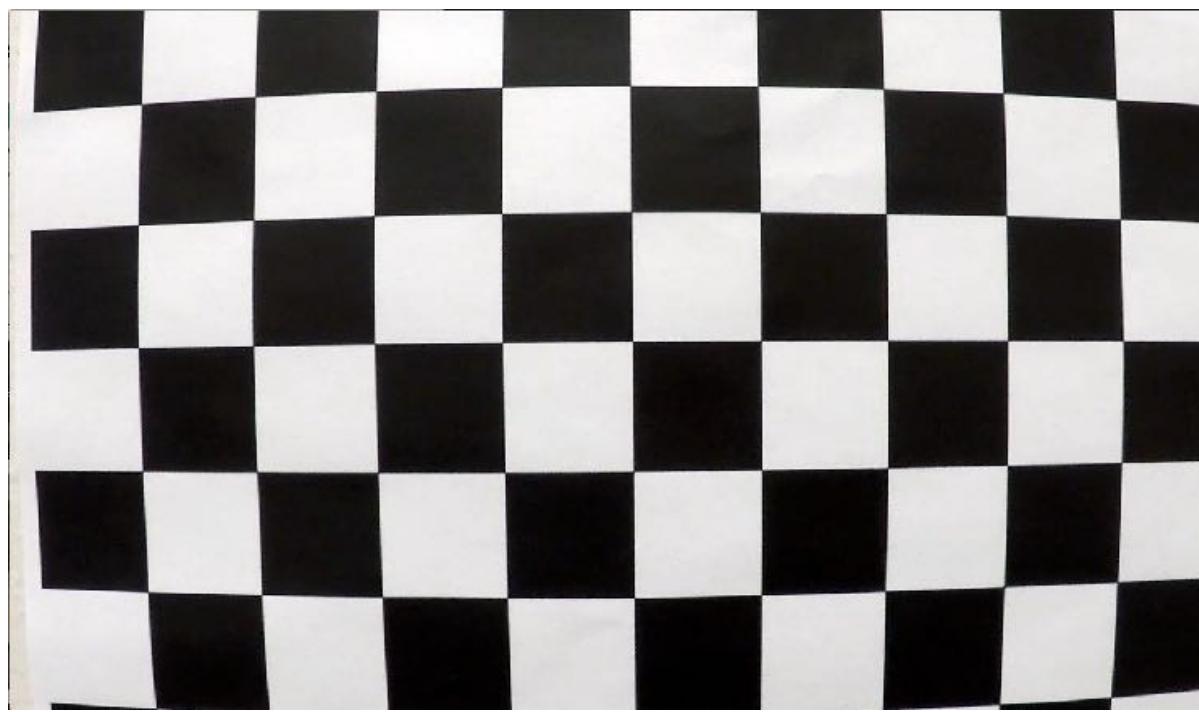
1. Calibrate camera to remove distortion
2. Warping images
3. Finding lane lines
4. Curve fitting
5. Overlap Results on Image

# Calibrate camera to remove distortion

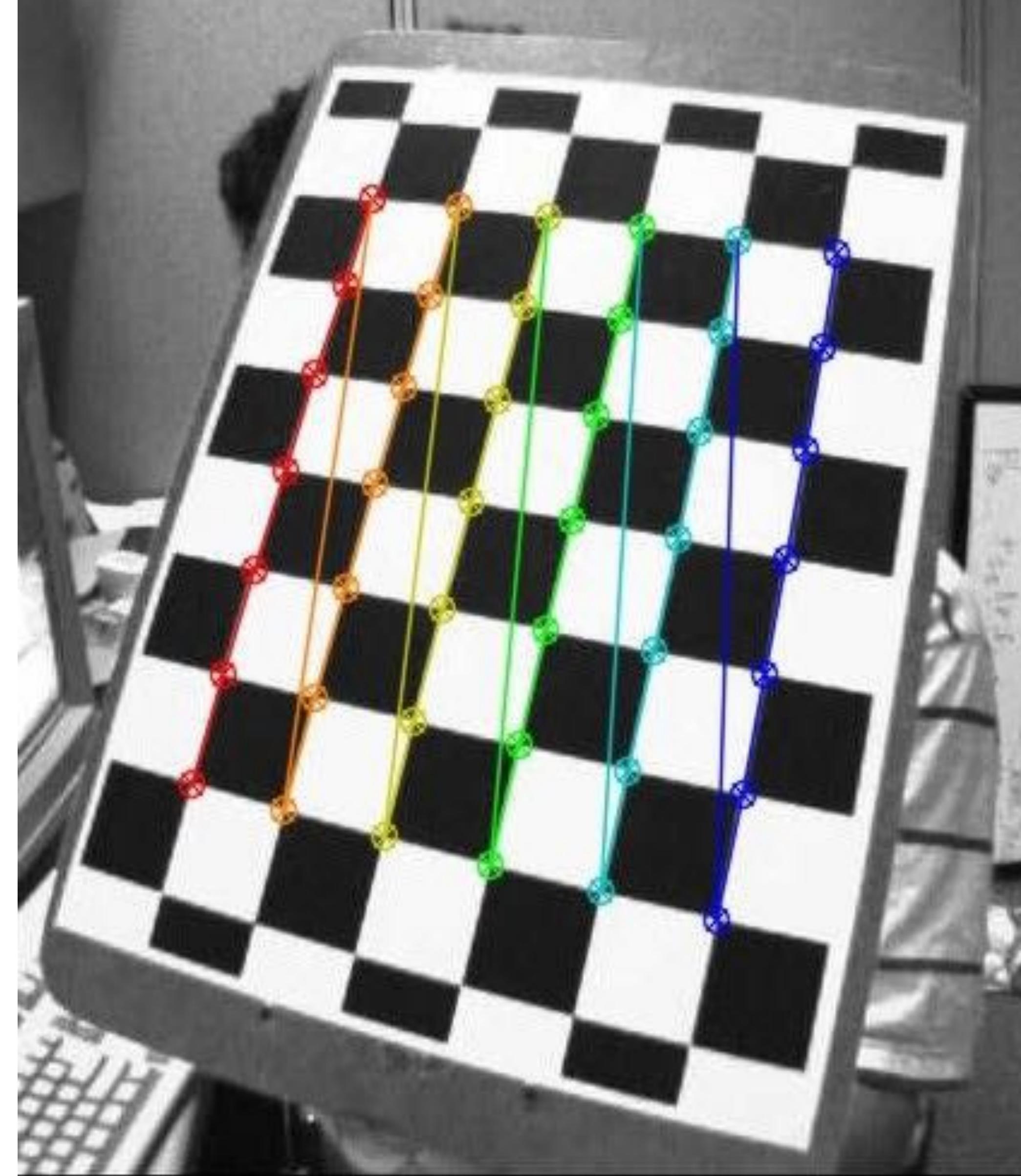
# Camera Lens Distortion



# OpenCV helps us correct for these distortions



```
ret, corners = cv2.findChessboardCorners(gray, (9,6), None)
```



# OpenCV: Locates the chessboard corners

Python: cv2.findChessboardCorners(image, patternSize[, corners[, flags]]) → retval, corners

Parameters:

- image – Source chessboard view. It must be an 8-bit grayscale or color image.
- patternSize – Number of inner corners per a chessboard row and column ( patternSize = cvSize(points\_per\_row,points\_per\_colum) = cvSize(columns,rows) ).
- corners – Output array of detected corners.
- flags – Various operation flags that can be zero or a combination of the following values:
  - CALIB\_CB\_ADAPTIVE\_THRESH Use adaptive thresholding to convert the image to black and white, rather than a fixed threshold level (computed from the average image brightness).
  - CALIB\_CB\_NORMALIZE\_IMAGE Normalize the image gamma with equalizeHist() before applying fixed or adaptive thresholding.
  - CALIB\_CB\_FILTER\_QUADS Use additional criteria (like contour area, perimeter, square-like shape) to filter out false quads extracted at the contour retrieval stage.
  - CALIB\_CB\_FAST\_CHECK Run a fast check on the image that looks for chessboard corners, and shortcut the call if none is found. This can drastically speed up the call in the degenerate condition when no chessboard is observed.

```

import numpy as np
import cv2 as cv
import glob

# termination criteria
criteria = (cv.TERM_CRITERIA_EPS + cv.TERM_CRITERIA_MAX_ITER, 30, 0.001)

# prepare object points, like (0,0,0), (1,0,0), (2,0,0) ....,(6,5,0)
objp = np.zeros((6*7,3), np.float32)
objp[:, :2] = np.mgrid[0:7,0:6].T.reshape(-1,2)

# Arrays to store object points and image points from all the images.
objpoints = [] # 3d point in real world space
imgpoints = [] # 2d points in image plane.

images = glob.glob('*.jpg')

for fname in images:
    img = cv.imread(fname)
    gray = cv.cvtColor(img, cv.COLOR_BGR2GRAY)

    # Find the chess board corners
    ret, corners = cv.findChessboardCorners(gray, (7,6), None)

    # If found, add object points, image points (after refining them)
    if ret == True:
        objpoints.append(objp)

        corners2 = cv.cornerSubPix(gray,corners, (11,11), (-1,-1), criteria)
        imgpoints.append(corners)

        # Draw and display the corners
        cv.drawChessboardCorners(img, (7,6), corners2, ret)
        cv.imshow('img', img)

cv.waitKey(500)

```

```

# termination criteria
criteria = (cv.TERM_CRITERIA_EPS + cv.TERM_CRITERIA_MAX_ITER, 30, 0.001)

# prepare object points, like (0,0,0), (1,0,0), (2,0,0) ....,(6,5,0)
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images = glob.glob('*.jpg')

for fname in images:
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    gray = cv.cvtColor(img, cv.COLOR_BGR2GRAY)

    # Find the chess board corners
    ret, corners = cv.findChessboardCorners(gray, (7,6), None)

    # If found, add object points, image points (after refining them)
    if ret == True:
        objpoints.append(objp)

        corners2 = cv.cornerSubPix(gray,corners, (11,11), (-1,-1), criteria)
        imgpoints.append(corners)

        # Draw and display the corners
        cv.drawChessboardCorners(img, (7,6), corners2, ret)
        cv.imshow('img', img)
        cv.waitKey(500)

cv.destroyAllWindows()

```

# Calibrate the camera

Finds the camera intrinsic and extrinsic parameters from several views of a calibration pattern.

```
ret, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(  
    objpoints, imgpoints, gray.shape[::-1],  
    None, None  
)
```

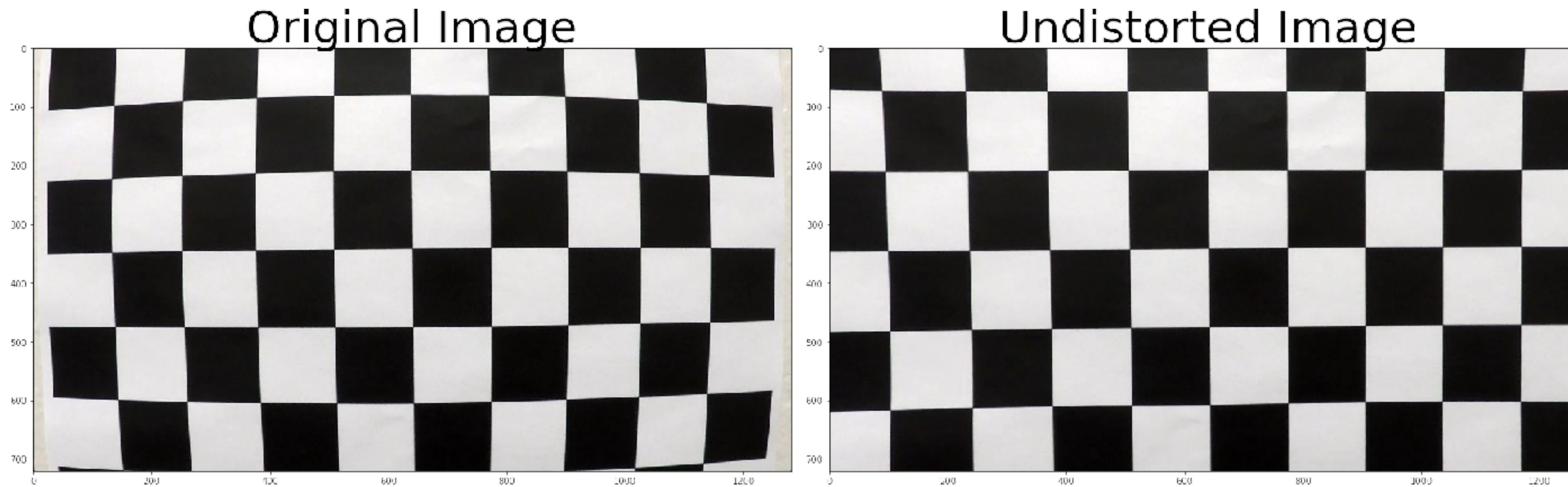
**Python:** `cv2.calibrateCamera(objectPoints, imagePoints, imageSize[, cameraMatrix[, distCoeffs[, rvecs[, tvecs[, flags[, criteria]]]]]])` → retval, cameraMatrix, distCoeffs, rvecs, tvecs

Finds the camera intrinsic and extrinsic parameters from several views of a calibration pattern.

- ▶ return values: cameraMatrix, distCoeffs, rvecs, tvecs
- ▶ **objectPoints**: a vector of vectors of calibration pattern points in the calibration pattern coordinate space
- ▶ **imagePoints**: a vector of vectors of the projections of calibration pattern points
- ▶ **imageSize**: Size of the image used only to initialize the intrinsic camera matrix.
- ▶ **cameraMatrix**: Output 3x3 floating-point camera matrix A
- ▶ **distCoeffs**: Output vector of distortion coefficients
- ▶ **rvecs** – Output vector of rotation vectors  
¶

# Use the camera calibration to undistort images

```
undist = cv2.undistort(img, mtx, dist,  
None, mtx)
```



C++: void `undistort(InputArray src, OutputArray dst, InputArray cameraMatrix, InputArray distCoeffs, InputArray newCameraMatrix=noArray() )`

Python: `cv2.undistort(src, cameraMatrix, distCoeffs[, dst[, newCameraMatrix]])` → dst

The function transforms an image to compensate radial and tangential lens distortion.

#### Parameters:

- **src** – Input (distorted) image.
- **dst** – Output (corrected) image that has the same size and type as **src** .
- **cameraMatrix** – Input camera matrix
- **distCoeffs** – Input vector of distortion coefficients ( $k_1, k_2, p_1, p_2[, k_3[, k_4, k_5, k_6]]$ ) of 4, 5, or 8 elements. If the vector is NULL/empty, the zero distortion coefficients are assumed.
- **newCameraMatrix** – Camera matrix of the distorted image. By default, it is the same as **cameraMatrix** but you may additionally scale and shift the result by using a different matrix.

$$A = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

# Warping images

# Need a “birds eye view” for curve fitting



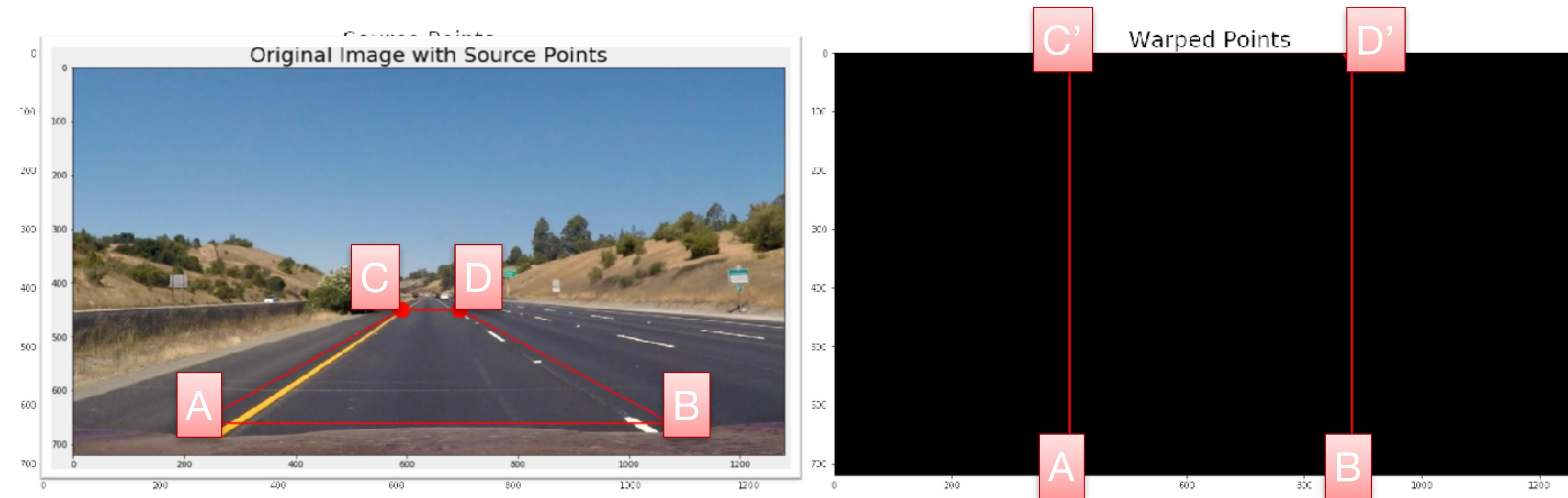
```
src={cv::Point2f(200, 650), cv::Point2f(1120, 650),
```

```
cv::Point2f(580, 450),cv::Point2f(760, 450)}}}
```

```
dst={cv::Point2f(200, 650), cv::Point2f(1120, 650),
```

```
cv::Point2f(200, 0),cv::Point2f(1120, 0)}}}
```

# Need a “birds eye view” for curve fitting



# create a transformation matrix

M = cv2.getPerspectiveTransform(src, dst)

```
# apply the transform to the original image warped = cv2.warpPerspective(img, M,  
img_size)
```

