1 Eigenvalues and eigenvectors

1.1 Linear independence

It is easy to prove that eigenspaces are disjoint. We said v is an eigenvector of T if

$$Tx = kx$$

for some k in K.

So if $Tx = kx$ and $Ty = k'y$, x and y are distinct, and $y \neq qx$, then k' and k cannot be equal, and the two vectors cannot be elements of the same eigenspace.

Note if $y + qx$ then $T(y) = T(qx) = k(qx) = ky$, and the two vectors do belong to the same eigenspace and have the same eigenvalue. So 'scaled versions' of the same vector belong to the same eigenspace.