Solving systems of linear equations over semirings

Asked 5 years, 7 months ago Active 1 year, 1 month ago Viewed 378 times

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So I have come across an issue where it would be very nice to solve systems of linear equations over semirings but I have no clue how to do that. Over a field I would use Gaussian elimination but I'm at a loss of how to find solution spaces to even rings much less semirings. In fact I'm not 100% sure this is decidable. The problem demands that I be able to do this for a wide class of semirings (preferably all) and not just 1 specific kind.

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Are systems of linear semiring equations decidable? If so what algorithms are known to compute solutions for them?

algorithms undecidability linear-algebra algebra

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edited Mar 18 '16 at 10:48
D.W. ♦ 134k 18 180 385

asked Mar 18 '16 at 2:56
Jake 3,658 17 33

1 Answer

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I don't think there's any general algorithm that works for arbitrary semirings. The requirement to be a semiring doesn't give us a lot to work with.

However, if you have a *closed* semiring, then there *are* algorithms for solving systems of linear equations over the semiring.

Closed semirings

A closed semiring is a semiring with a closure operator, denoted $*$, which satisfies the equation

$$a^* = 1 + a \times a^* = 1 + a^* \times a.$$

A closed semiring is also known as a star semiring.

The intuition is that a^* is intended to be the sum of the infinite series

$$1 + a + a^2 + a^3 + \dots$$

For instance, the regular languages form a closed semiring under union and concatenation; the $*$ operator is the Kleene star. The real numbers form a closed semiring under addition and multiplication; the $*$ operator is $a^* = 1/(1 - a)$.

Systems of linear equations over a closed semiring

Now, if you have that kind of structure, then there *is* an analog of Gaussian elimination. In particular, if you have a linear system of equations

$$Ax + b = x$$

where x is a vector of variables over the closed semiring, b is a vector of constants, and A is a matrix of constants, then this has the solution

$$x = A^* b.$$

The closure operator on matrices takes a bit of work to define, but it can be computed efficiently using an analog of Gaussian elimination.

For a careful development of the theory, I recommend the following papers:

Stephen Dolan. Fun with Semirings: A functional pearl on the abuse of linear algebra. International Conference on Functional Programming, ICFP '13.

Daniel J. Lehmann. Algebraic structures for transitive closure. Theoretical Computer Science, vol 4 pp.59--76, 1977.

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edited Sep 2 '20 at 3:21

answered Mar 18 '16 at 10:43

D.W. ♦
134k 18 180 385

Is there any significance in using first a lower-case b and then an uppercase B in the equations? – Matthias Sep 2 '20 at 3:13

@Matthias, oops, no, it was just sloppiness on my part. Sorry, fixed – D.W. ♦ Sep 2 '20 at 3:35