NEOCLASSICAL GROWTH MODEL

1. The model

Look at (dynamic) **competitive equilibrium** (CE) in an economy with many identical households (HH) and many idential firms.

CE adds a new element: HH, firms are price takers and prices must be determined.

Difference from the Solow model: saving is guided by maximization of an intertemporal utility function. Advantages of this approach:

1. More useful for studying the effects of fiscal policy.

   In the Solow model it is not clear if, say, a tax on consumption, labor or capital income will raise or lower the saving rate.

   Here we see what the households will choose to do.

2. Allows welfare comparisons across equilibrium outcome paths.

3. Shows that the conclusions of the Solow model are robust: they change very little if we assume utility-maximizing behavior instead of a fixed savings rate.

   So we can run chnages through the Solow model first, to get a rough idea about their consequences.

The "givens" are

technology: CRS production function $F(K, N)$ and depreciation rate $\delta > 0$;

intertemporal pref. (additively separable over time): $u(c, 1-n)$ and $\rho > 0$.

2. Economic environment

Infinite time horizon, $t \geq 0$.

Agents: HH, firms, gov't. HH and firms are **price takers** (competitive markets!)

Commodities: labor, capital services, and one produced good, used for consumption and investment.

Ownership and market structure:

Three possibilities:

1. HH own capital and operate their technology "in the back yard."

   With identical households and a CRS tech., there are no "gains from trade."

   But, it is difficult to put taxes in this setting.

2. HH own capital and make investment decisions.

   Firms hire labor and capital services from HH in rental markets.

   HH own firms, but with CRS there are no profits:

   factor payments exhaust output.

3. Firms own capital and invest to maximize firm value. HH's own firms.

#2 and #3 are equivalent.

We will use #2, which puts all the interesting decisions in the HH.

We will use a linear cost of investment, $\Psi(I) = pI$.

If convex costs of adjustment are wanted, retain CRS by using

$$\Psi(I, k) = k\psi\left(I/k\right).$$

This is standard in business cycle models (which are usually in discrete time).

2. Economic environment (cont)

Markets: for labor, capital services, and current output.

Prices: $\{w(t), R(t),\ t \geq 0\}$ must clear all markets at all dates.

wage $w(t)$ and rental rate $R(t)$ are in **contemporaneous goods**;

Later we can also back out interest rates. Define $p(t)$ as price of **goods at** t,

so $p(t)/p(0)$ is an **interest factor.** W.l.o.g. normalize $p(0) = 1$.

Use $p(t)$ to determined interest rates $r(t)$, $t \geq 0$.

Firms: hire capital and labor, taking prices $\{w, R\}$ as given.

Their objective is to maximize profits.

Gov't: levies flat-rate taxes (τ_k, τ_n, τ_c) on capital and labor income, and consumption.

The proceeds $\{S(t)\}$ are rebated lump sum.

Households: make consumption, investment (savings) and labor supply decisions,

taking taxes rates (τ_k, τ_n, τ_c), prices $\{w, R\}$ and subsidy $\{S\}$ as given.

Their objective is to maximize the PDV of (lifetime) HH utility.

CE: Given gov't policy (τ_k, τ_n, τ_c), a set of quantities $\{c(t), n(t), I(t), k(t)\}$,

prices $\{w(t), R(t),\ t \geq 0\}$ and subsidies $\{S(t)\}$ that

—are consistent with HH maximization;

—are consistent with firm maximization;

—satisfy budget balance, at every date t, for the gov't;

—clear all markets at every date.

2. Economic environment (cont)

Interpretations: (a) Contracts for all exchanges are signed at $t = 0$. At later dates factors are supplied, production occurs, goods are delivered, and consumption and investment occur.

(b) All agents make plans for the infinite future at $t = 0$. They forecast all future prices correctly.

3. Firms' decisions

Continuum of measure one of (competitive) firms, all with the same CRS technology.

At every date t, each firm hires labor and capital, given prices (w, R).

It's (stationary) technology F is strictly increasing, strictly quasi-concave, twice continuously differentiable, and h.o.d. one (CRS).

$$y = F(k, n),$$

and satisfies Inada conditions at $k = 0$ and $n = 0$.

In addition $\lim_{k\to\infty} F(k, n) = 0$.

Their problem is **static**: they choose inputs at each date to maximize profits:

$$\max_{k,n} \left[F(k, n) - r(t)k - w(t)n\right].$$

Competition implies that factors are paid their marginal products.

CRS implies that factor payments exhaust output, so firms have no profits.

The returns to capital and labor are

$$\begin{aligned} R(t) &= F_K\left[k(t), n(t)\right] \qquad (1) \\ w(t) &= F_L\left[k(t), n(t)\right]. \end{aligned}$$

4. Households' decisions

Continuum of measure one of infinitely lived households, with the same preferences and initial endowment of capital k_0. No population growth.

