Second TA Session: Microfoundations

Macroeconomics 2

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Highlights

- Representative Firm
  - Key objectives
  - Inada condition
  - Steady state level of investment and adjustment costs
- Representative Household
  - Key objectives
  - Optimal decision rule
- Homework discussion

Background

- Representative firm essentially seeks to maximize the following:

$$\max_{N_t, K_t \in \mathbb{R}^+} \sum_{t=0}^{\infty} \beta^t \left[F(N_t, K_t) - wN_t - I_t\right] \tag{1}$$

$$\text{s.t.} \quad I_t = K_{t+1} - K_t + \delta K_t \tag{2}$$

- $\beta = \left(\frac{1}{1+r}\right)$ is the simplified notation of the discounting term.
- In this problem, the amount of capital becomes state variable (i.e. variable that evolve over time), and investment is residually determined
- Question: why don't we use this framework in class?
- Answer: we do not cover the theory of optimal control or dynamic programming in undergraduate level

Firm's decision function covered in this class

- Firms instead are assumed to maximize the following

$$\max_{N_t, K_t \in \mathbb{R}^+} \sum_{t=0}^{\infty} \beta^t \left[PF(N_t, K_t) - WN_t - P_I I_t - bP_I I_t^2 \right] \quad (3)$$

$$\text{s.t.} \quad I_t = K_{t+1} - K_t + \delta K_t \quad (4)$$

- $\beta = \left(\frac{1}{1+r}\right)$ is the simplified notation of the discounting term.
- In this problem, investment becomes the control variable; we seek to find profit-maximizing level of investment in order to maximize profit

Inada conditions: a primer

- Whether we perform static or dynamic optimization for firm's problem, we need to ensure that the underlying function obeys Inada conditions
- Named after Ken-ichi Inada
- Inada conditions:
  - The value function $f(\mathbf{x}) = 0$ if $\mathbf{x} = 0$
  - The value function is concave; $\partial f(\mathbf{x})/\partial x_i > 0$ and $\partial^2 f(\mathbf{x})/\partial x_i^2 < 0$
  - $\lim_{x_i \to 0} \partial f(\mathbf{x})/\partial x_i = +\infty$
  - $\lim_{x_i \to +\infty} \partial f(\mathbf{x})/\partial x_i = 0$

Inada conditions: why is it essential

- Inada condition insures interior solution/rules out corner solution $\longrightarrow$ makes optimization problem relevant
- Example: suppose we have production function in the form of $y = A(K + L)$
- It can be trivially shown that although it satisfies the first condition, it violates the second, third and fourth condition
- Implication: suppose we want to show that the firm maximizes profit $\pi = A(K + L) - wL - rK$, we can show that if $w = r$, the profit-maximizing L and K becomes undefined, and if $w > r$, firm will switch their factors of production entirely towards capital

Steady State Investment and Adjustment Cost

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- We can derive (and I am not interested with full derivation) the profit-maximizing investment decision rule as follows:

$$I_t - \left(\frac{1+r}{1-\delta}\right) I_{t-1} + \frac{F_K - (r+\delta)}{2b(1-\delta)} = 0 \tag{5}$$

- By some algebraic manipulation, we can get the following

$$I_t = \frac{1}{2b}\left(\frac{F_K}{r+\delta} - 1\right) \tag{6}$$

- Let's do some analysis: what would happen if we set the adjustment cost factor to 0?

Background

- Representative household essentially seeks to maximize the following:

$$\max_{C_t \in \mathbb{R}^+} \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ s.t.} \tag{7}$$

$$C_t + (A_{t+1} - A_t) = rA_t + w_t \tag{8}$$

- $\beta = \left(\frac{1}{1+r}\right)$ is the simplified notation of the discounting term.
- In this problem, the amount of consumption is the control variable and the stock of asset is the state variable

Optimal Decision Rule for Household

- We do not solve for the steady state level of consumption; that is beyond the scope of this class
- Instead, we solve for optimal *path* of consumption over time
- Solving for optimal path of consumption as stated in equation (7) and (8) by assuming functional form of $U(\cdot) = \ln(\cdot)$, we obtain

$$C_t - C_{t-1} = \left(\frac{r - \rho}{1 + \rho} C_{t-1} \right) \tag{9}$$

- If $r - \rho < 0$, we see that the optimal path of consumption is decreasing; people consume more in the beginning and consume less towards the end

Optimal Decision Rule for Household

- Optimal consumption path with decreasing C_t makes a lot of sense if we are talking about pure endowment economy
- Does not make much sense if we are talking about closed system in which consumption and investment is directly interchangeable; treatment of this will be further discussed when we are talking about Solow growth model

Snippets on Optimal Control

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- If you are interested about dynamic system and how modern macro treats the interaction of firm and household in the economy, you may want to learn about Hamiltonian
- Basically the dynamic version of Lagrangian (only slightly harder than Lagrangian)
- Instead of presenting the problem as discrete-time optimization problem, to use Hamiltonian, we change the problem into continuous-time optimization problem
- Alternatively, we can use recursive optimization by using Bellman equation

Homework Discussions