Original Image with Source Points



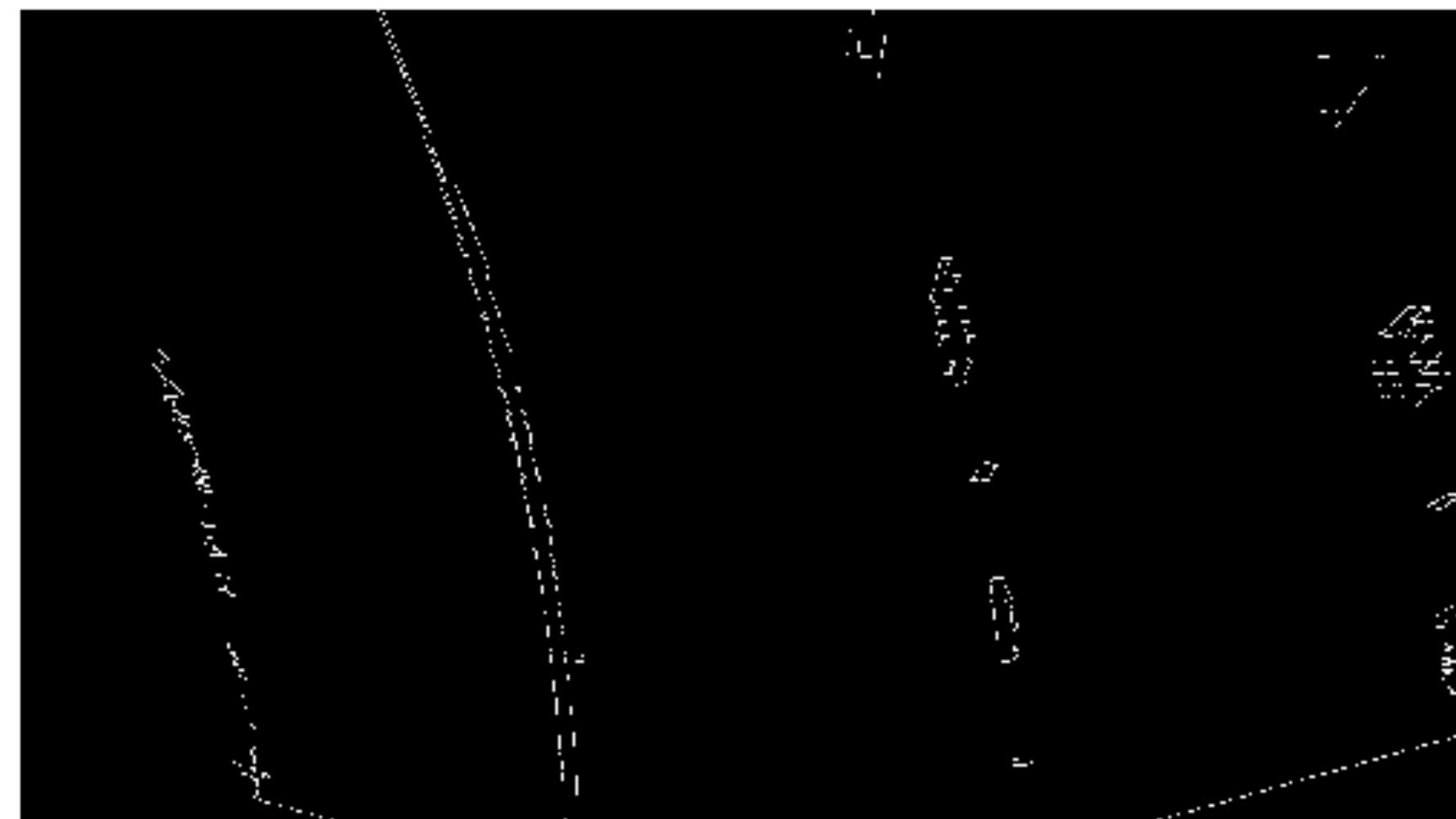
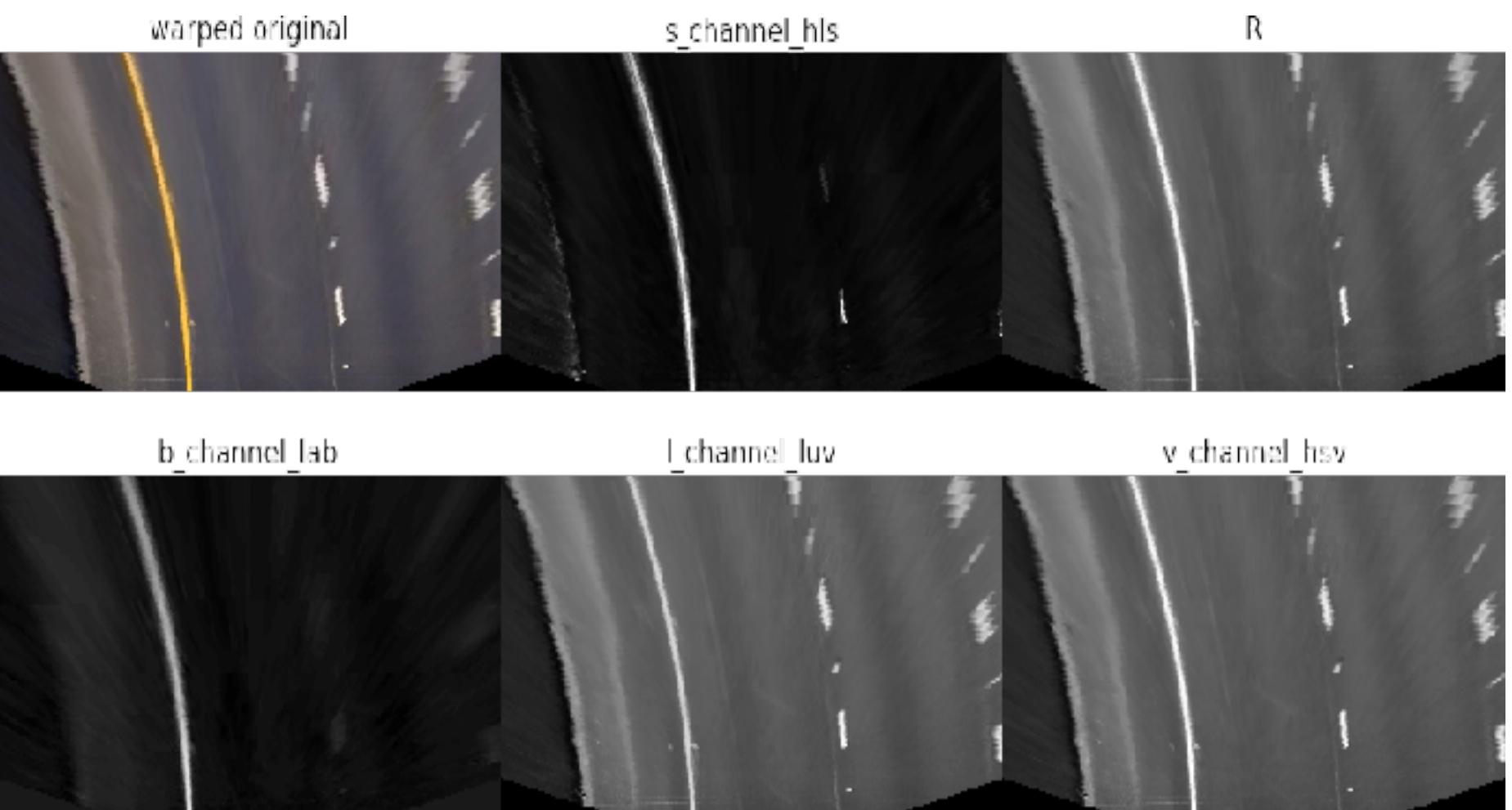
Warped Perspective



# Finding the lane lines

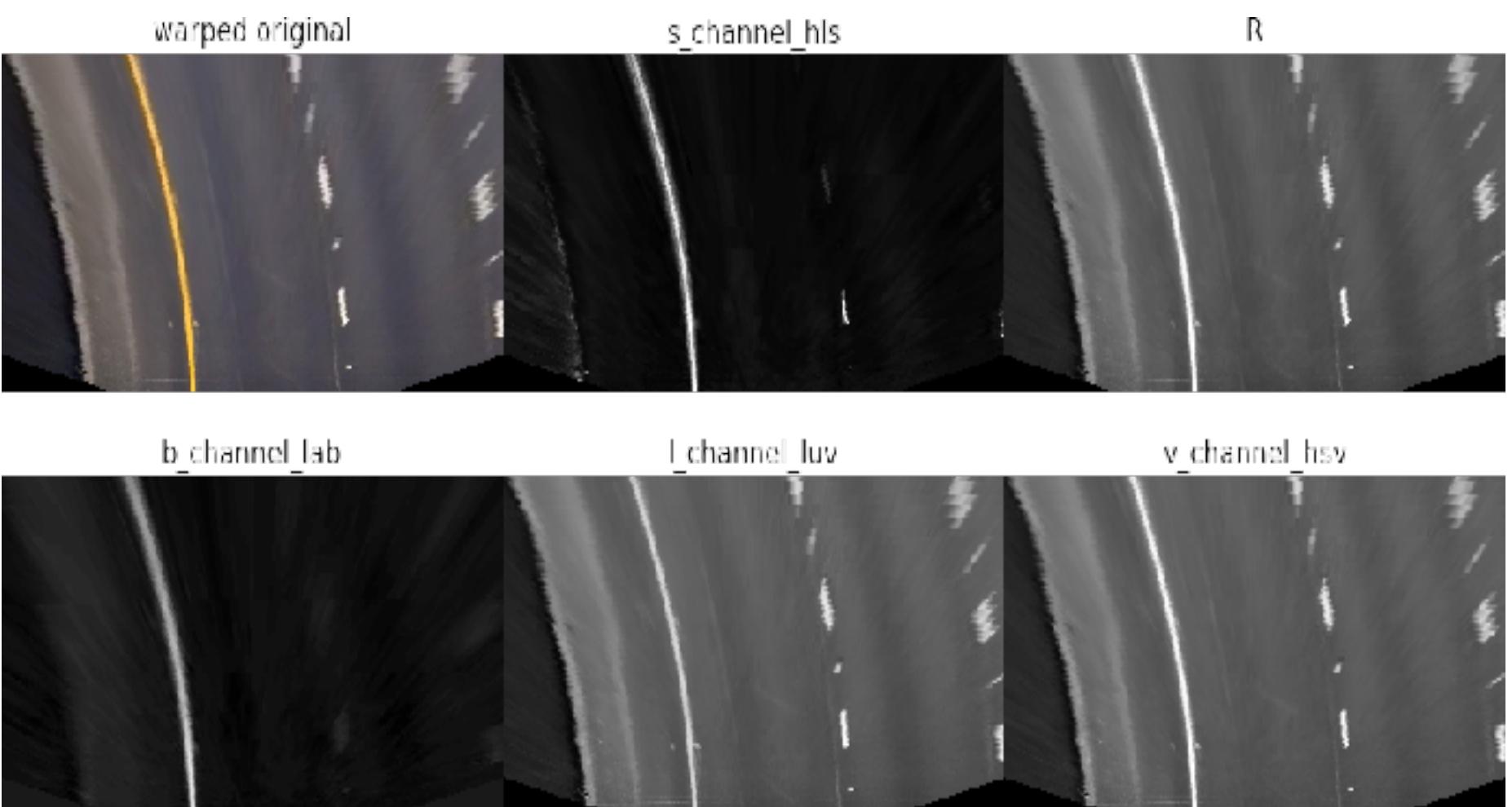
# Edge Detection

## Color Selection



# Edge Detection

## Color Selection



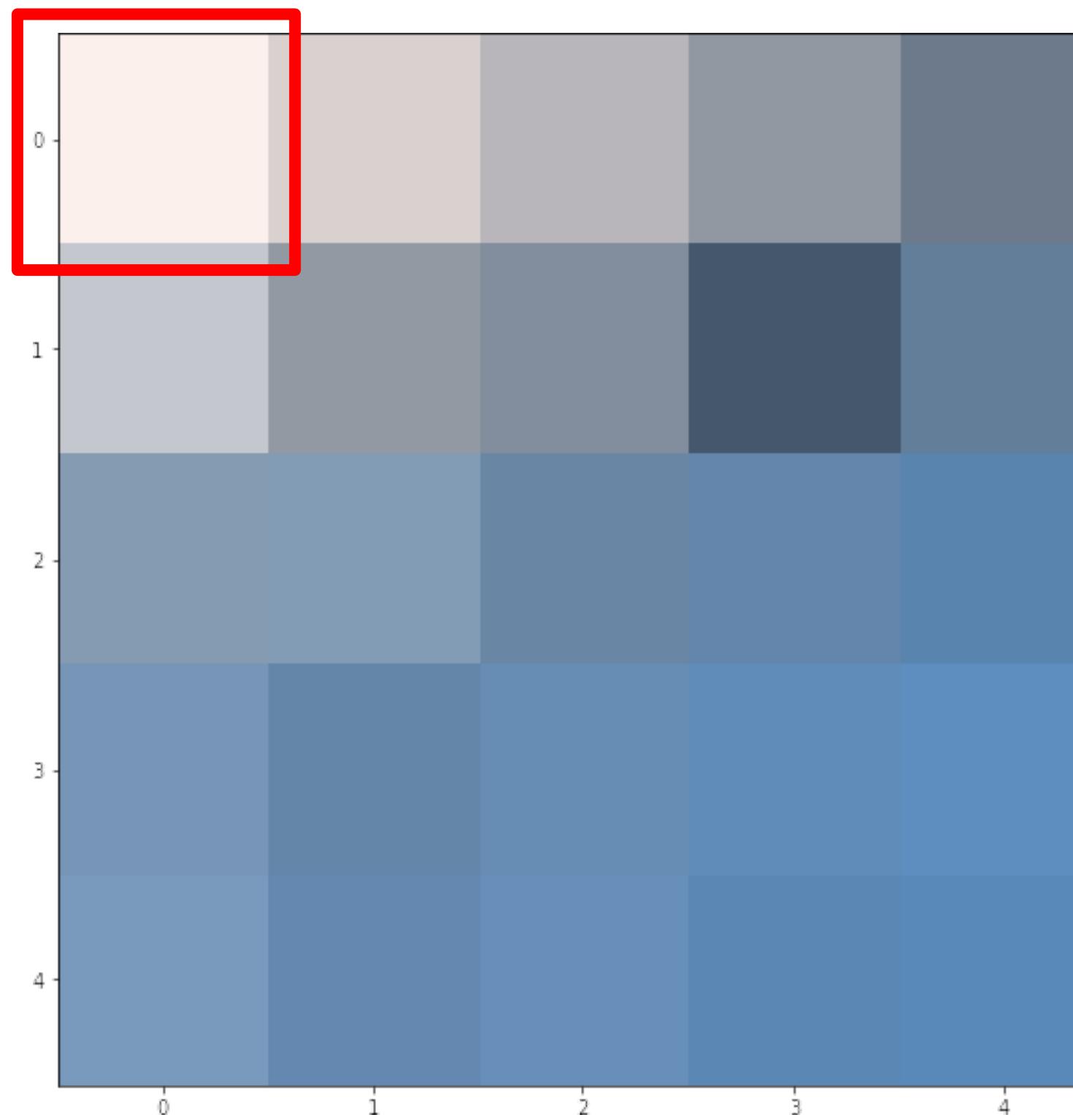
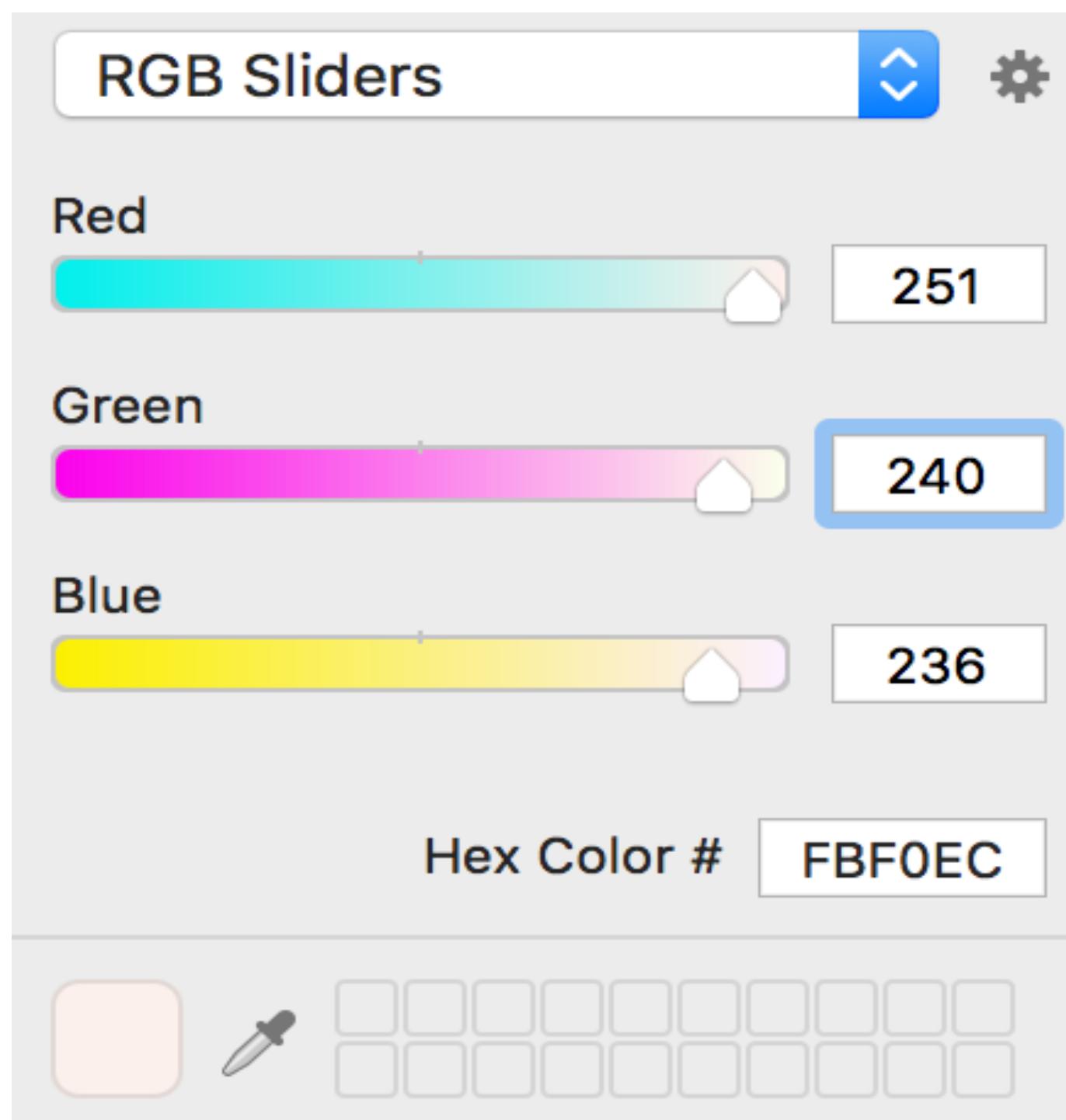
# Anatomy of an image



