

## **SELF ORGANIZING NETWORKS**

Self-organized clustering may be defined as a mapping through which N-dimensional pattern spaces are mapped into a smaller number of points in an output space. The mapping is achieved autonomously by a system without supervision, i.e., the clustering is achieved in a self-organizing manner.

In a self-organizing network, the weights of the neurons represent a class of pattern. Input patterns are presented to all the neurons and each neuron produces an output. The value of each neuron's output is used as a measure of how close are the input and the stored patterns (neuron's weights). A competitive learning strategy is used to select the neuron whose weight vector matches closely with the input vector.

Self-organizing refers to the ability to learn and organize information without being given correct answers for the input pattern (unsupervised learning).

### **FIXED WEIGHT COMPETITIVE NETS**

They are additional structures included in networks of multi output in order to force their output layers to make a decision as to which one neuron will fire. This mechanism is called competition. When competition is complete, only one output neuron has nonzero output. Symmetric (fixed) weight nets are: (*Maxnet*, *Mexican Hat Net* and *Humming Net*).

#### **1- Maxnet**

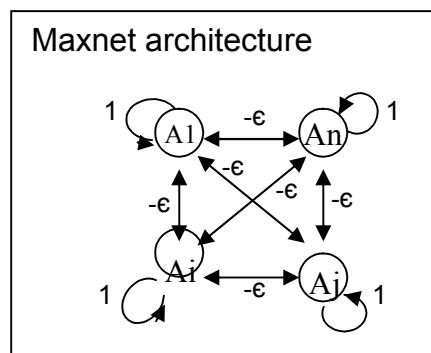
- -Maxnet is based on winner-take-all policy.
- -The n-nodes of Maxnet are completely connected.

- -There is no need for training the network, since the weights are fixed.
- -The Maxnet operates as a recurrent recall network that operates in an auxiliary mode.

Activation functions

$$f(\text{net}) = \begin{cases} \text{net} & \text{if } \text{net} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Where  $\epsilon$  is usually positive less than 1 number.



### Maxnet Algorithm

Step 1: Set activations and weights,

$a_j(0)$  is the starting input value to node  $A_j$

$$\omega_{ij} = \begin{cases} 1 & \text{for } i = j \\ -\epsilon & i \neq j \end{cases}$$

Step 2: If more than one node has nonzero output, do step 3 to 5.

Step 3: Update the activation (output) at each node for

$$j = 1, 2, 3, \dots, n$$

$$a_j(t+1) = f[a_j(t) - \epsilon \sum_i a_i(t)] \quad i \neq j$$

$\epsilon < 1/m$  where  $m$  is the number of competitive neurons

Step 4: Save activations for use in the next iteration.

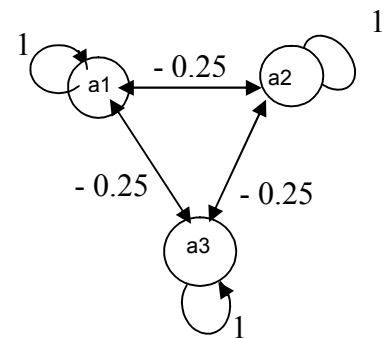
$$a_j(t+1) \rightarrow a_j(t)$$

Step 5: Test for stopping condition. If more than one node has a nonzero output then Go To step 3, Else Stop.

**Example:** A Maxnet has three inhibitory weights a 0.25 ( $\epsilon = 0.25$ ). The net is initially activated by the input signals [0.1 0.3 0.9]. The activation function of the neurons is:

$$f(\text{net}) = \begin{cases} \text{net} & \text{if net} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the final winning neuron.



**Solution:**

First iteration: The net values are:

$$a1(1) = f[0.1 - 0.25(0.3+0.9)] = 0$$

$$a2(1) = f[0.3 - 0.25(0.1+0.9)] = 0.05$$

$$a3(1) = f[0.9 - 0.25(0.1+0.3)] = 0.8$$

Second iteration:  $a1(2) = f[0 - 0.25(0.05+0.8)] = 0$

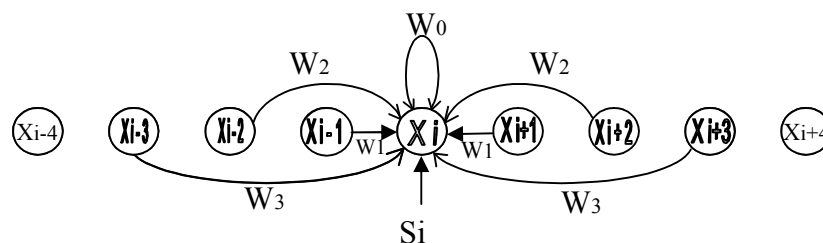
$$a2(2) = f[0.05 - 0.25(0 + 0.8)] = 0$$

$$a3(2) = f[0.8 - 0.25(0+0.05)] = 0.7875$$

Then the 3<sup>rd</sup> neuron is the winner.

