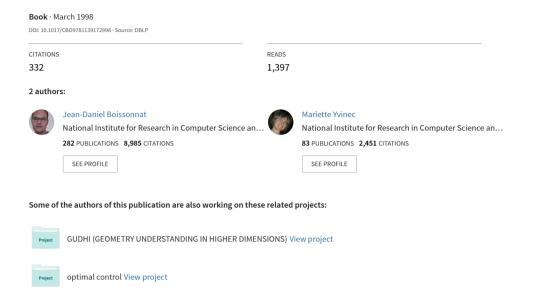
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Algorithmic Geometry



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Jean-Daniel Boissonnat Mariette Yvinec INRIA Sophia-Antipolis, France

Translated by Hervé Brönnimann INRIA Sophia-Antipolis, France



Table of contents

Prefac	е	χv
Transla	ator's preface	xix
Ackno	wledgments	xxi
Part I	- Algorithmic tools	1
Chapte	er 1 – Notions of complexity	3
_	The complexity of algorithms	
	1.1.1 The model of computation	
	1.1.2 Notions of complexity	
	1.1.3 Asymptotic behavior, notation	. 7
1.2	Optimality, lower bounds	. 9
	1.2.1 The complexity of a problem	
	1.2.2 The example of sorting: decision trees	
	1.2.3 Lower bounds by transforming one problem into another	
1.3	Bibliographical notes	. 12
Chapte	er 2 – Basic data structures	13
2.1		. 14
	2.1.1 Lists, heaps, and queues	. 14
	2.1.2 Dictionaries and priority queues	. 15
2.2	Balanced search trees	. 16
	2.2.1 Graphs, trees, balanced trees	
	2.2.2 Red-black trees	
2.3	Dictionary on a finite universe	
2.4	Exercises	
2.5	Bibliographical notes	. 30
Chapte	er 3 – Deterministic methods used in geometry	32
3.1	The divide-and-conquer method	. 33
	3.1.1 Overview	. 33

		3.1.2 An example: sorting n numbers using merge-sort	
	3.2	The sweep method	
		3.2.1 Overview	
		3.2.2 An example: computing the intersections of line segments	
	3.3	Vertical decompositions	
		3.3.1 Vertical decompositions of line segments	40
		3.3.2 Vertical decompositions and simplified decompositions	42
	3.4	Exercises	43
	3.5	Bibliographical notes	44
Cl	hapte	er 4 – Random sampling	46
	4.1	Definitions	46
		4.1.1 Objects, regions, and conflicts	46
		4.1.2 Random sampling	
	4.2	Probabilistic theorems	
		4.2.1 The sampling theorem	
		4.2.2 The moment theorem	55
	4.3	Exercises	
	4.4	Bibliographical notes	62
Cŀ	apte	er 5 – Randomized algorithms	63
	5.1	The randomized incremental method	64
	5.2	Off-line algorithms	65
		5.2.1 The conflict graph	65
		5.2.2 An example: vertical decomposition of line segments	69
	5.3	On-line algorithms	75
		5.3.1 The influence graph	75
		5.3.2 An example: vertical decomposition of line segments	81
	5.4	Accelerated incremental algorithms	84
		5.4.1 The general method	84
		5.4.2 An example: vertical decomposition of a polygon	87
	5.5	Exercises	88
	5.6	Bibliographical notes	92
Cł	apte	er 6 – Dynamic randomized algorithms	95
	6.1	The probabilistic model	96
	6.2	The augmented influence graph	97
	6.3	Randomized analysis of dynamic algorithms	101
	6.4	Dynamic decomposition of a set of line segments	110
	6.5	Exercises	
	6.6	Bibliographical notes	123

Part II – Convex hulls 125				
Chapte	Chapter 7 – Polytopes 127			
7.1	Definitions	127		
	7.1.1 Convex hulls, polytopes			
	7.1.2 Faces of a polytope			
	7.1.3 Polarity, dual of a polytope			
	7.1.4 Simple and simplicial polytopes			
7.2	The combinatorics of polytopes			
	7.2.1 Euler's relation	141		
	7.2.2 The Dehn–Sommerville relations	144		
	7.2.3 The upper bound theorem	145		
	7.2.4 Cyclic polytopes	147		
7.3	Projective polytopes, unbounded polytopes	148		
	7.3.1 Projective spaces			
	7.3.2 Oriented projective spaces	153		
	7.3.3 Projective polytopes, unbounded polytopes	156		
7.4	Exercises			
7.5	Bibliographical notes	168		
Chapte	er 8 – Incremental convex hulls	L 69		
8.1	Representation of polytopes			
8.2	Lower bounds			
8.3	Geometric preliminaries			
8.4	A deterministic algorithm	176		
8.5	On-line convex hulls	180		
8.6	Dynamic convex hulls	186		
8.7	Exercises	195		
8.8	Bibliographical notes	197		
Chapte	er 9 - Convex hulls in two and three dimensions	198		
9.1	Representation of 2- and 3-polytopes	199		
9.2	Divide-and-conquer convex hulls in dimension 2			
9.3	Divide-and-conquer convex hulls in dimension 3			
9.4	Convex hull of a polygonal line	214		
9.5	Exercises			
	Bibliographical notes			
Chapte	er 10 – Linear programming	223		
	Definitions			
	Randomized linear programming			
	Convex hulls using a shelling			
	Exercises			
10.5	Bibliographical notes	239		