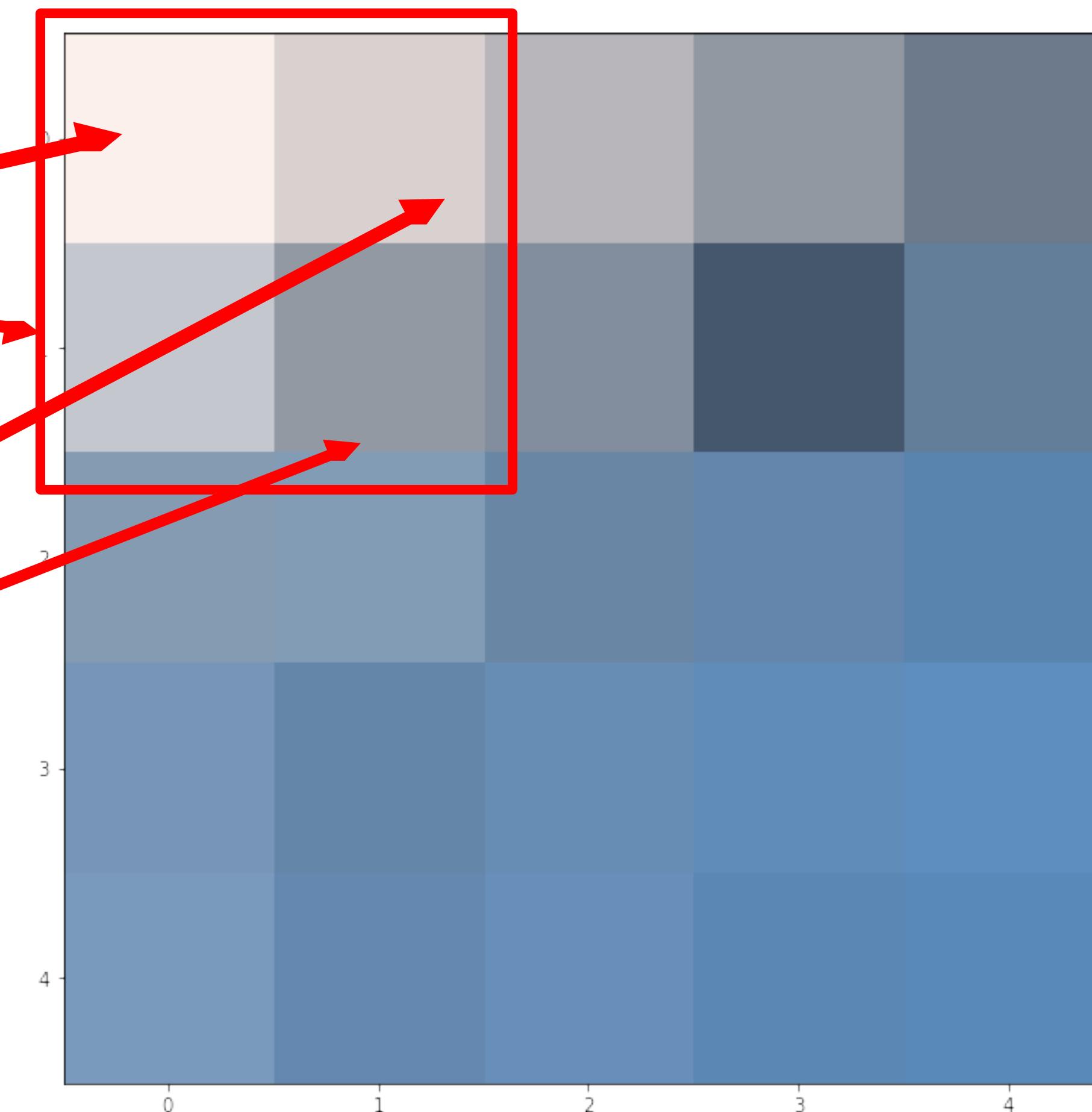
```
> img[0,0,:] # first pixel
```

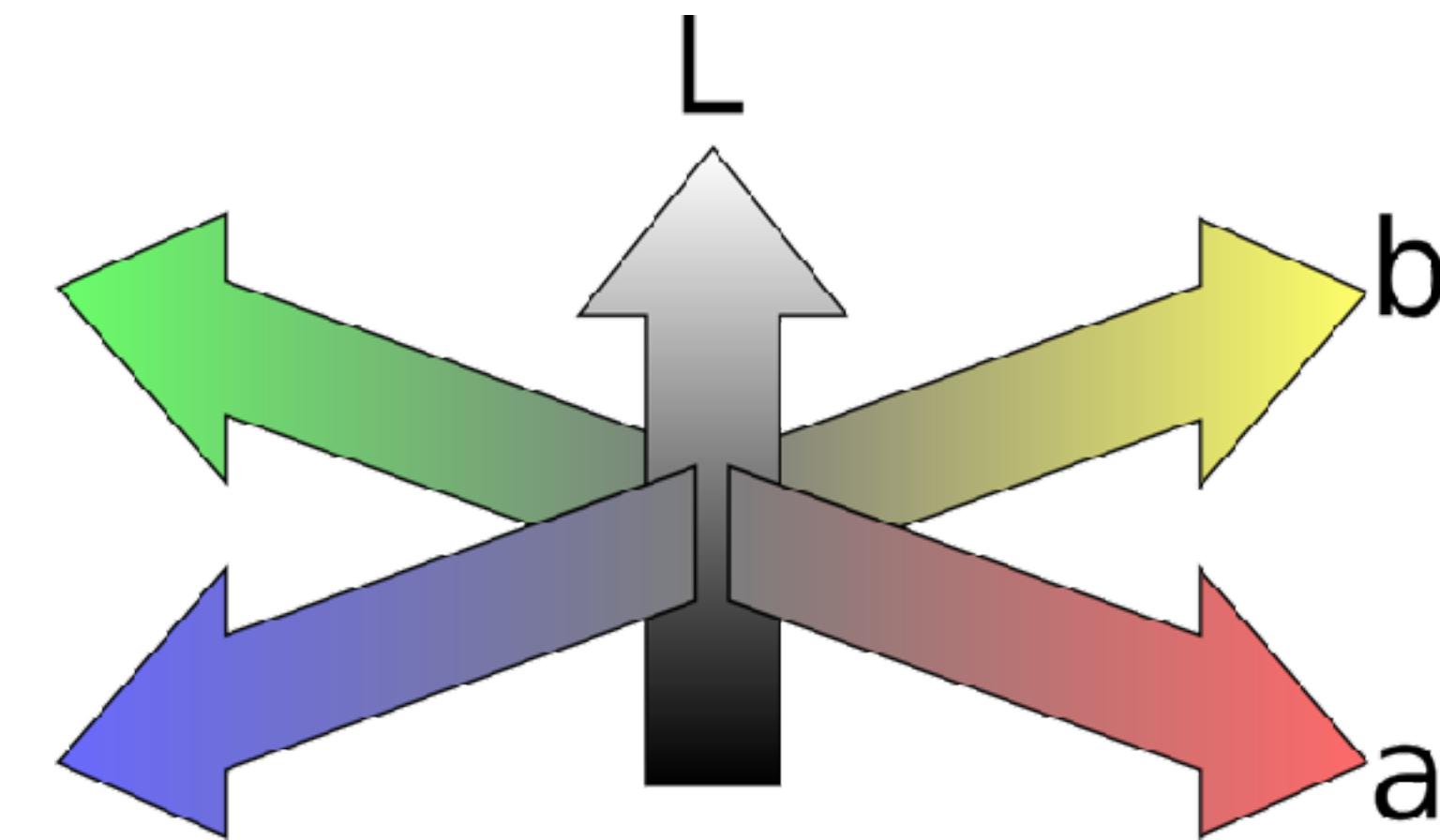
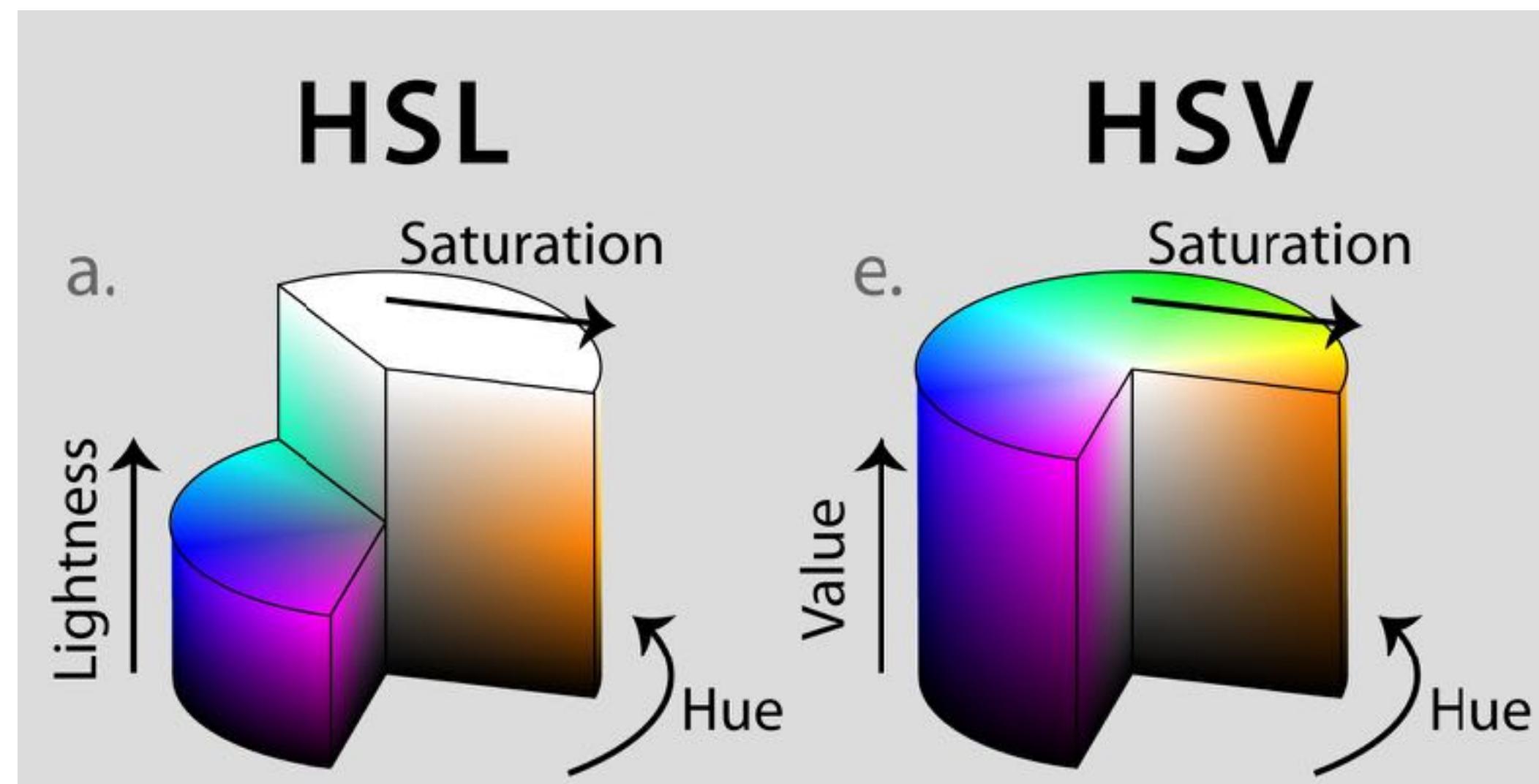
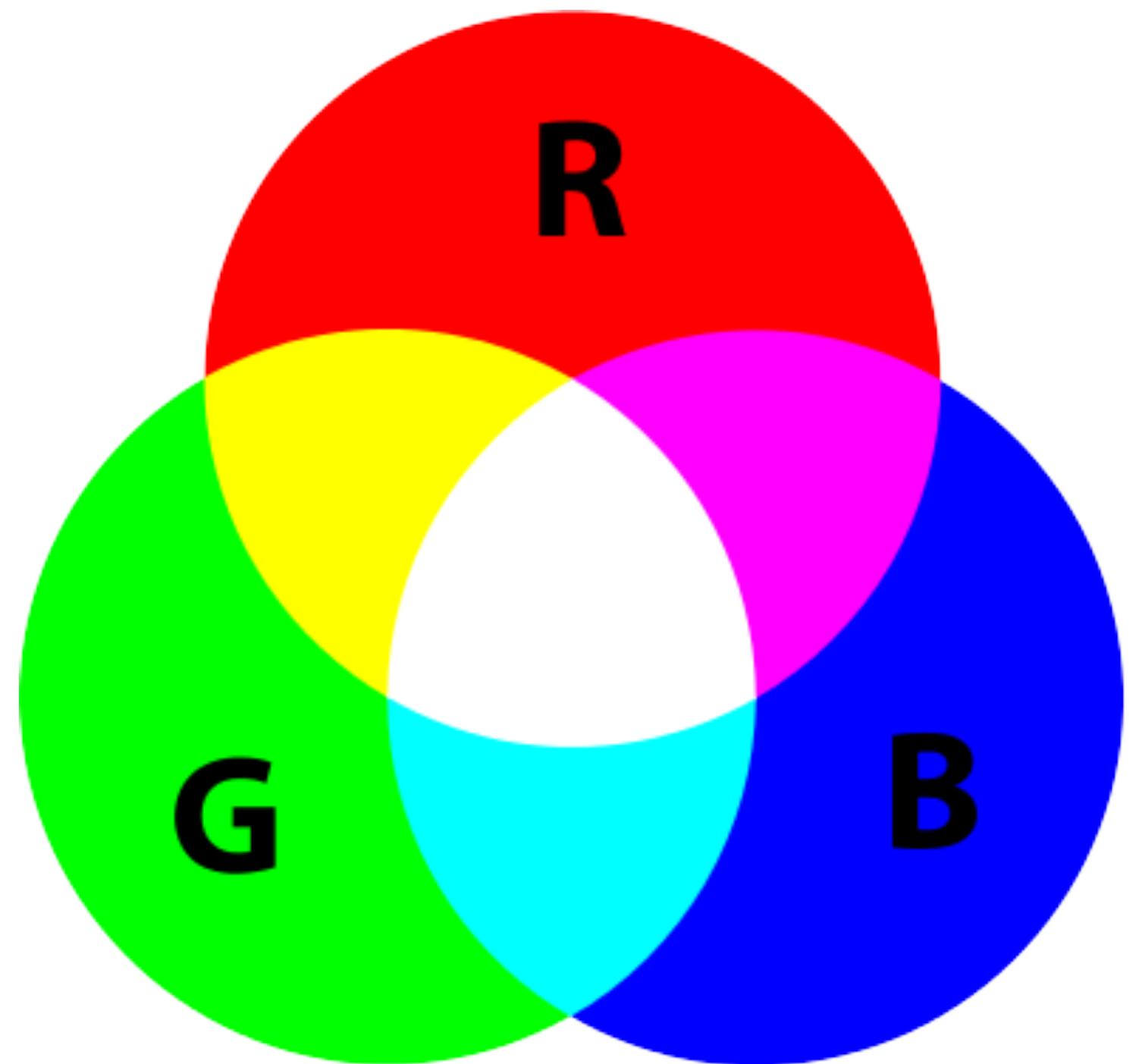
```
array([251, 240, 236])
```

Red      Green      Blue



```
> img[0:2,0:2,:]  
array([ [  
    [251, 240, 236], ,  
    [217, 209, 207] ], ,  
    [[196, 199, 206], ,  
    [146, 153, 163]] ], )
```





# OpenCV makes changing color space easy

```
# convert RGB image to Hue Light Saturation (HLS)
```

```
HLS = cv2.cvtColor(RGB, cv2.COLOR_RGB2HLS)
```

```
> HLS[0,0,:]
```

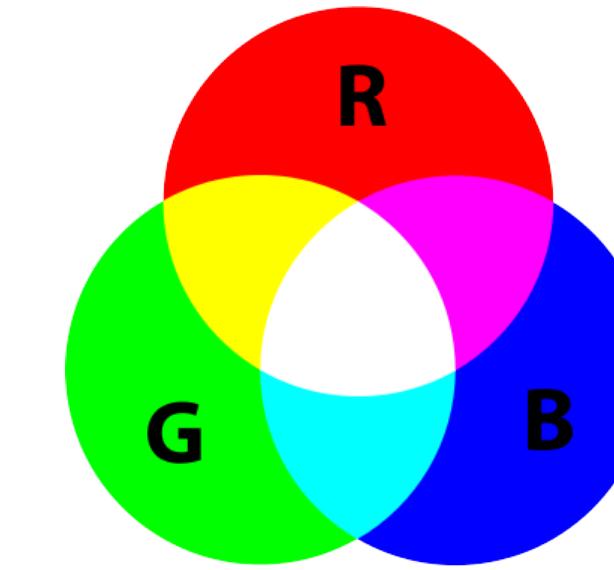
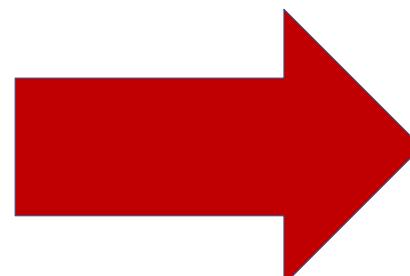
```
array([ 8, 244, 166])
```



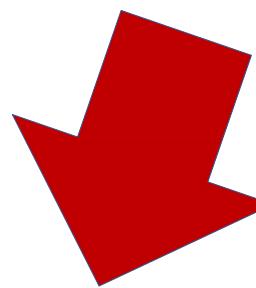
```
> RGB[0,0,:]
```

```
array([251, 240, 236])
```