## 2- Mexican Hat Network

- It is more contrast enhancing subnet than the Maxnet.
- Each neuron is connected to other neurons through excitatory and inhibitory links.
- Neurons which are in close proximity are connected through excitatory (positive) links and are called "*cooperative neighbors*"
- Neurons of farther away positions are connected through inhibitory (negative), links and are called "*competitive neighbors*".
- Neurons beyond the competitive neighbors are unconnected.
- Each neuron in a particular layer receives an external signal in addition to the interconnection signals.



### Activation function

$$f(\text{net}) = \begin{cases} \text{net} & \text{net} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$K = i + R$$

$$X_i = f \left[ S_i + \sum_{K=i-R}^{K=i+R} W_K X_K(t-1) \right]$$

$$K = i - R$$

- The interconnections of each neuron have two symmetrical regions around it. The region near the neuron within radius (R1) has positive weights, and the region outside R1 and within R2, where connections are negative.

## **TERMINOLOGY**

R1: Radius of reinforcement region  
 R2: Radius of connection, ( $R1 < R2$ )  
 $W_k$ : Weights of  $X_i$  neighbors  
      $W_k$  is positive if  $0 < k \leq R1$   
      $W_k$  is negative if  $R1 < k \leq R2$   
      $W_k$  is zero if  $k > R2$   
 X: Vector of activations  
 X\_old: Vector of activations at previous step  
 t\_max: Total number of iterations of contrast enhancement  
 Si: External signal

## **ALGORITHM**

Step 1: Set values of R1, R2 and t\_max (No. of iterations).  
     Initialize weights:  
      $W_k = C1$  for  $k = 0, 1, 2, R1$ , where C1 is random  $> 0$ .  
      $W_k = C2$  for  $k = R1+1, R1+2, \dots, R2$  (C2 is random  $> 0$ ).  
  
 Step 2: (t = 0)  
     Present the external signal vector S:  $X=S$

Save activations in X\_old array for  $i = 1, 2, n$ , to be used in the next step.

$$X\_old = X = (x_1, x_2, x_n) \quad t = 1$$

Step 3: Calculate the net input for  $i = 1, 2, n$ .

$$(net)_i = X_i = C_1 \sum (x\_old)_{i+k} - C_2 \sum (x\_old)_{i-k}$$

Step 4: Apply activation function and save current activations in X\_old to be used in the next iteration.

Step 5: Increment counter  $t$ , as  $t = t + 1$ .

Step 6: Test for stopping condition:

If  $t < t\_max$ , then continue (Go to step 3), else stop.

**Note:** The positive and negative reinforcement have the effect of increasing the activation of units with large initial values and reducing that of units with small initial activations respectively.

**Example:** A Mexican Hat net consists of seven input units. The net is initially activated by the input signal vector [0.0 0.4 0.7 1.0 0.7 0.4 0.0]. The activation function of the neuron is:

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 2 \end{cases}$$

The max. number of iterations is three.

Solution:

Step 1:

Let  $R1=1$ ,  $R2=2$ ,  $C1=.7$ , and  $C2= -0.3$  (initialization).

Step 2: (t = 0)

$$X = S = [0.0 \ 0.4 \ 0.7 \ 1.0 \ 0.7 \ 0.4 \ 0.0]$$

$X\_old = [0.0 \ 0.4 \ 0.7 \ 1.0 \ 0.7 \ 0.4 \ 0.0]$ , (presenting external signal vector and saving activations in  $X\_old$  array).