Part III – Triangulations	
Chapter 11 – Complexes and triangulations	243
11.1 Definitions	243
11.1.1 Simplices, complexes	
11.1.2 Topological balls and spheres, singularities	
11.1.3 Triangulations	
11.1.4 Polygons and polyhedra	
11.2 Combinatorics of triangulations	
11.2.1 Euler's relation for topological balls and spheres	
11.2.2 The complexity of 2-complexes	
11.2.3 The complexity of 3-triangulations	
11.3 Representation of complexes, duality	
11.4 Exercises	
11.5 Bibliographical notes	
- ·	
Chapter 12 – Triangulations in dimension 2	263
12.1 Triangulation of a set of points	
12.1.1 The complexity of computing a triangulation	
12.1.2 An incremental algorithm	264
12.2 Constrained triangulations	
12.3 Vertical decompositions and triangulations of a polygon	267
12.3.1 Lower bound	
12.3.2 Triangulating monotone polygons	269
12.3.3 Vertical decomposition and triangulation of a polygon	274
12.4 Exercises	281
12.5 Bibliographical notes	287
Chapter 13 – Triangulations in dimension 3	289
13.1 Triangulation of a set of points	290
13.1.1 The size of a triangulation	
13.1.2 The split theorem	293
13.1.3 An incremental algorithm	296
13.2 Constrained triangulations	300
13.3 Vertical and simplicial decompositions	302
13.3.1 Vertical decomposition of a polyhedral region	302
13.3.2 Simplicial decomposition of a polyhedron of genus 0	
13.4 Exercises	317
13.5 Bibliographical notes	318
Part IV - Arrangements	319
Chapter 14 – Arrangements of hyperplanes	321
14.1 Definitions	321

	Combinatorial properties	
14.3	The zone theorem	. 325
14.4	Incremental construction of an arrangement	. 330
	14.4.1 The case of dimension 2	
	14.4.2 The case of dimensions higher than 2	
14.5	Levels in hyperplane arrangements	
	14.5.1 Definitions	
	14.5.2 Combinatorial properties of levels	
	14.5.3 Computing the first k levels in an arrangement	. 335
14.6	Exercises	. 343
14.7	Bibliographical notes	. 350
Chapte	er 15 – Arrangements of line segments in the plane	352
15.1	Faces in an arrangement	. 353
15.2	Davenport-Schinzel sequences	. 353
	The lower envelope of a set of functions	
	15.3.1 Complexity	. 356
	15.3.2 Computing the lower envelope	
15.4	A cell in an arrangement of line segments	. 358
	15.4.1 Complexity	. 359
	15.4.2 Computing a cell	. 362
15.5	Exercises	. 368
15.6	Bibliographical notes	. 371
Chapte	er 16 – Arrangements of triangles	373
	Faces in an arrangement	. 374
16.2	Decomposing an arrangement of triangles	. 374
	16.2.1 Vertical decomposition	. 375
	16.2.2 Convex decomposition	. 377
16.3	The lower envelope of a set of triangles	. 379
	16.3.1 Complexity	
	16.3.2 Vertical decomposition	. 383
	16.3.3 Computing the lower envelope	. 384
16.4	A cell in an arrangement of triangles	
	16.4.1 Complexity	
	16.4.2 Vertical decomposition	
	16.4.3 Computing a cell	
16.5	Exercises	. 402
16.6	Bibliographical notes	. 404
Part V	7 – Voronoi diagrams	405
Chapte	er 17 – Euclidean metric	407
17.1	Definition	. 407

	17.2	Voronoi diagrams and polytopes	. 408
		17.2.1 Power of a point with respect to a sphere	
		17.2.2 Representation of spheres	
		17.2.3 The paraboloid \mathcal{P}	
		17.2.4 Polarity	
		17.2.5 Orthogonal spheres	
		17.2.6 Radical hyperplane	
. •		17.2.7 Voronoi diagrams	
	17.3	Delaunay complexes	
		17.3.1 Definition and connection with Voronoi diagrams	. 416
		17.3.2 Delaunay triangulations	. 418
		17.3.3 Characteristic properties	. 418
		17.3.4 Optimality of Delaunay triangulations	. 421
		Higher-order Voronoi diagrams	
	17.5	Exercises	. 428
	17.6	Bibliographical notes	. 431
Ch	apte	er 18 – Non-Euclidean metrics	433
	18.1	Power diagrams	. 434
		18.1.1 Definition and computation	
		18.1.2 Higher-order power diagrams	. 436
	18.2	Affine diagrams	
		18.2.1 Affine diagrams and power diagrams	
		18.2.2 Diagrams for a general quadratic distance	
	18.3	Weighted diagrams	. 439
		18.3.1 Weighted diagrams with additive weights	. 439
		18.3.2 Weighted diagrams with multiplicative weights	. 442
	18.4	L_1 and L_{∞} metrics	. 445
	18.5	Voronoi diagrams in hyperbolic spaces	. 449
		18.5.1 Pencils of spheres	. 449
		18.5.2 Voronoi diagrams in hyperbolic spaces	.450
	18.6	Exercises	.454
	18.7	Bibliographical notes	. 457
Ch	apte	er 19 – Diagrams in the plane	459
	19.1	A sweep algorithm	. 459
	19.2	Voronoi diagram of a set of line segments	. 464
		19.2.1 Definition and basic properties	. 464
		19.2.2 A sweep algorithm	
		19.2.3 An incremental algorithm	
•		19.2.4 The case of connected segments	. 476
		19.2.5 Application to the motion planning of a disk	. 479
	19.3	The case of points distributed in two planes	. 481
		19.3.1 The two planes are parallel	. 481

19.3.2 The two planes are not parallel 19.4 Exercises 19.5 Bibliographical notes	488
References	492
Notation	508
Index	513