# Isolate each color channel



```
red = img[ :, :, 0 ]  
green = img[ :, :, 1 ]  
blue = img[ :, :, 2 ]
```



Red Channel



Green Channel

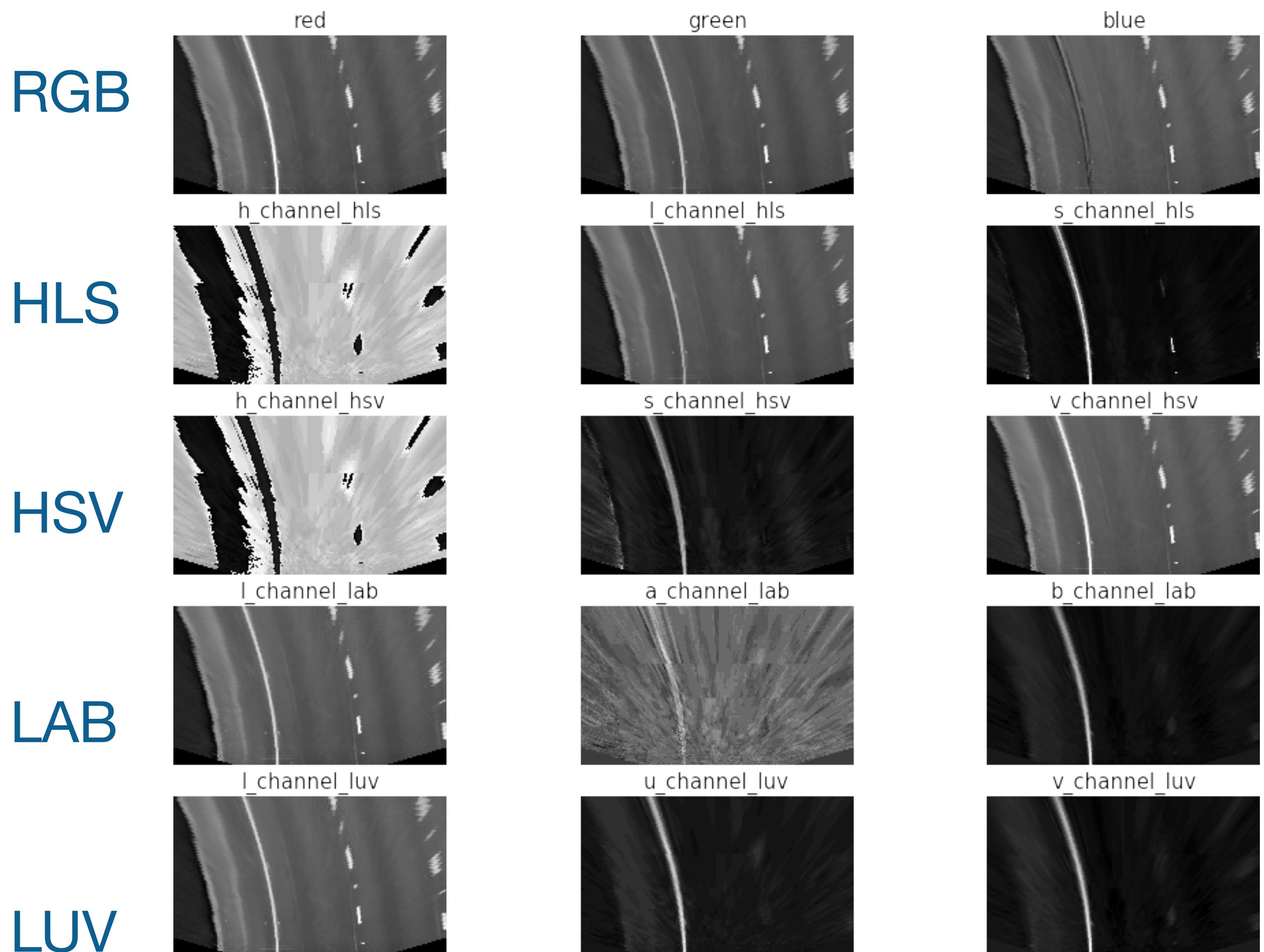


Blue Channel

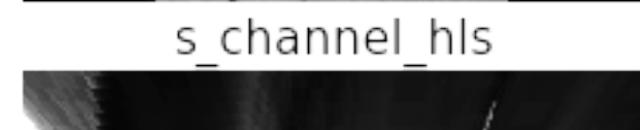
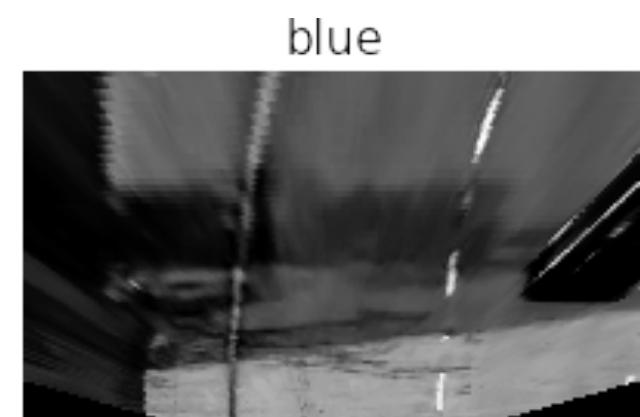
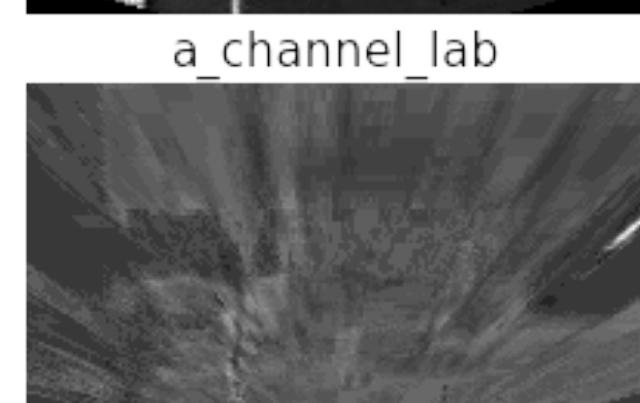
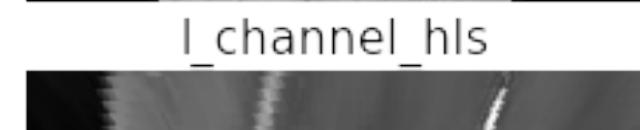
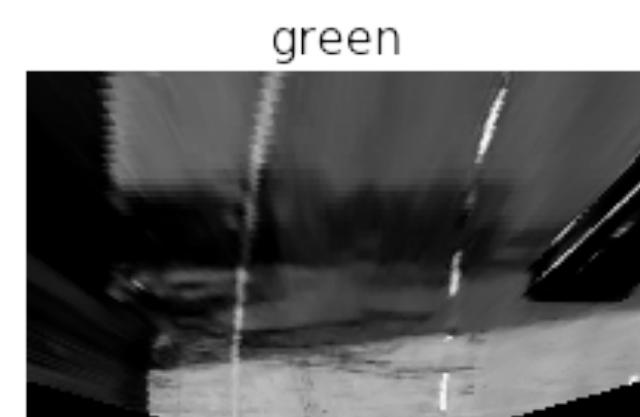
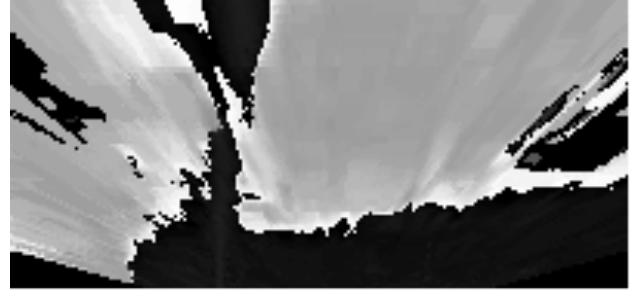
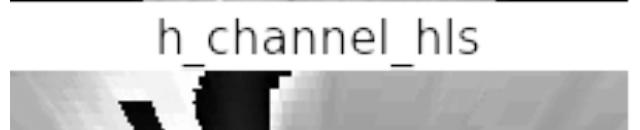


## Yellow Channel





RGB



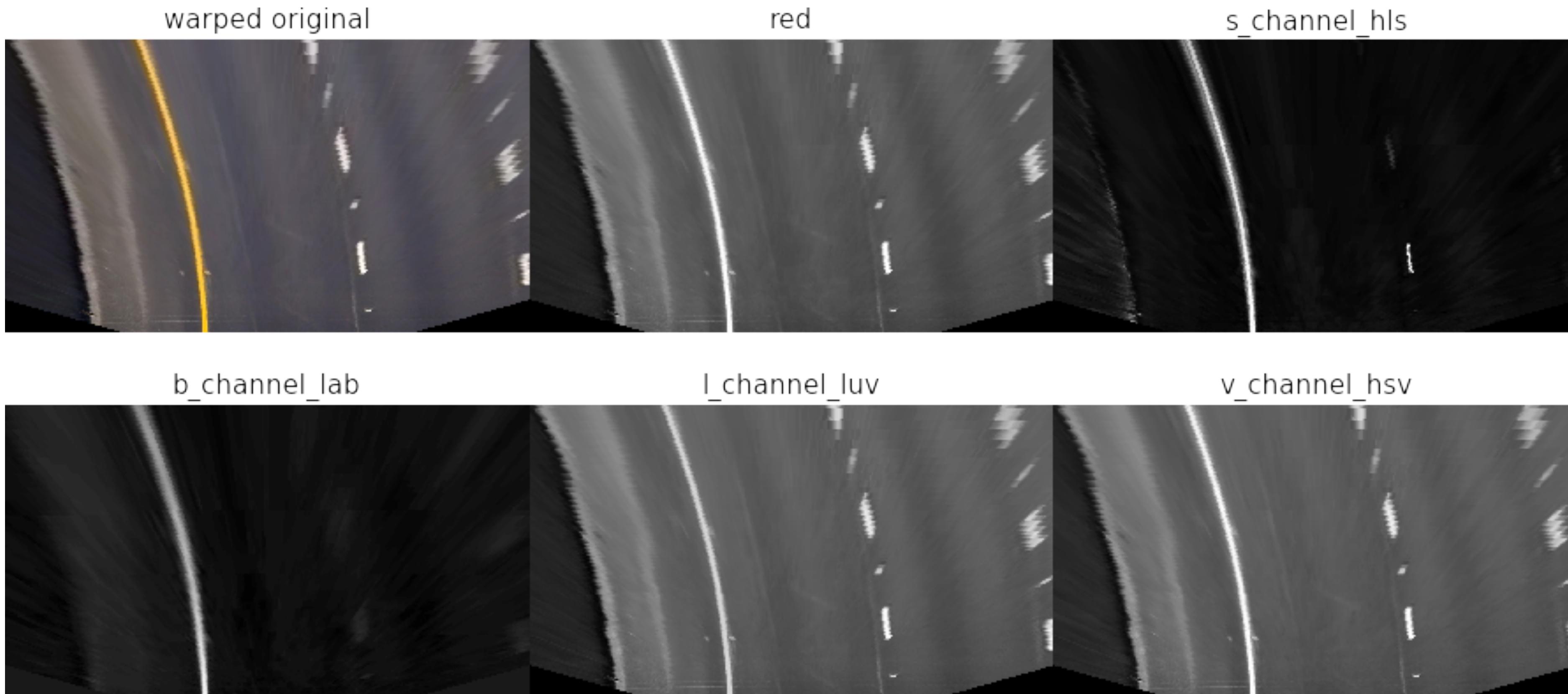
HLS

HSV

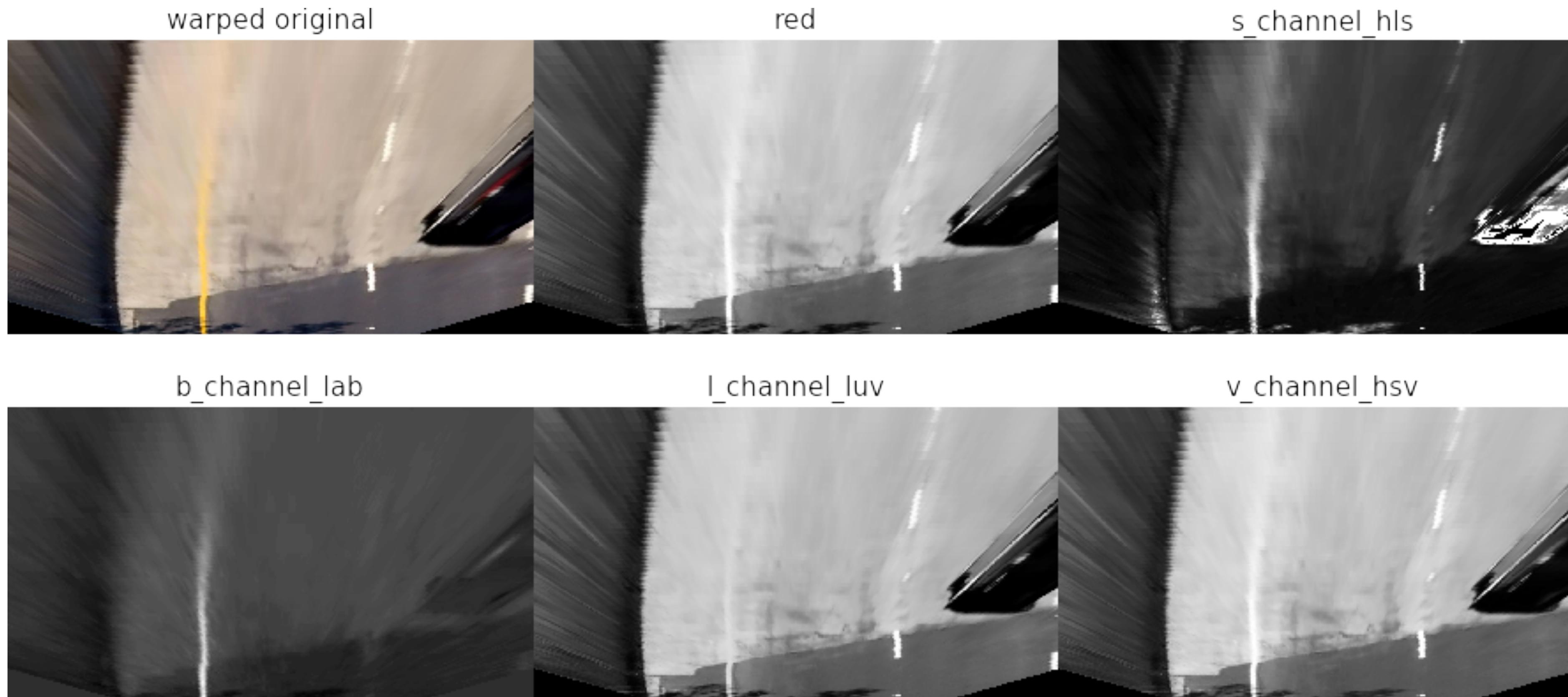
LAB

LUV

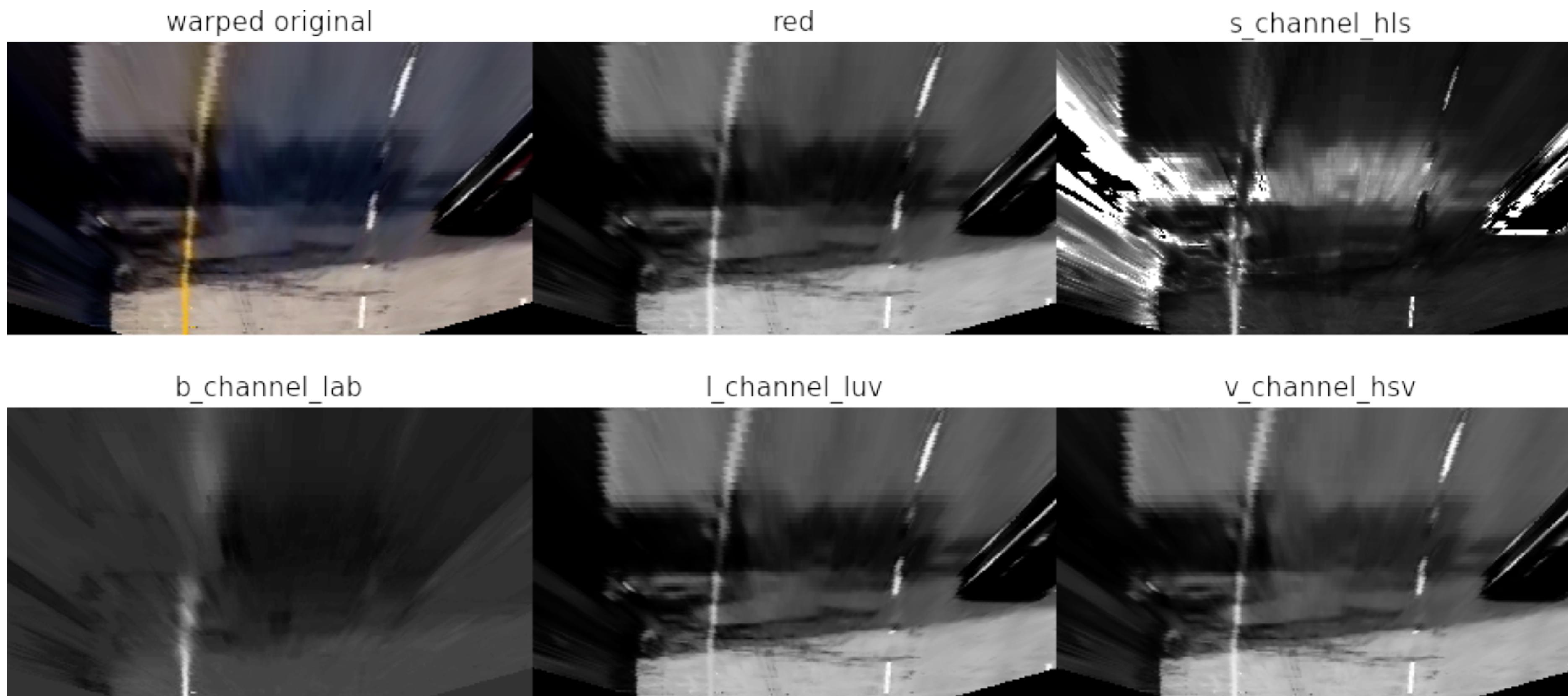
# The best ones...