$$t = 1$$

Step 3:

$$X1 = 0.7(0.0+0.4) - 0.3(0.7) = 0.07$$

$$X2 = 0.7(0.0+0.4+0.7) - 0.3(1.0) = 0.47$$

$$X3 = 0.7(0.4+0.7+1.0) - 0.3(0.0+0.7) = 1.26$$

$$X4 = 0.7(0.7+1.0+0.7) - 0.3(0.4+0.4) = 1.44$$

$$X5 = 0.7(1.0+0.7+0.4) - 0.3(0.7+0.0) = 1.26$$

$$X6 = 0.7(0.7+0.4+0.0) - 0.3(1.0) = 0.47$$

$$X7 = 0.7(0.4+0.0) - 0.3(0.7) = 0.07$$

Step 4:

$$X\_old = [0.07 \ 0.47 \ 1.26 \ 1.44 \ 1.26 \ 0.47 \ 0.07]$$

Step 5:

$$t = t+1 = 2$$

Step 6:

t = 2, Go to step 3

Step 3:

$$X\_old = [0.07 \ 0.47 \ 1.26 \ 1.44 \ 1.26 \ 0.47 \ 0.07]$$

$$X1 = 0.7(0.07+0.47) - 0.3(1.26) = 0$$

$$X2 = 0.7(0.07+0.47+1.26) - 0.3(1.44) = 0.828$$

$$X3 = 0.7(0.47+1.26+1.44) - 0.3(0.07+1.26) = 1.82$$

$$X4 = 0.7(1.26+1.44+1.26) - 0.3(0.47+0.47) = 2.49$$

$$X5 = 0.7(1.44+1.26+0.47) - 0.3(1.26+0.07) = 1.82$$

$$X_6 = 0.7(1.26+0.47+0.07) - 0.3(1.44) = 0.828$$

$$X_7 = 0.7(0.47 + 0.07) - 0.3(1.26) = 0$$

Step 4:

$$X_{old} = [0.0 \ 0.828 \ 1.82 \ 2.49 \ 1.82 \ 0.828 \ 0.0]$$

Step 5:

$t = t+1 = 3$ , then stop.

The network's outputs are shown for  $t = 0, 1$  and  $2$ .

### 3- Hamming Net

Hamming net is a maximum likelihood classifier net. It is used to determine an exemplar vector which is most similar to an input vector. The measure of similarity is obtained from the formula:

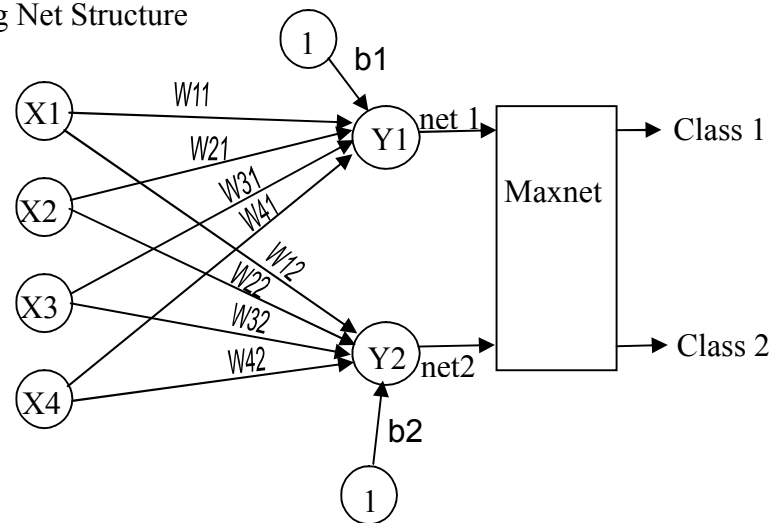
$$x.y = a - D = 2a - n, \text{ since } a + D = n$$

Where  $D$  is the hamming distance (number of component in which vectors differ),  $a$  is the number components in which the components agree and  $n$  is the number of each vector components.

When weight vector of a class unit is set to be one half of the exemplar vector, and bias to be  $(n/2)$ , the net will find the unit closest exemplar by finding the unit with maximum net input. Maxnet is used for this purpose.



Hamming Net Structure



$$W_{ij} = e_i(j)/2$$

Where  $e_i(j)$  is the  $i$ 'th component of the  $j$ 'th exemplar vector.

## Terminology

M : number of exemplar vectors

N : number of input nodes (input vector components)

$E(j)$  :  $j$ 'th exemplar vector

## Algorithm

Step 1: Initialize the weights

$w_{ij} = e_i(j)/2 = i$ 'th component of the  $j$ 'th exemplar,  
(  $i = 1, 2, \dots, n$ , and  $j = 1, 2, \dots, m$  )

