

## Homework 2

Macroeconomics 2

Beta Yulianita Gitaharie/Eugenia Mardanugraha

TA: Alvin Ulido Lumbanraja

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Late Penalty Applies

### 1. (50 points) Theory of the Firm

Consider an infinitely-lived representative firm that maximizes the present value of their intertemporal profit:

$$\max_{N_t, K_t \in \mathbb{R}^+} \sum_{t=0}^{\infty} \beta^t [PF(N_t, K_t) - WN_t - P_I I_t - bP_I I_t^2] \quad (1)$$

$$\text{s.t. } I_t = K_{t+1} - K_t + \delta K_t \quad (2)$$

Where  $\beta = \left(\frac{1}{1+r}\right)$  is the simplified notation of the discounting term.

#### Questions:

- Characterize the steady state level of investment from equation (1) and (2). DO NOT assume any functional form of  $F(\cdot)$  just yet
- If we assume perfect substitution between labor and capital, and that the production function follows  $Y_t = A(K_t + N_t)$ , does the Inada condition hold? Explain, mathematically and economically, why we want Inada condition to hold!
- Explain what would happen to steady state level of investment if adjustment cost parameter decline? (i.e.  $\hat{b} < b$ )
- What would happen to (i) the investment decision rule and (ii) steady state level of investment if the central bank announces **nominal** interest rate cut? Explain your answer in detail!
- Intuitively, why is price level not relevant in determining the steady-state level of investment?

### 2. (50 points) Theory of the Household

Consider an infinitely-lived representative household that maximizes the present value of their intertemporal utility subject to intertemporal constraint:

$$\max_{C_t \in \mathbb{R}^+} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.}$$
$$C_t + (A_{t+1} - A_t) = rA_t + w_t$$

Where  $\beta = \left(\frac{1}{1+\rho}\right)$  is the simplified notation of the discounting term.

#### Questions:

- Characterize the optimal decision rule for  $C_t$ , assuming  $U(\cdot) = \ln(\cdot)$
- What would happen to the optimal decision rule if we increase the initial wealth from  $A_0$  to  $\hat{A}_0$ , where  $\hat{A}_0 > A_0$ ? (Hint: this is a trick question. As a guide, consider what would happen if the parameters change. What conditions should hold for your answer to be true?)

- (c) Logically, if  $r > \rho$ , that is the interest rate is higher than the household discount rate, a household should be incentivized to consume at a later date. Why, then, does the consumer still choose to consume a positive value of  $c_t$  at any point in time? What prevents them from just choosing to consume  $c_t = 0$  and consume the bulk of their wealth at a later date, since their money will grow infinitely large at a later date?
- (d) Illustrate graphically (that is, draw a graph) that shows the optimal path for  $C_t$  from  $t = 0$  to  $t = 100$ . Assume the following ( $w_t = \bar{w}$  indicates constant wage level over time)

Parameter	$\rho$	$A_0$	$r$	$\bar{w}$	$c_0$
Values	0.04	10	0.02	1	1.5

Furthermore, assume without proof that the initial consumption ( $c_0 = 1.5$ ) is on the optimal trajectory of consumption, such that the sequence of  $\{c_t\}_{t=0}^{\infty}$  is the optimal consumption path. It is highly recommended that you graph this path in Excel, Stata, R, Python, or any other software that can do the calculation for you. You do not need to submit any code you use to generate the graph