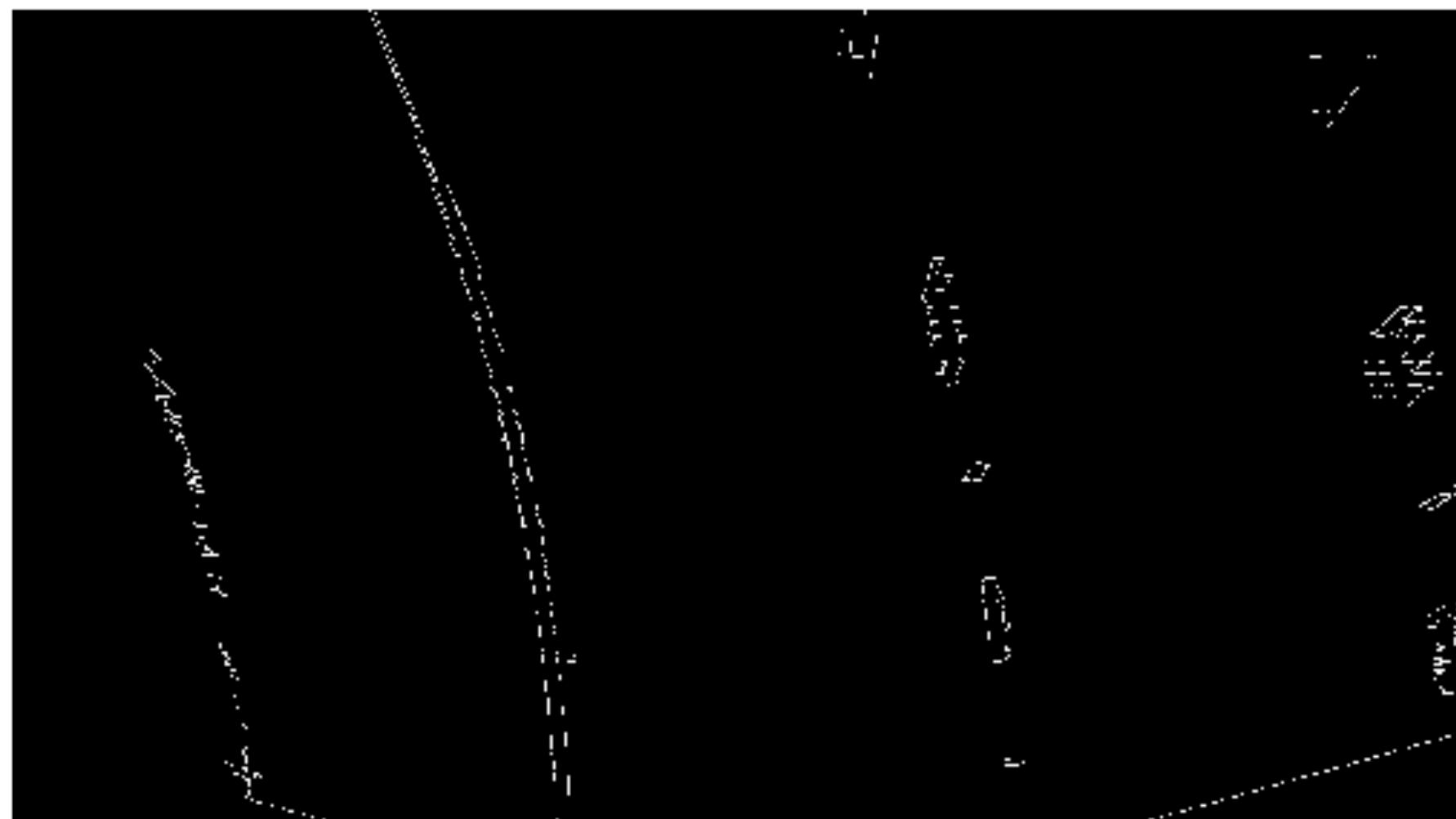


# But some images are harder than others



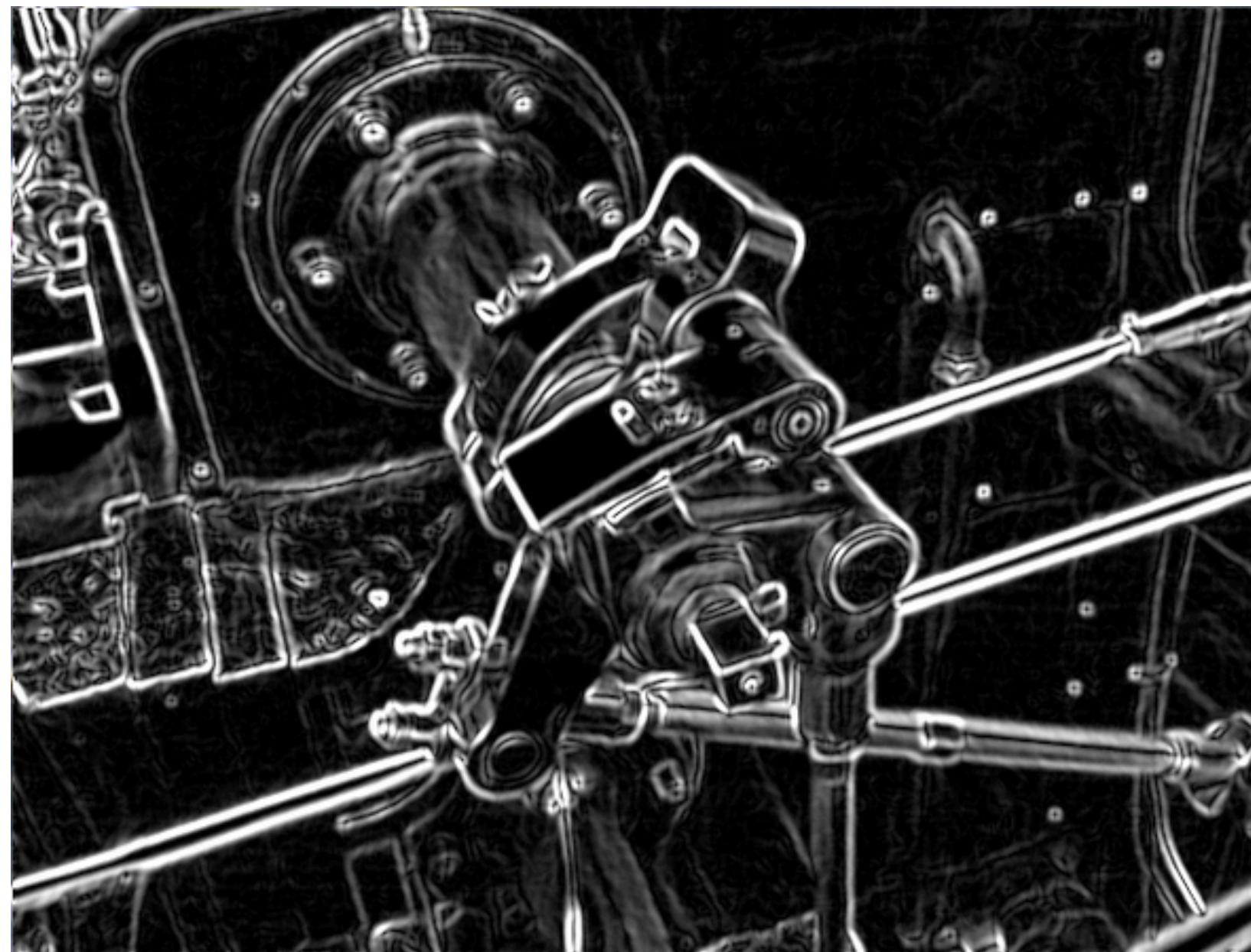
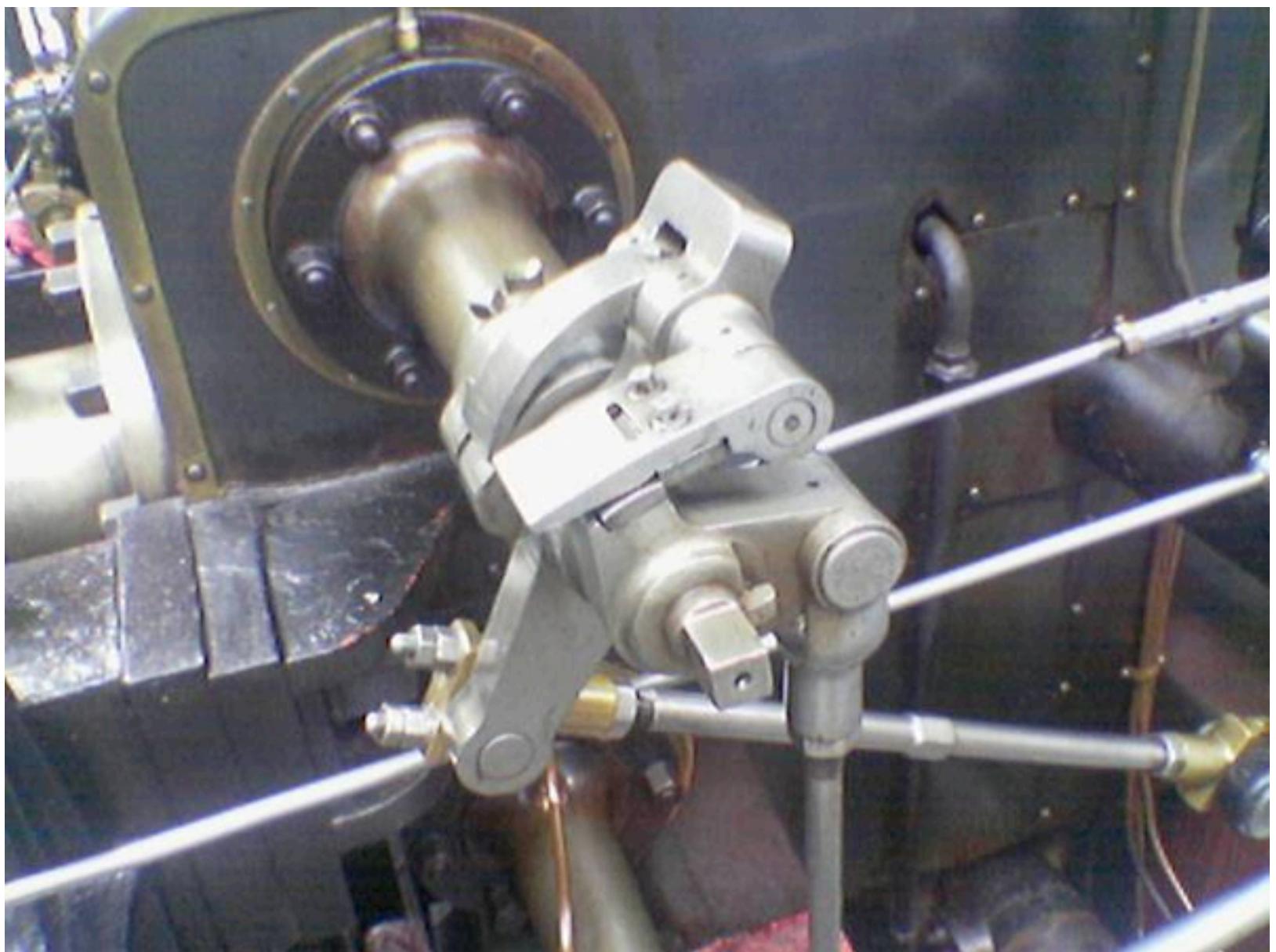
# And some are really hard



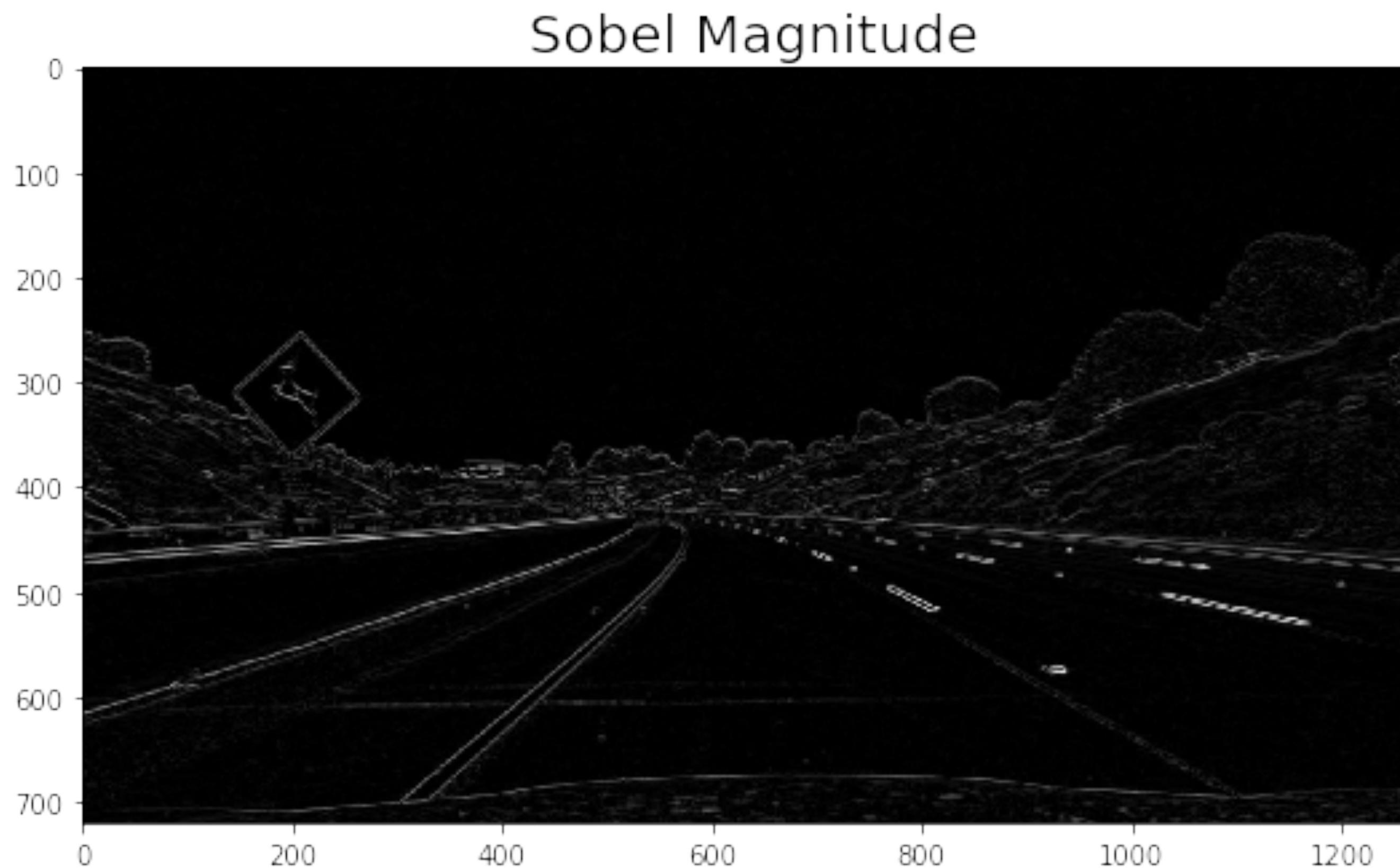


# Sobel operator

Sobel operator creates an image emphasizing edges. Technically, it is a discrete differentiation operator, computing an approximation of the gradient of the image intensity function.



# We can use built in OpenCV functions to find edges



Sobel operator creates an image emphasising edges. Technically, it is a discrete differentiation operator, computing an approximation of the gradient of the image intensity function.

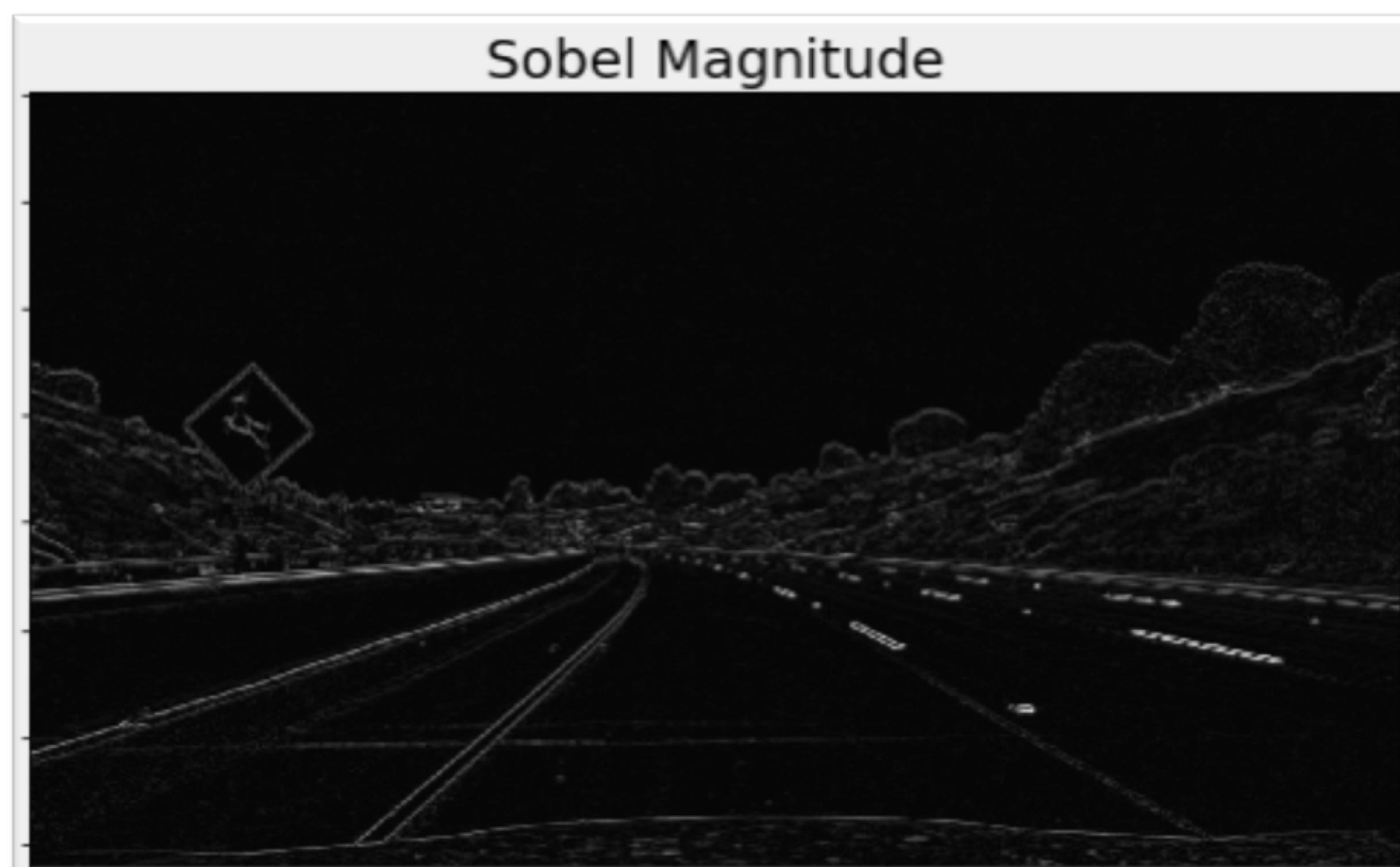
Sobel X



Sobel Y

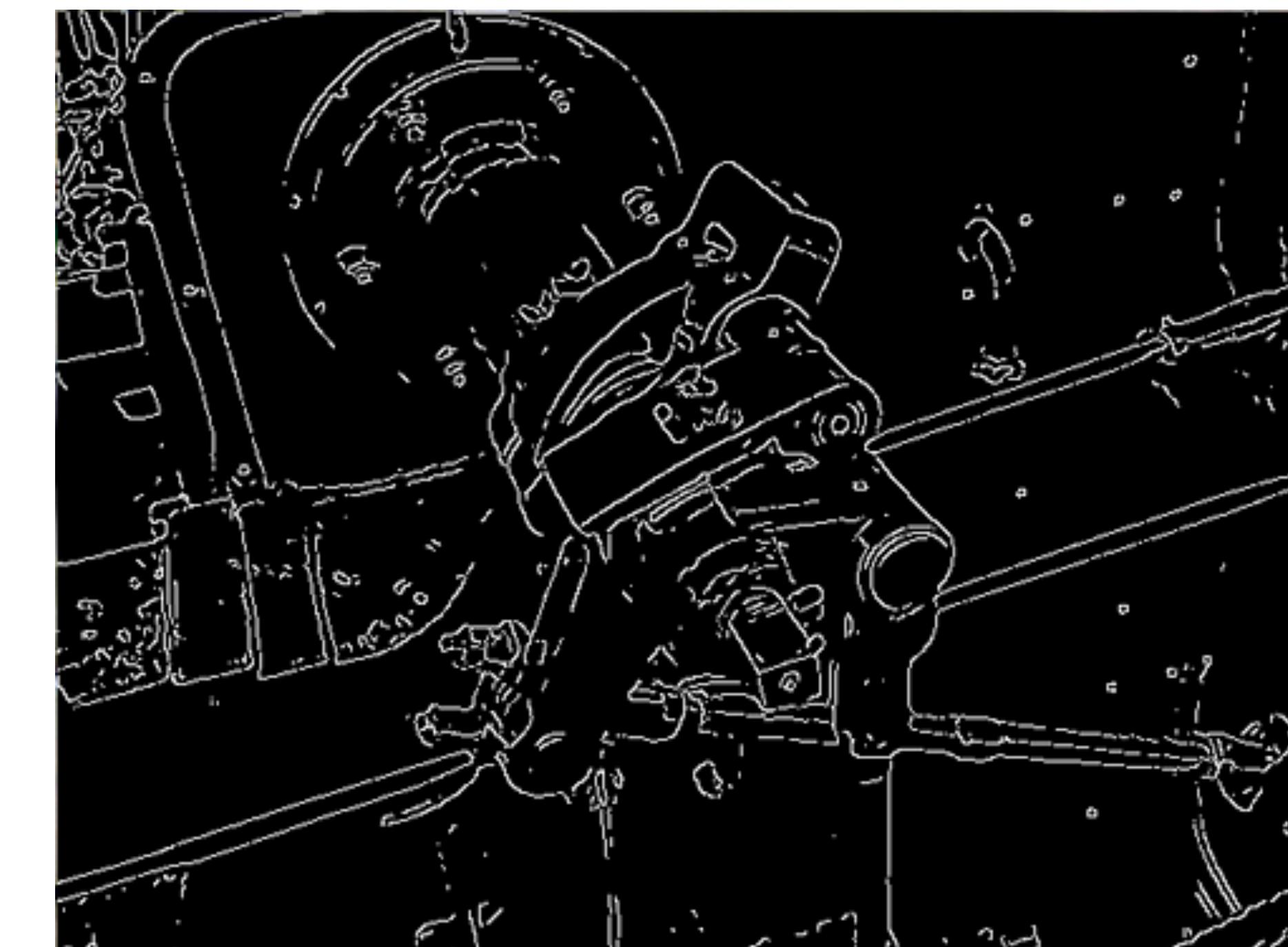
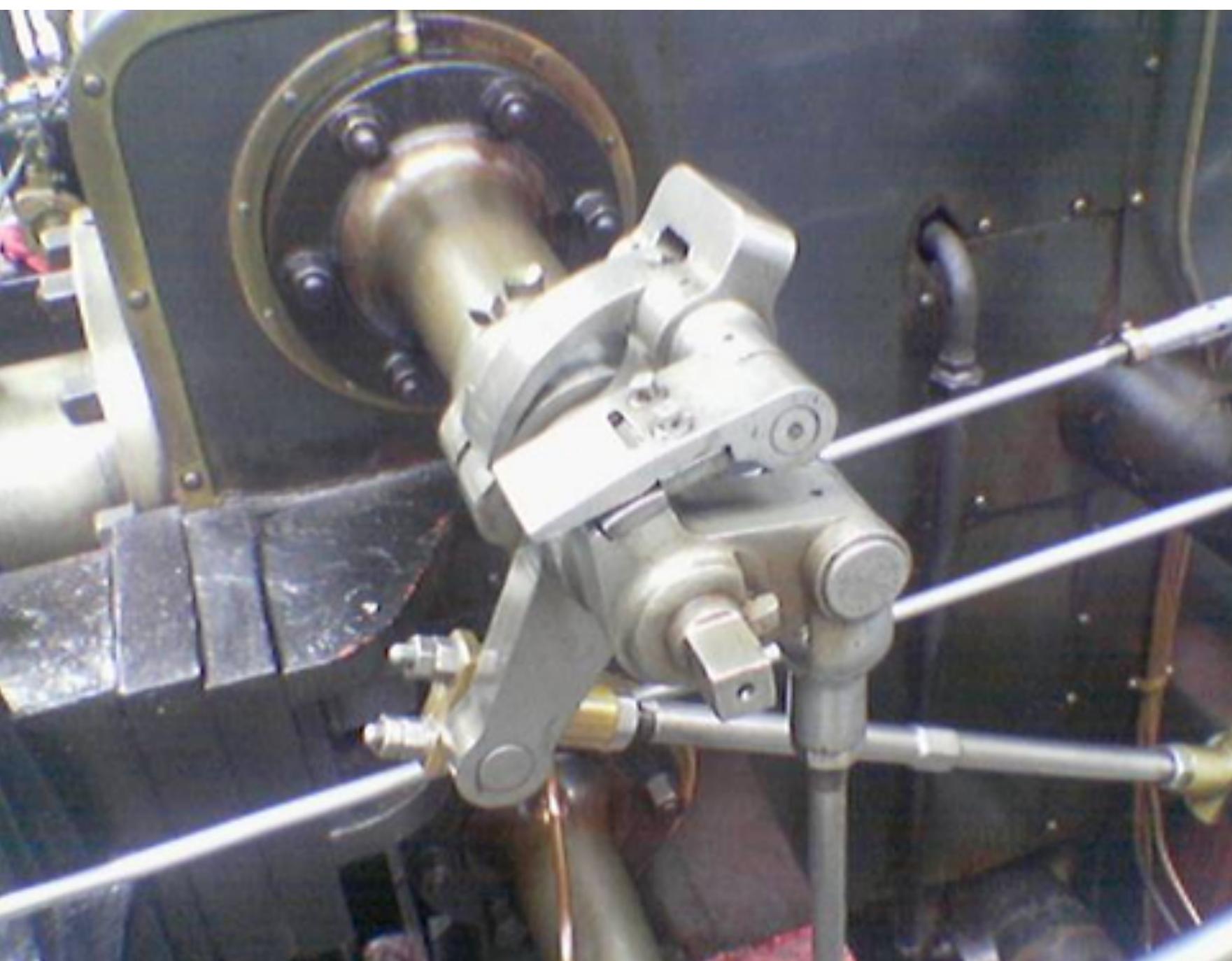