Initialize bias values,  $b_j = n/2$

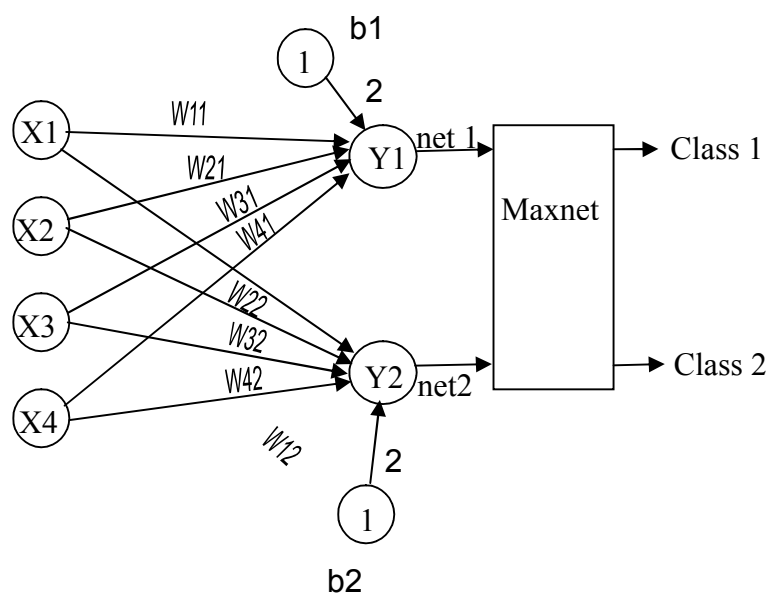
For each input vector  $X$  do steps 2 to 4

Step 2: Compute net input to each output unit  $Y_j$  as:

$$Y_{inj} = b_i + \sum e_{ij} x_{ij} \quad (i = 1, 2, \dots, n, j = 1, 2, \dots, m)$$

Step 3: Maxnet iterations are used to find the best match exemplar.

**Example:** Given the exemplar vector  $e(1)=(-1 \ 1 \ 1 \ -1)$  and  $e(2)=(1 \ -1 \ 1 \ -1)$ . Use Hamming net to find the exemplar vector close to bipolar input patterns  $(1 \ 1 \ -1 \ -1)$ ,  $(1 \ -1 \ -1 \ -1)$ ,  $(-1 \ -1 \ -1 \ 1)$  and  $(-1 \ -1 \ 1 \ 1)$ .



### Solution:

Step 1: Store the exemplars in the weights as:

$w_{ij} = e_i(j)/2 = i\text{'th component of the } j\text{'th exemplar,}$

$$W = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & -0.5 \\ 0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix}$$

Since  $e(1) = (-1 \ 1 \ 1 \ -1)$  and  $e(2) = (1 \ -1 \ 1 \ -1)$ .

$b_j = n/2 = 2$

Step 2: Apply 1<sup>st</sup> bipolar input (1 1 -1 -1)

$$\begin{aligned} Y_{in1} &= b_1 + \sum x_i w_{i1} \\ &= 2 + (1 \ 1 \ -1 \ -1) \cdot (-0.5 \ 0.5 \ 0.5 \ -0.5) \\ &= 2 \end{aligned}$$

$$\begin{aligned} Y_{in2} &= b_2 + \sum x_i w_{i2} \\ &= 2 + (1 \ 1 \ -1 \ -1) \cdot (0.5 \ -0.5 \ 0.5 \ -0.5) \\ &= 2 \end{aligned}$$

Hence, the first input pattern has the same Hamming distance  
 $HD = 2$  with both exemplar vectors.

Step 3: Apply the second input vector (1 -1 -1 -1)

$$Y_{in1} = 2 + (1 \ -1 \ -1 \ -1) \cdot (-0.5 \ 0.5 \ 0.5 \ -0.5) = 1$$

$$Y_{in2} = 2 + (1 \ -1 \ -1 \ -1) \cdot (0.5 \ -0.5 \ 0.5 \ -0.5) = 3$$

Since  $y_2 > y_1$ , then the second input best matches with the second exemplar  $e(2)$ .

Step 4: Apply input pattern no. 3 (-1 -1 -1 1)

$$Y_{in1} = 2 + (-1 \ -1 \ -1 \ 1) \cdot (-0.5 \ 0.5 \ 0.5 \ -0.5) = 1$$

$$Y_{in2} = 2 + (-1 \ -1 \ -1 \ 1) \cdot (0.5 \ -0.5 \ 0.5 \ -0.5) = 1$$

Hence we have Hamming similarity.

Step 5: Consider the last input vector (-1 -1 1 1)

$$Y_{in1} = 2 + (-1 \ -1 \ 1 \ 1) \cdot 0.5 \ (-1 \ 1 \ 1 \ -1) = 2$$

$$Y_{in2} = 2 + (-1 \ -1 \ 1 \ 1) \cdot 0.5 \ (1 \ -1 \ 1 \ -1) = 2$$

Hence we have Hamming similarity.