Problem Statement: Analyzing and Implementing FFT Algorithms for Audio Signal Processing

Objective

The goal of this project is to provide hands-on experience with implementing the **Discrete Fourier Transform (DFT)** and the **Fast Fourier Transform (FFT)** algorithms, specifically focusing on the **Decimation-in-Time (DIT)** and **Decimation-in-Frequency (DIF)** approaches. You will compare the performance of these algorithms against the naive DFT and apply them to analyze an audio signal.

Tasks

1. **Naive DFT Implementation**:
    - Write a MATLAB function to compute the **DFT** of a given signal using the naive formula ($O(N^2)$ complexity).
    - Analyze the computational cost by measuring the execution time for different signal lengths.
2. **DIT FFT Implementation**:
    - Implement the **Decimation-in-Time (DIT) FFT** algorithm recursively.
    - Visualize the block diagram for **N = 8**, clearly indicating the stages of decomposition and the **butterfly operations**.
    - Demonstrate the use of **twiddle factors** in the implementation.
3. **DIF FFT Implementation**:
    - Implement the **Decimation-in-Frequency (DIF) FFT** algorithm.
    - Compare the results and computational cost with the DIT FFT.
4. **Frequency Analysis of Audio Signal**:
    - Use the provided noisy audio signal (`noisy_signal.wav`) and clean audio signal (`clean_signal.wav`) for analysis.
    - Apply your FFT implementations to the audio signals and plot the **magnitude spectrum**.
    - Identify the dominant frequency components and discuss any differences observed between the noisy and clean signals.
5. **Comparative Analysis**:
    - Compare the performance of the **naive DFT**, **DIT FFT**, and **DIF FFT** in terms of computational cost ($O(N^2)$ vs $O(N \log_2 N)$).
    - Provide a brief analysis of the efficiency and accuracy of each method.

Deliverables

1. MATLAB code files:
    - `naive_dft.m`
    - `dit_fft.m`
    - `dif_fft.m`
    - A main script for testing the functions and analyzing the audio signals.
2. A brief report (2-3 pages) that includes:
    - Explanation of the implemented algorithms.

- Block diagram for DIT FFT (N = 8).
- Computational cost analysis.
- Plots of the magnitude spectrum for the audio signals.
- Conclusions based on the comparative analysis.

Detailed Marking Rubric

Criteria	Description	Points
Code Correctness	Accurate implementation of the naive DFT, DIT FFT, and DIF FFT algorithms.	30 points
Block Diagram	Correct and detailed block diagram for DIT FFT (N = 8), showing all stages and butterfly operations.	15 points
Computational Analysis	Clear comparison of the computational costs, with proper timing measurements and analysis.	15 points
Frequency Analysis	Accurate frequency analysis of the provided audio signals, including well-labeled magnitude spectrum plots.	20 points
Report Quality	Well-structured report with clear explanations, diagrams, and analysis.	10 points
Creativity and Extensions	Additional features or creative approaches (e.g., optimizing the FFT code, using built-in MATLAB functions for comparison).	10 points

Total: 100 Points

Expectations

- Your MATLAB code should be **well-documented** with comments explaining each major step.
- The **report** should be concise, clear, and demonstrate a good understanding of the FFT concepts.
- Discuss your implementation, including any challenges you faced and how you overcame them.