Sobel Magnitude



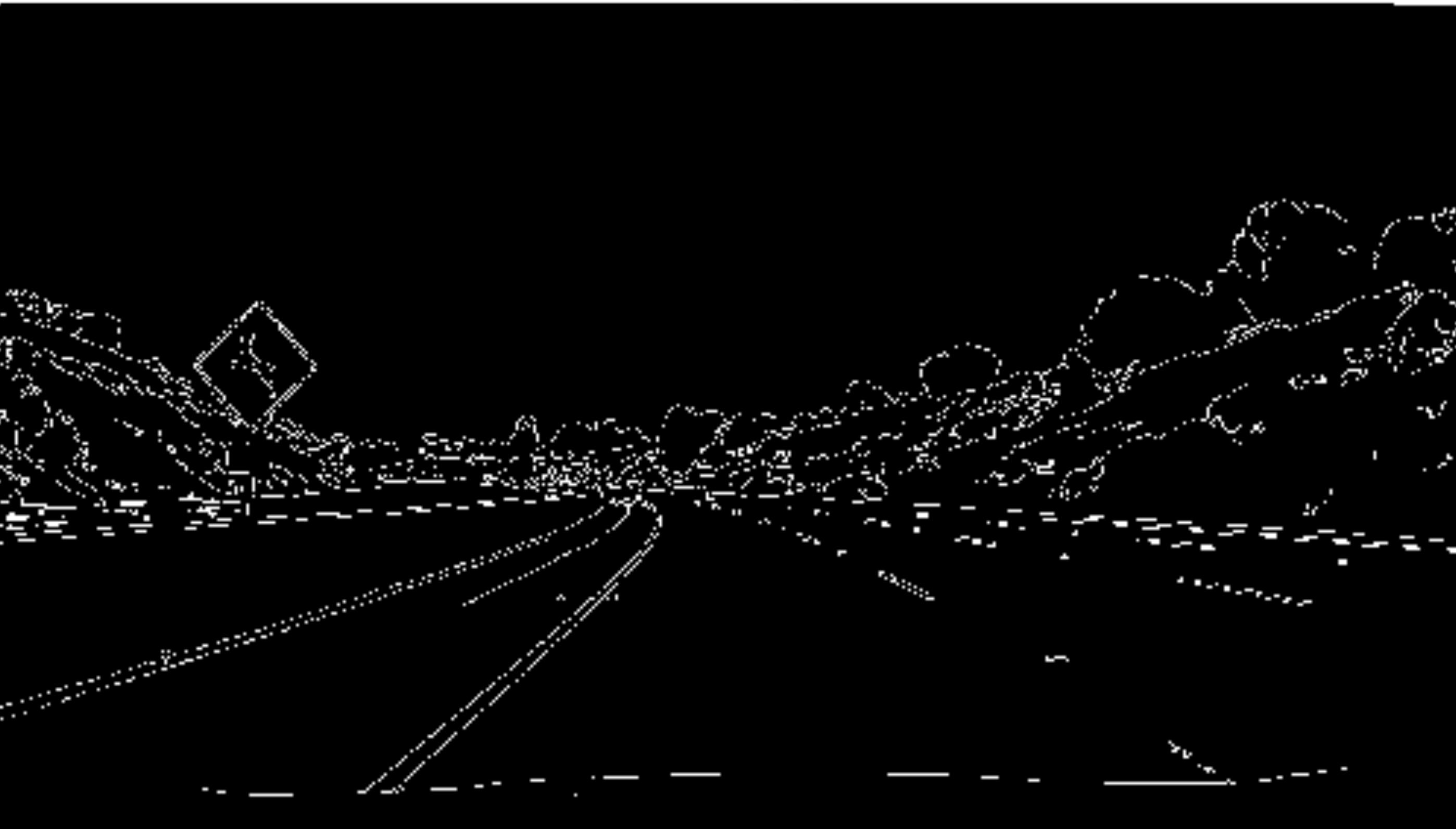
# Canny edge detection

Canny edge detection is a technique to extract useful structural information from different vision objects and dramatically reduce the amount of data to be processed.



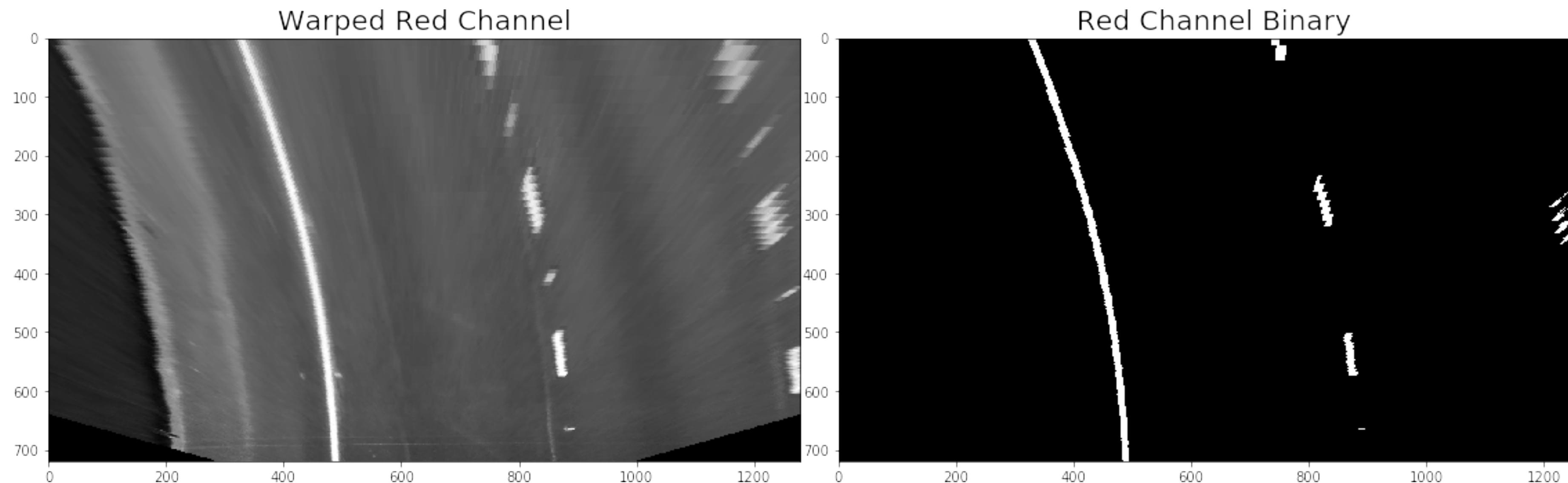
# Canny edge detection

Canny



# Use binary thresholding to isolate “hot” pixels

```
channel = warped[:, :, 0] thresh = (200, 255)  
output = np.zeros_like(channel)  
output[(channel >= thresh[0]) & (channel <= thresh[1])] = 1
```



# Combine binary threshold images

```
combined_binary = np.zeros_like(red) combined_binary[ (red == 1) | (sobel == 1) ] = 1
```

