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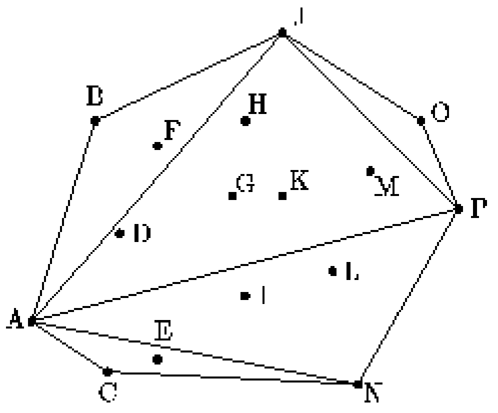
Planar Convex-Hull

Our next example solves the planar convex hull problem: Given *n* points in a plane, find which of them lie on the perimeter of the smallest convex region that contains all points. This example shows another use of nested parallelism for divide-and-conquer algorithms. The algorithm we use is a parallel *Quickhull* [20], so named because of its similarity to the Quicksort algorithm. As with Quicksort, the strategy is to pick a ``pivot'' element, split the data based on the pivot, and recurse on each of the split sets. Also as with Quicksort, the pivot element is not guaranteed to split the data into equally sized sets, and in the worst case the algorithm requires $O(n^2)$ work, however in practice the algorithm is often very efficient.



Click on the image to see and run the code.

Figure 10: Example of the *Quickhull* algorithm. Each sequence in the example shows one step of the algorithm. Since *A* and *P* are the two *x* extrema, the line *AP* is the original split line. *J* and *N* are the farthest points in each subspace from *AP* and are, therefore, used for the next level of splits. The values outside the brackets are hull points that have already been found.

Figure 10 shows the code and an example of the Quickhull algorithm. The algorithm is based on the recursive routine `hsplit`. This function takes a set of points in the plane ($\langle x, y \rangle$ coordinates) and two points `p1` and `p2` that are known to lie on the convex hull and returns all the points that lie on the hull clockwise from `p1` to `p2`, inclusive of `p1`, but not of `p2`. In Figure 10, given all the points `[A, B, C, ..., P]`, `p1 = A` and `p2 = P`, `hsplit` would return the sequence `[A, B, J, O]`. In `hsplit`, the order of `p1` and `p2` matters, since if we switch `A` and `P`, `hsplit` would return the hull along the other direction `[P, N, C]`.

The `hsplit` function first removes all the elements that cannot be on the hull because they lie below the line between `p1` and `p2` (which we denote by `p1-p2`). This is done by removing elements whose cross product with the line between `p1` and `p2` is negative. In the case `p1 = A` and `p2 = P`, the points `[B, D, F, G, H, J, K, M, O]` would remain and be placed in the sequence `packed`. The algorithm now finds the point, `pm`, furthest from the line `p1-p2`. The point `pm` must be on the hull since as a line at infinity parallel to `p1-p2` moves toward `p1-p2`, it must first hit `pm`. The point `pm` (`J` in the running example) is found by taking the point with the maximum cross-product. Once `pm` is found, `hsplit` calls itself twice recursively using the points `(p1, pm)` and `(pm, p2)` (`(A, J)` and `(J, P)` in the example). When the recursive calls return, `hsplit` flattens the result, thereby appending the two subhulls.

The overall `convex-hull` algorithm works by finding the points with minimum and maximum `x` coordinates (these points must be on the hull) and then using `hsplit` to find the upper and lower hull. Each recursive call has constant depth and *O(n)* work. However, since many points might be deleted on each step, the work could be significantly less. As with Quicksort, the worst case costs are *W* = *O(n2)* and *D* = *O(n)*. For *m* hull points the best case times are *O(log m)* depth and *O(n)* work. It is hard to state the average case time since it depends on the distribution of the inputs. Other parallel algorithms for the convex-hull problem run in *D* = *O(log n)*, and *W* = *O(n)* in the worst case [16], but have larger constants.

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