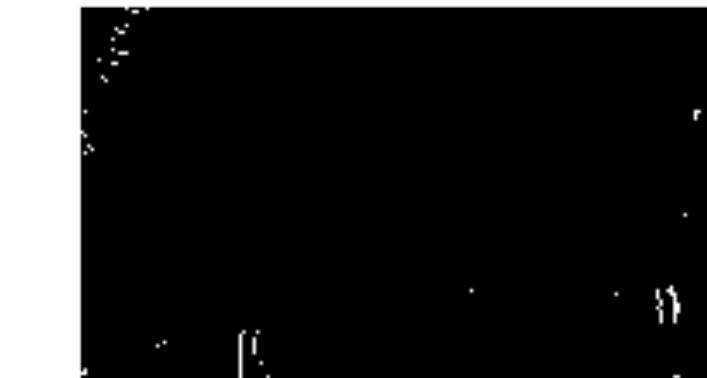
original ./test\_images/test4.jpg



RGB R Channel



Sobel



Final Binary



original ./test\_images/test5.jpg



RGB R Channel



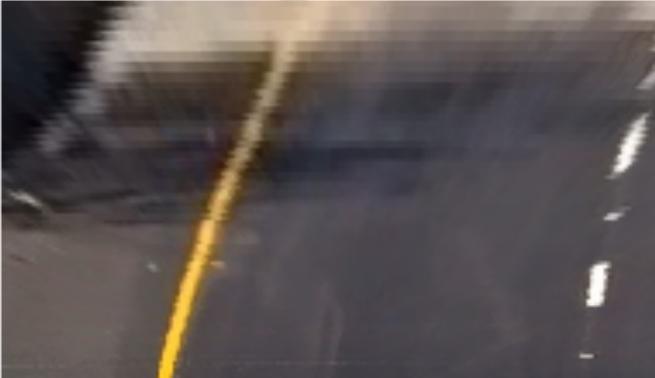
Sobel



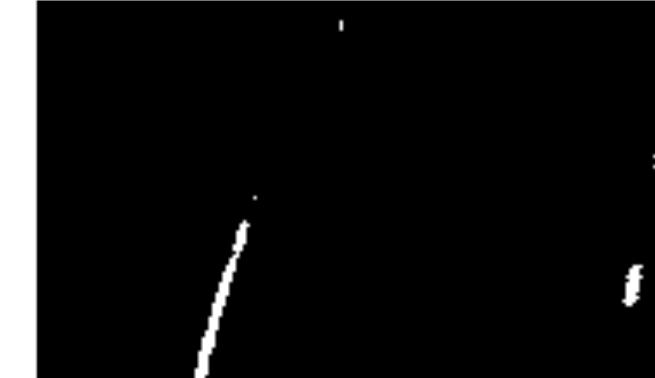
Final Binary



original ./test\_images/test6.jpg



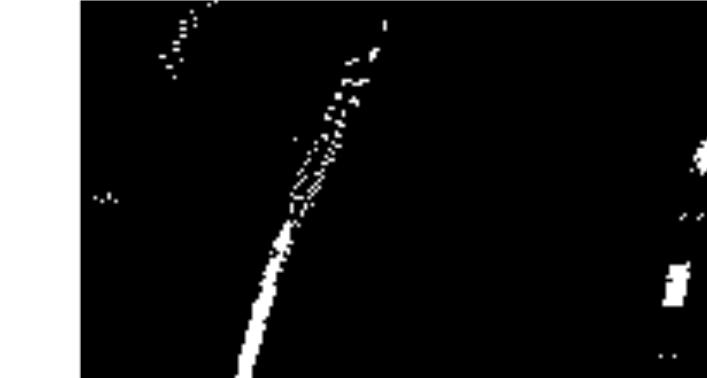
RGB R Channel



Sobel

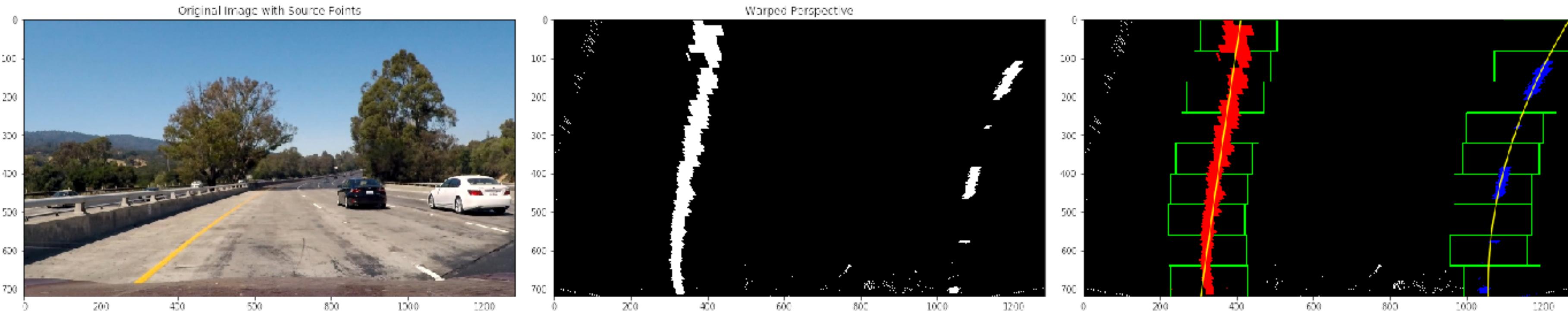


Final Binary

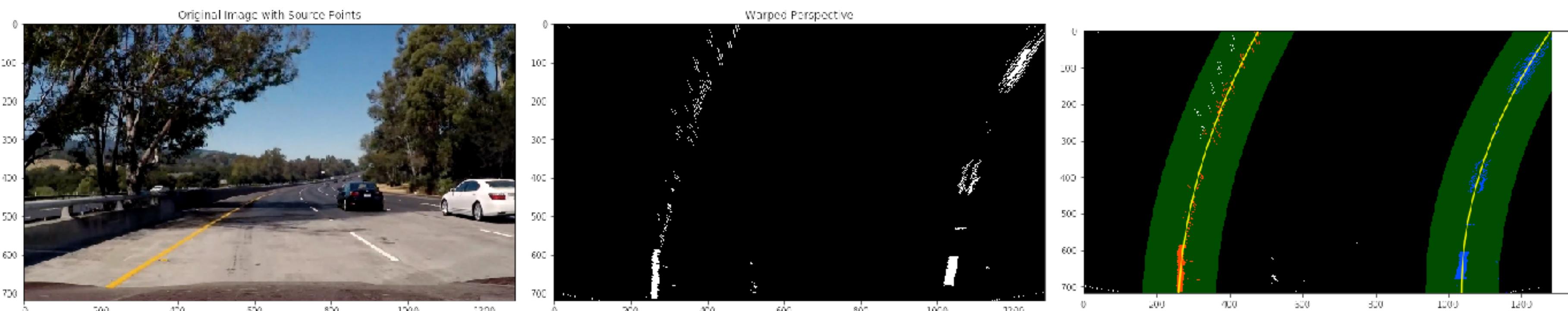


# Curve Fitting

# Method 1



# Method 2

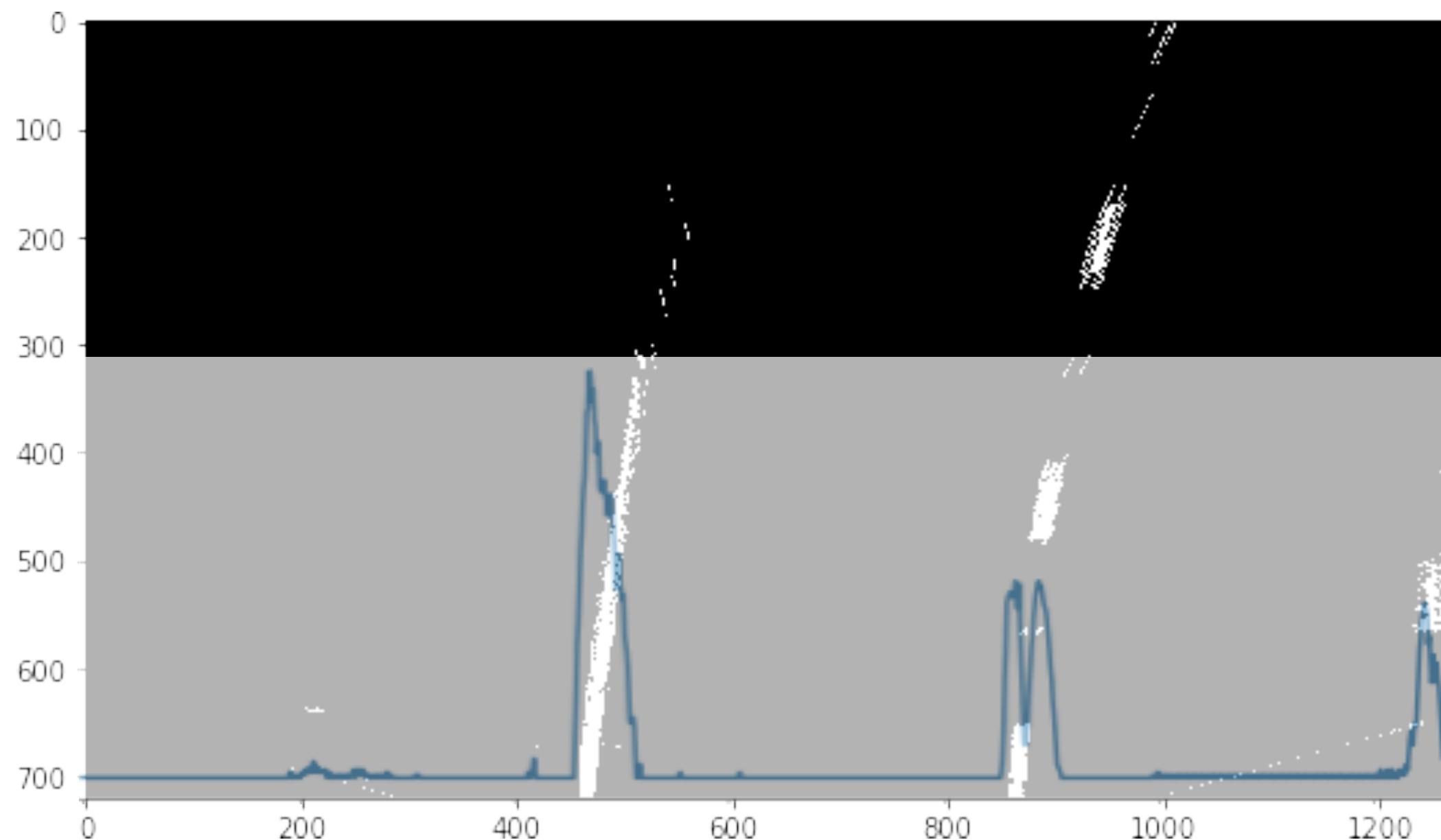


# Step 1: Find the start of the line

```
import numpy as np
```

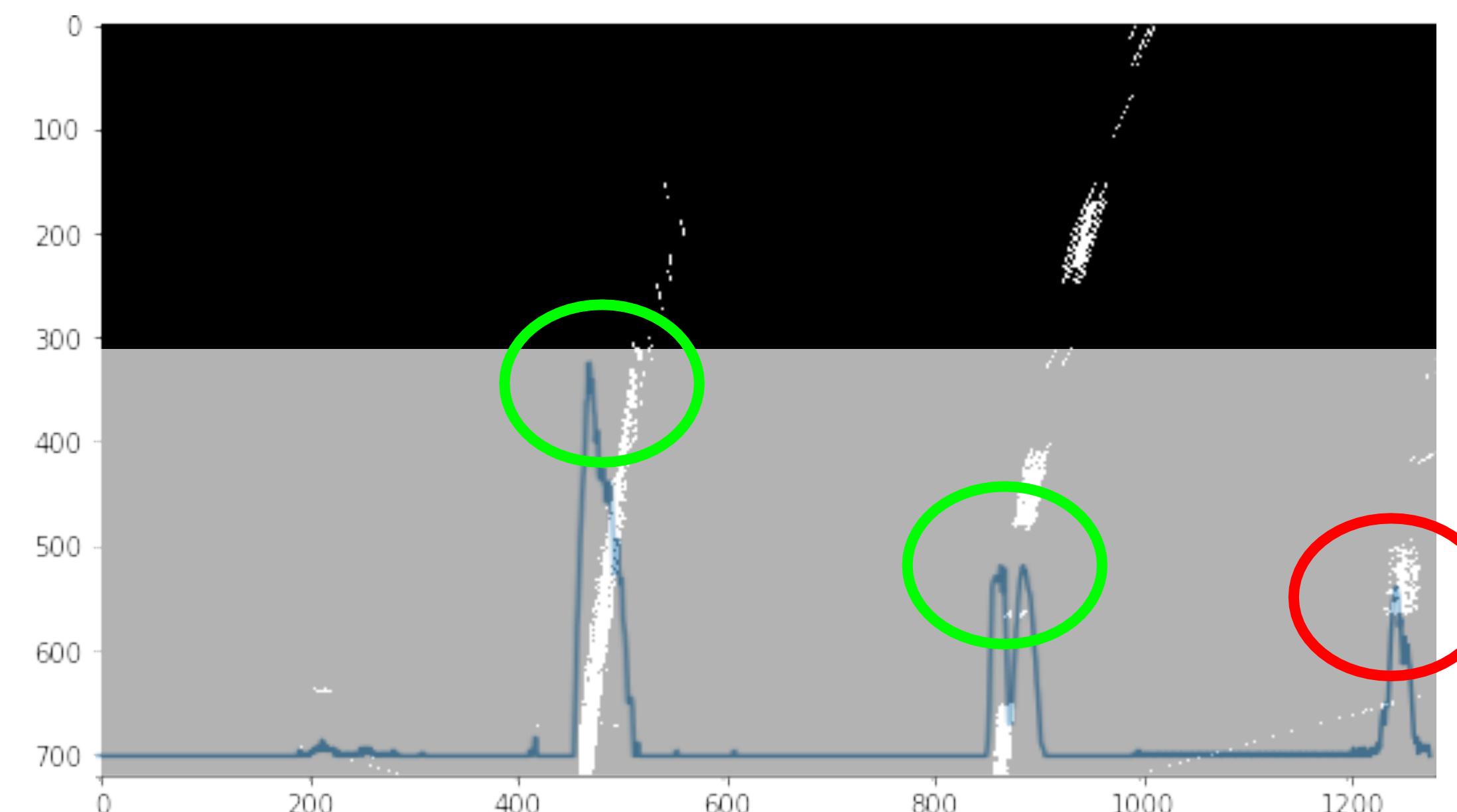
```
histogram = np.sum(img[img.shape[0]//2:,:,:], axis=0)
```

Sum white pixels at x  
location

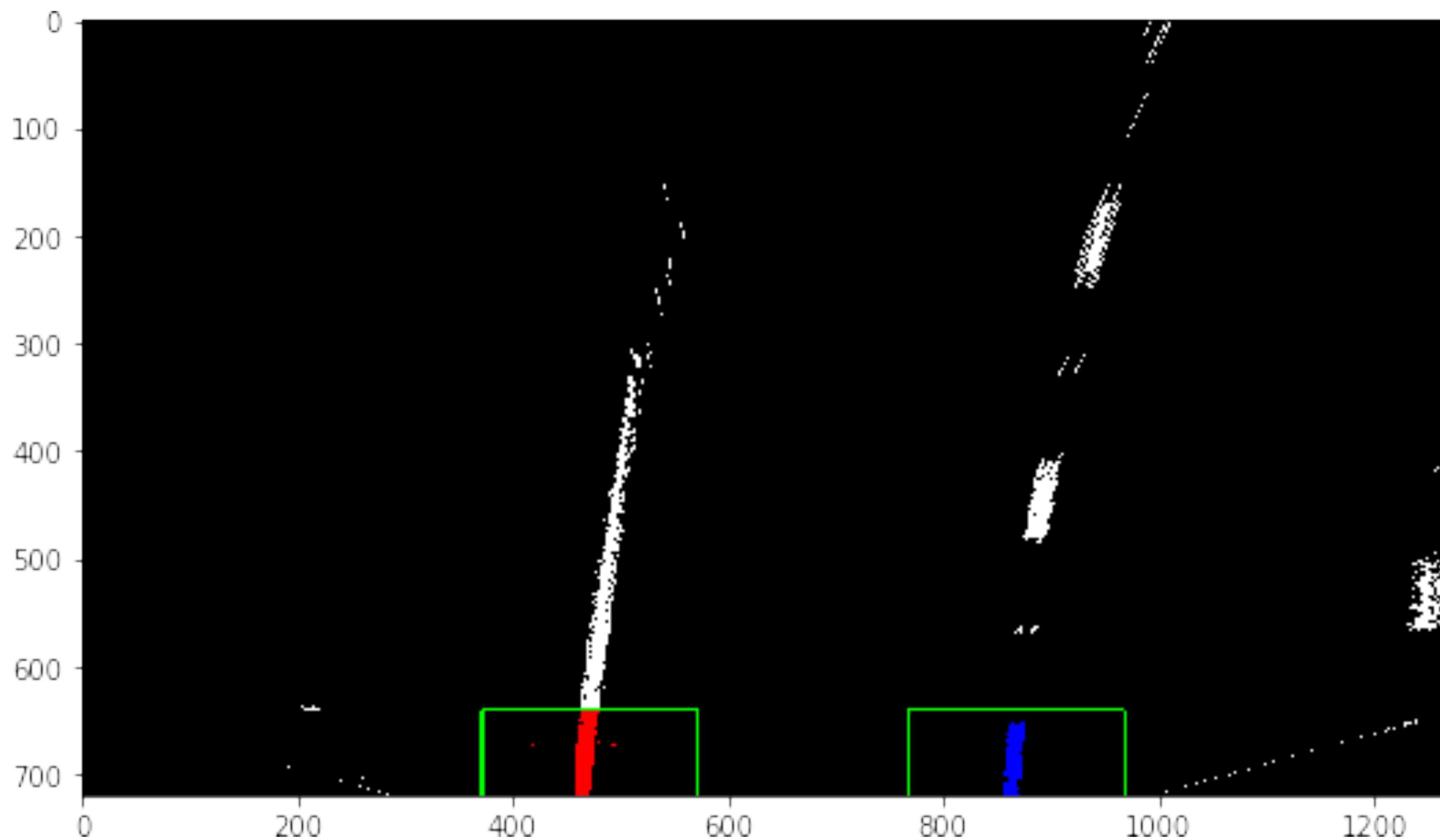


# Step 1: Find the start of the line

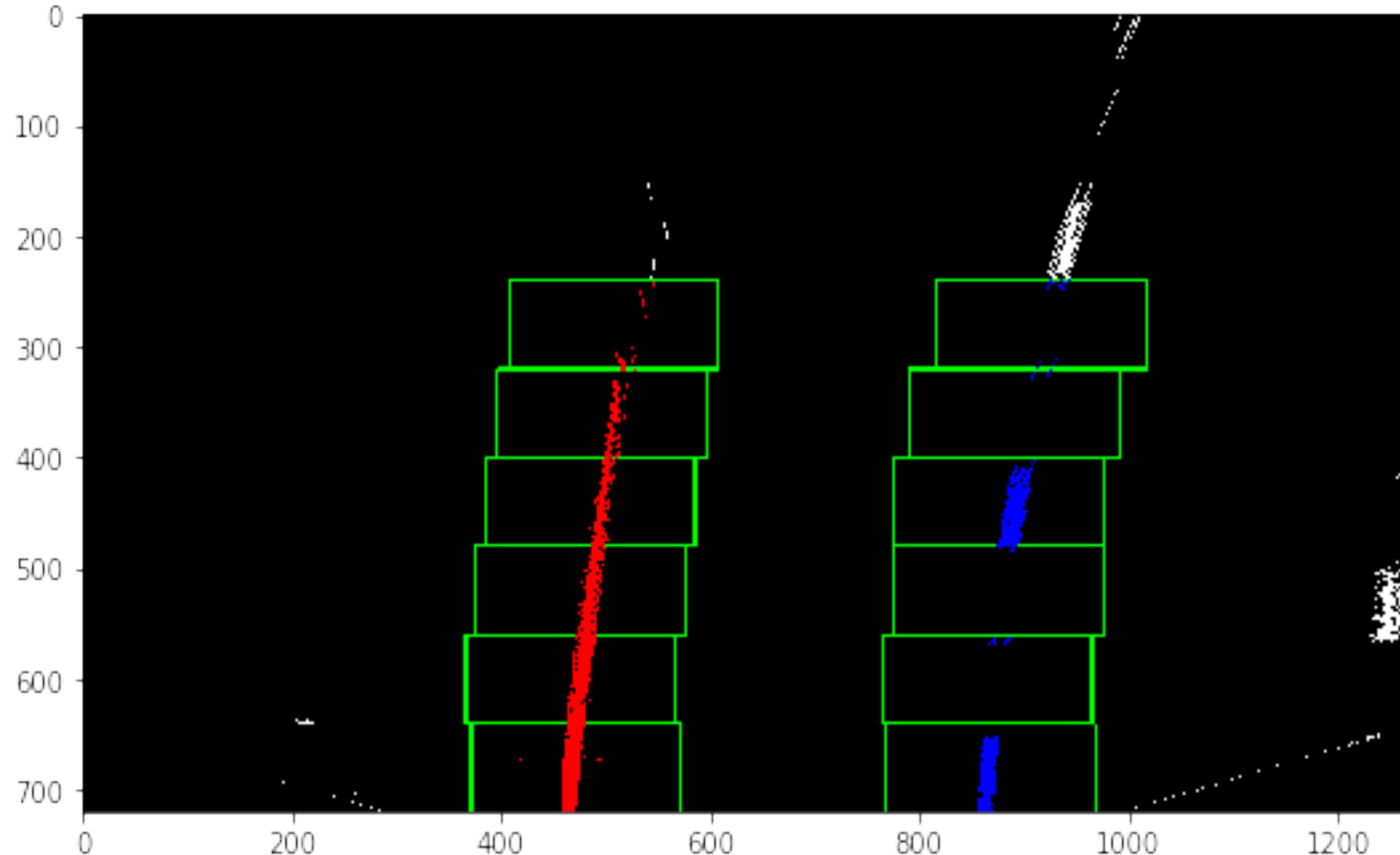
```
midpoint = ar(histogram.shape[0]/2)
leftx_base = np.argmax(histogram[:midpoint])
rightx_base = np.argmax(histogram[midpoint:]) + midpoint
```



# Step 2: Create windows around these maxima

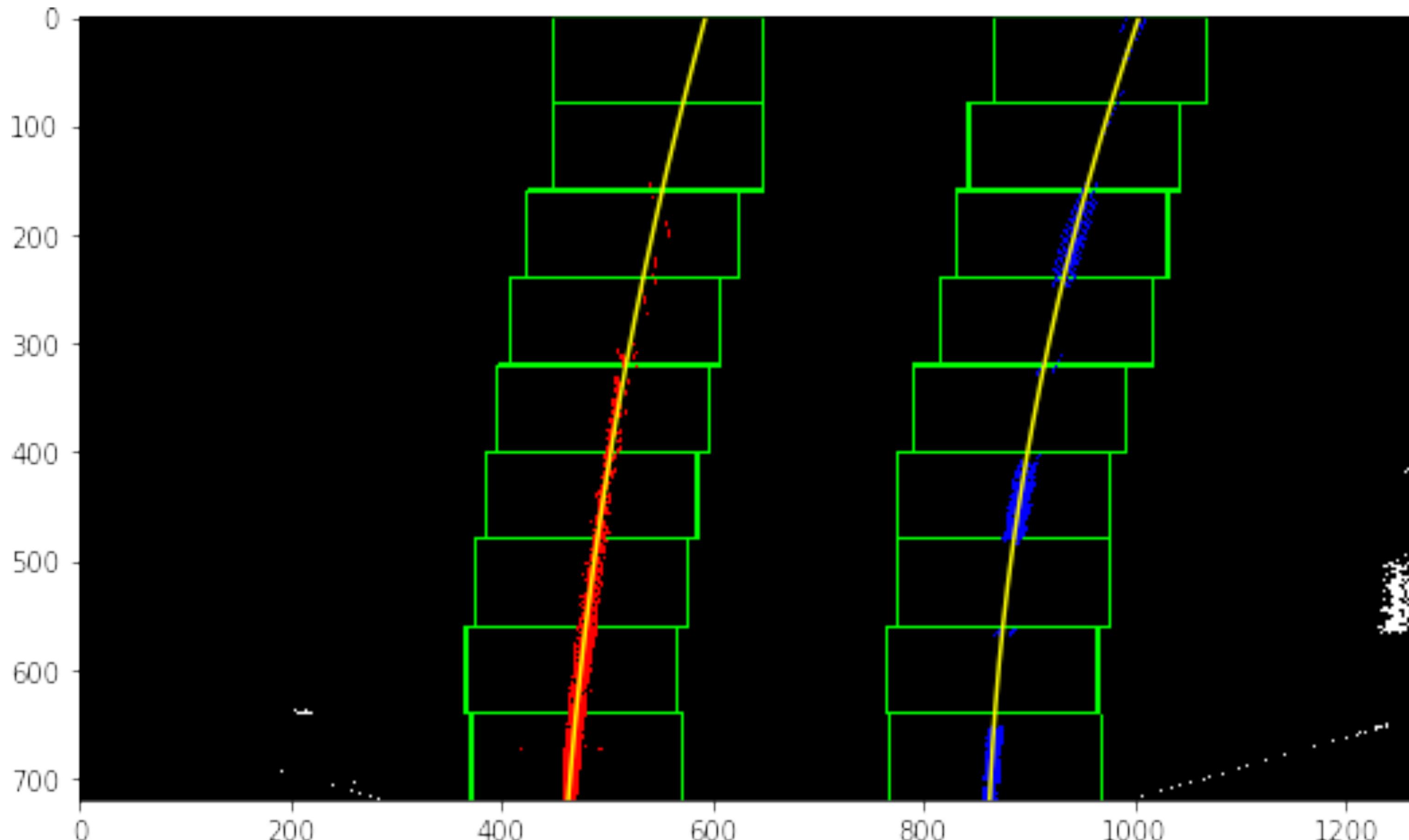


# Step 3: Stack and move the windows

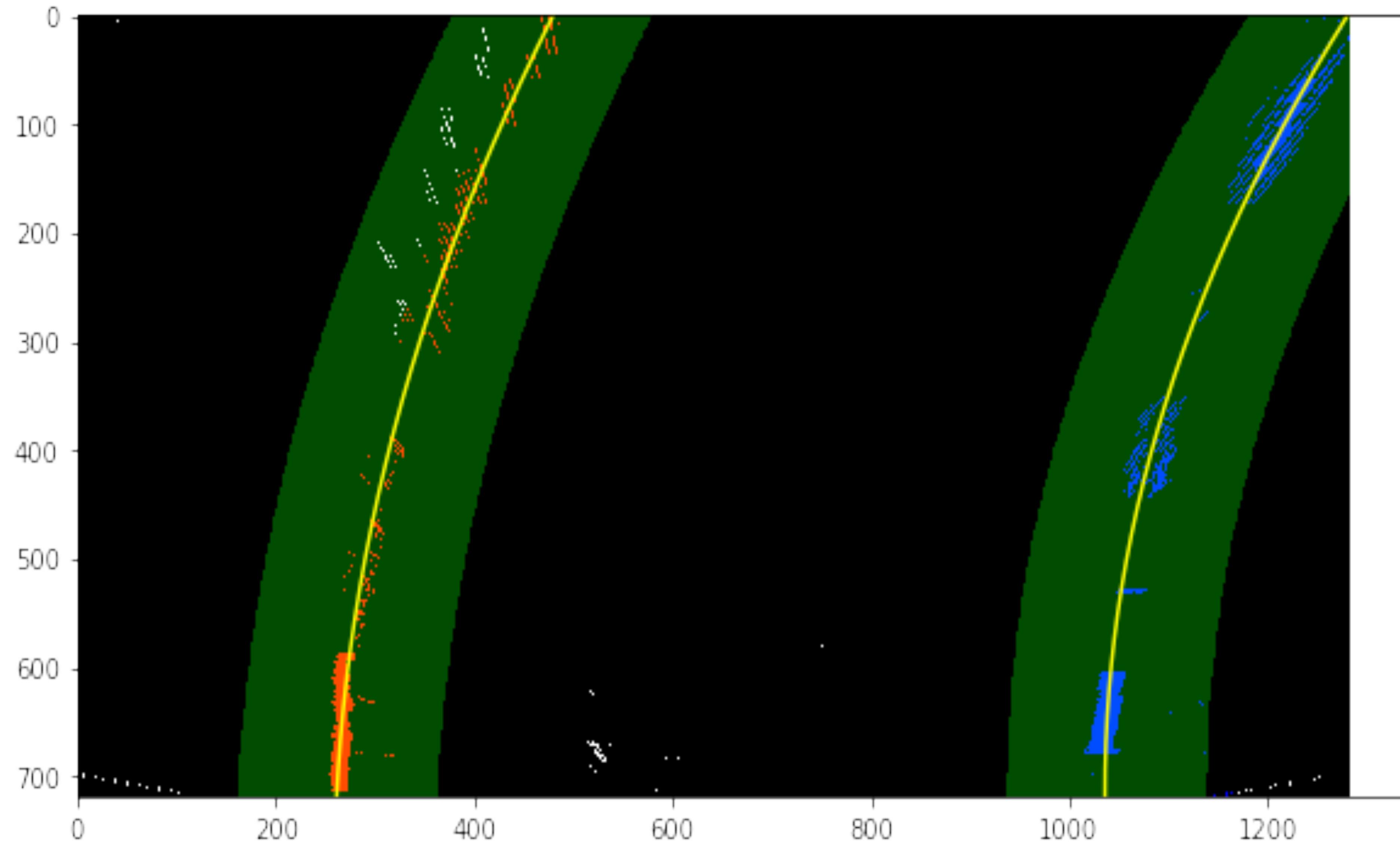


## Step 4: Polyfit all the points from windows

```
left_fit = np.polyfit(lefty, leftx, 2)  
right_fit = np.polyfit(righty, rightx, 2)
```

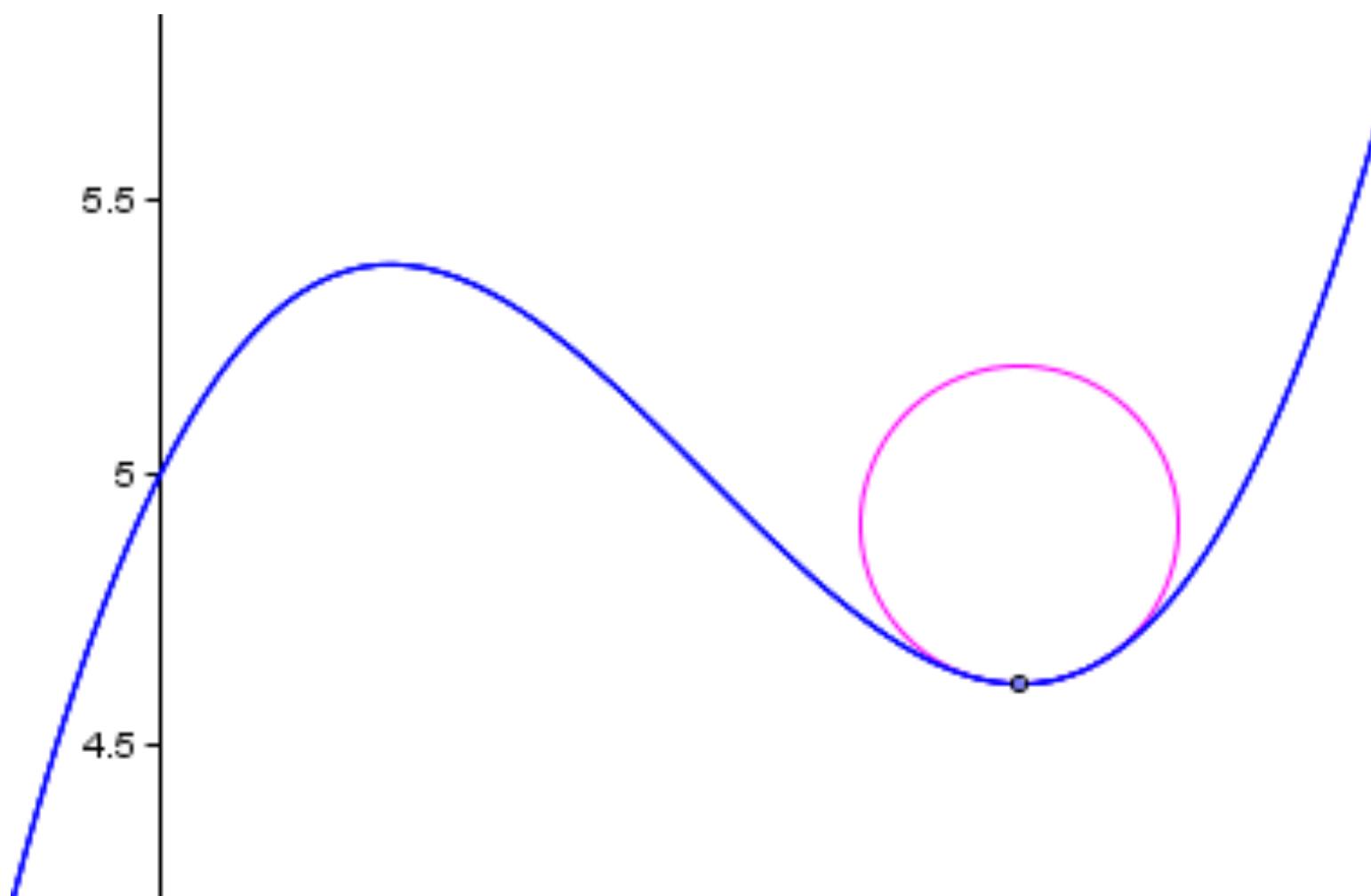


# Method 2 uses an existing fit



# Overlap Results on Image

# Calculating radius of curvature



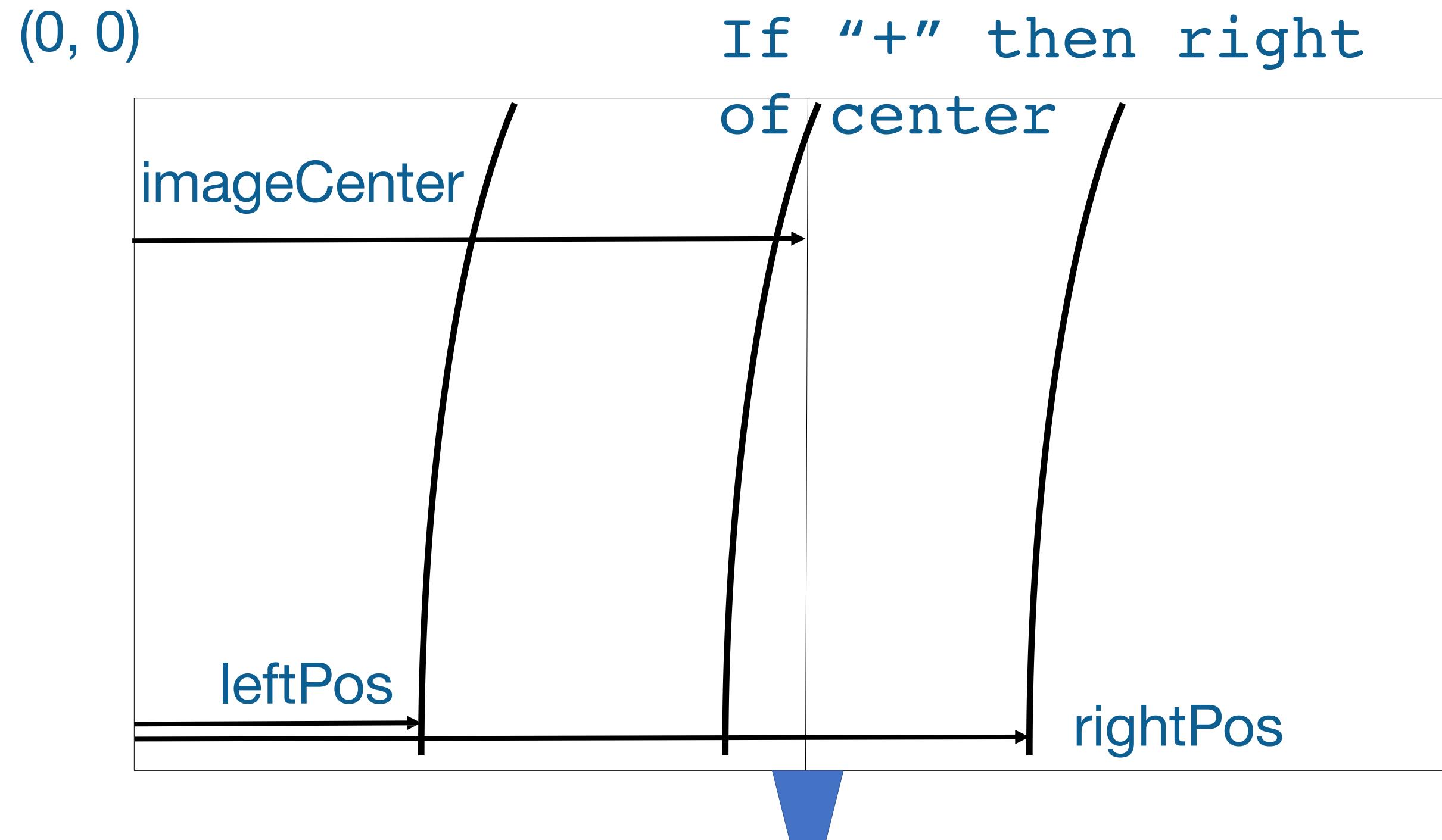
$$\text{Radius of curvature} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Polyfit equation:  $y = Ax^2 + Bx + C$   
fit = [A, B, C]

```
curve_rad = ((1 + (2*fit[0]*y_eval +  
fit[1])**2)**1.5) / np.absolute(2*fit[0])
```

# Distance to lane center

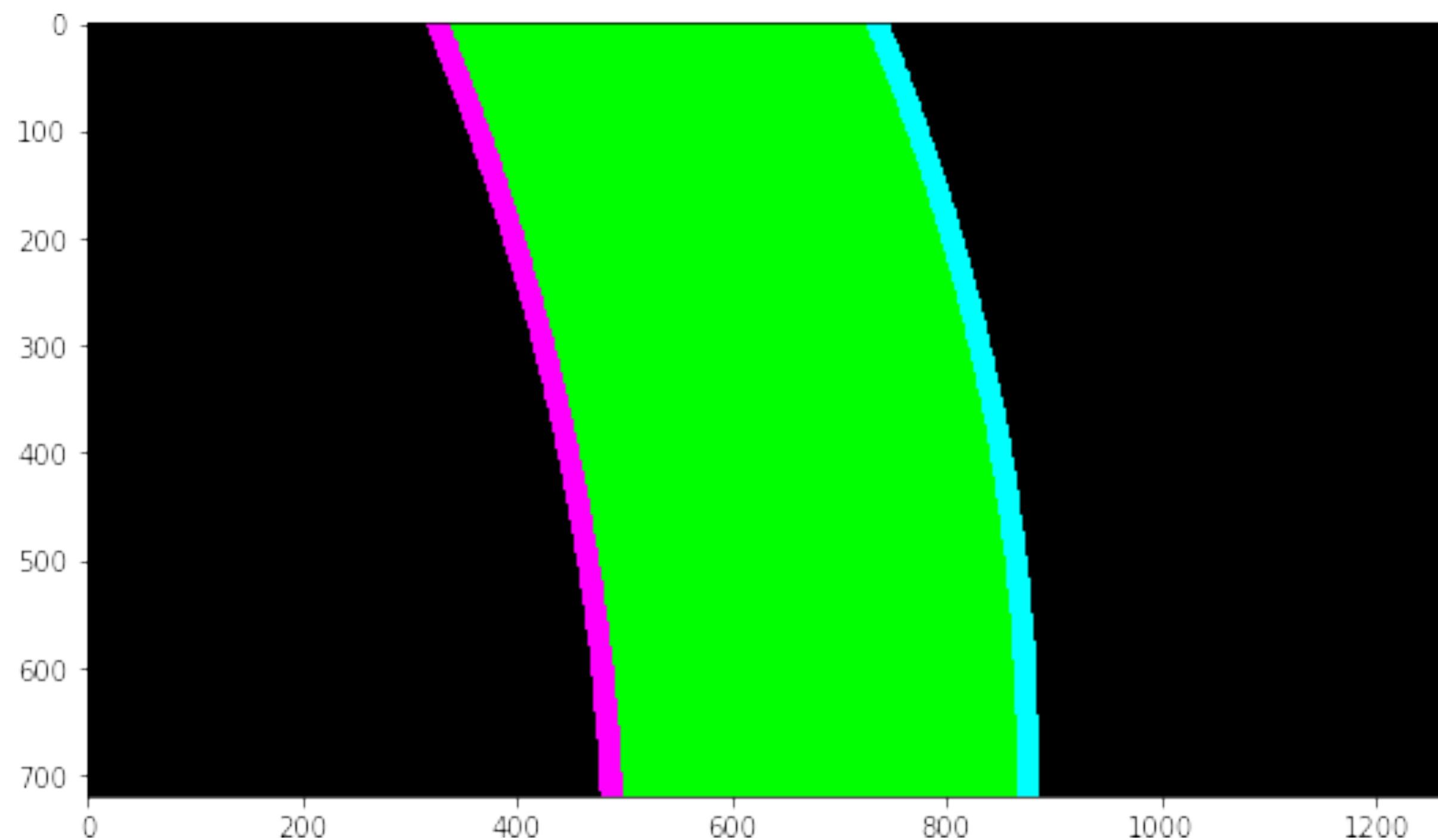
```
laneCenter = (rightPos - leftPos) / 2 + leftPos  
distanceToCenter = laneCenter - imageCenter
```



# Plotting the lines and the drive area

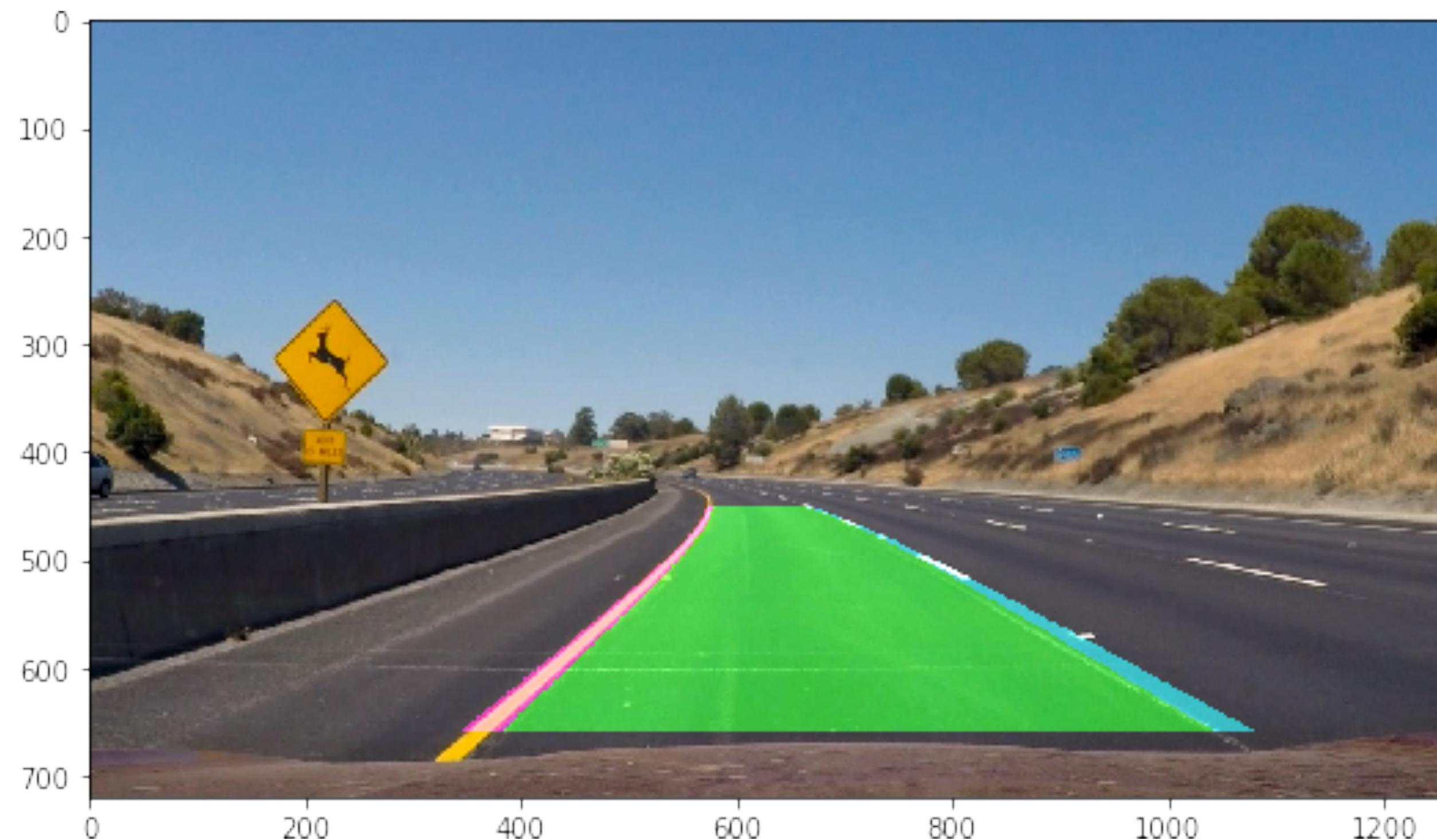
```
cv2.fillPoly(color_warp, np.int_([pts]), (0,255, 0))
```

```
cv2.polyline(color_warp, np.int32([pts_left]), ...)
```



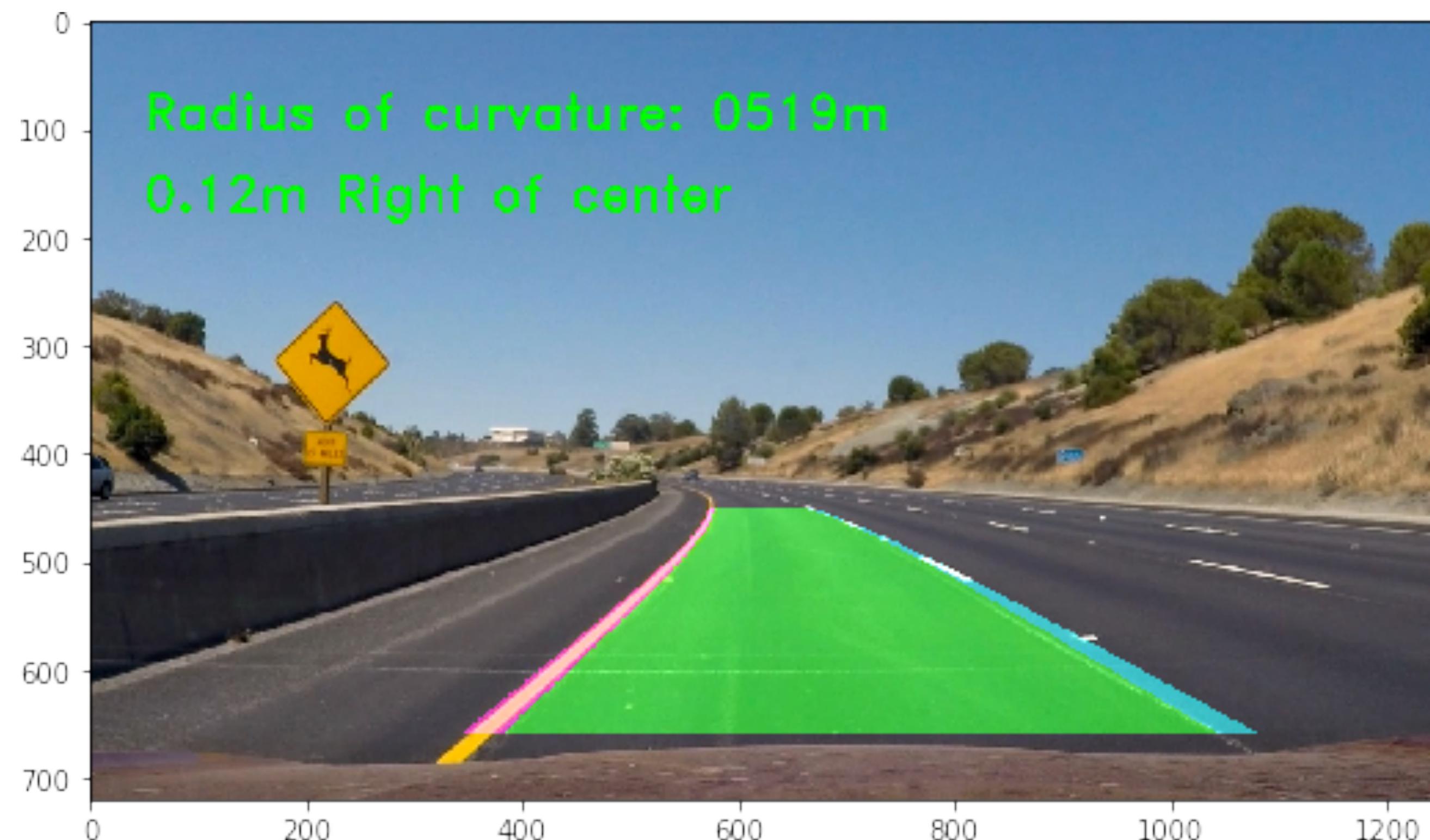
# Unwarp the image and add to the original

```
result = cv2.addWeighted(img, 1, newwarp, 0.5, 0)
```



# Add the radius and center distance

```
cv2.putText(img, text, (50,100), font, 1.5, (0,255, 0),  
2, cv2.LINE_AA)
```



# Building more

- ▶ Image warping
- ▶ Dynamic color channels
- ▶ Speed
- ▶ Smoothing

# References

- ▶ Test images and videos come from the udacity open source self- driving car repo:  
<https://github.com/udacity/self-driving-car>