Utility is additively separable over time, with a constant rate of time preference.

$\{c(t), I(t), n(t), \quad t \geq 0\}$ are consumption, investment, and labor supply per person; $1-n$ is leisure.

Household utility is

$$\max_{\{c(t),I(t),n(t)\}} \int_0^\infty e^{-\rho t} u\left[c(t), 1-n(t)\right] dt, \tag{2}$$

where $\rho > 0$; and where u is strictly increasing, strictly concave, and twice continuously differentiable,

satisfies **Inada conditions** at $c = 0$ and $1 - n = 0$, and

has the feature that both consumption and leisure are **normal goods**.

Recall: both goods are normal if the functions (c^*, ℓ^*) defined by

$$[c^*(y), \ell^*(y)] \equiv \arg\max_{c,\ell} u(c, \ell) \qquad \text{s.t.} \qquad c + w\ell \leq y,$$

are increasing in y.

Interpretation: an increase in (outside) income induces increases in both consumption and leisure.

Budget constraint: HH takes as given tax rates (constant over time) and future prices (factor returns) and subsidy: $\{w(t), R(t), S(t) \;\; t \geq 0\}$.

Initial capital stock $k_0 > 0$, is given.

Then the HH budget constraint is

$$(1+\tau_c)\, c(t) + I(t) = (1-\tau_k)\, R(t)k(t) + (1-\tau_n)\, w(t)n(t) + S(t), \quad \text{all } t,$$

and its stock of real assets evolves as

$$\dot{k}(t) = I(t) - \delta k(t), \quad \text{all } t.$$

For simplicity we will combine these (eliminate $I(t)$) to get

$$\dot{k}(t) = [(1-\tau_k)\, R(t) - \delta]\, k(t) + (1-\tau_n)\, w(t)n(t) - (1+\tau_c)\, c(t) + S(t), \quad \text{all } t. \quad (3)$$

The household's problem is to maximize (2) subject to (3).

5. Government

The government balances its budget at all times, so

$$S(t) = \tau_k R(t)k(t) + \tau_n w(t)n(t) + \tau_c c(t), \quad \text{all } t. \quad (4)$$

6. Market clearing

Aggregates: with a continuum of HH and a continuum of firms, each of measure one, per capita quantities—including the subsidy $S(t)$—are also aggregates.

Market clearing for factors of production requires the (k, n) offered by the HH is one the firm chooses.

Market clearing for goods requires

$$c(t) + I(t) = F\,[k(t), n(t)]\,.$$

We will see below that this equation (by Walras Law) is redundant.

7. Competitive Equilibrium

DEFINITION. A **competitive equilibrium**, given the initial capital stock $k_0 > 0$ and tax rates (τ_k, τ_n, τ_c), consists of paths for prices $\{w(t), R(t)\}$, the subsidy $\{S(t)\}$ and quantities $\{c(t), n(t), k(t)\}$ such that:

1. At each date t, the factor input ratio $k(t)/n(t)$ is cost-minimizing for firms, given factor prices $w(t), R(t)$ [in other words, (1) holds];
2. The allocation $\{c(t), n(t), k(t),\ t \geq 0\}$ is utility maximizing for the HH given factor prices and subsidy $\{w(t), R(t), S(t)\}$, the tax rates (τ_k, τ_n, τ_c), and the initial capital stock k_0.

   That is, the allocation $\{c(t), n(t), k(t),\ t \geq 0\}$ maximizes (2) subject to (3).
3. The government's budget is balanced at all dates, so (4) holds.
4. Markets (for factors and output) clear at all dates.

8. The household's problem

The current value Hamiltonian for the household's problem is

$$H = u(c, 1-n) + \lambda \left\{ \left[(1-\tau_k) R - \delta \right] k + (1-\tau_n) wn - (1+\tau_c) c \right\} + S. \tag{5}$$

Hence the FOCs are

$$\begin{aligned} u_1(c, 1-n) - (1+\tau_c)\lambda &= 0, \\ -u_2(c, 1-n) + \lambda (1-\tau_n) w &= 0 \end{aligned} \tag{6}$$

$$\dot{\lambda} - \rho\lambda = -\lambda \left[(1-\tau_k) R - \delta \right],$$

or

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta - (1-\tau_k) R. \tag{7}$$

The LOM (3) for capital must also hold,

$$\dot{k}(t) = \left[(1-\tau_k) R(t) - \delta \right] k(t) + (1-\tau_n) w(t) n(t) - (1+\tau_c) c(t) + S(t), \quad \text{all } t.$$

Also the transversality conditions should hold,

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t) \geq 0, \quad \text{and} \quad \lim_{t\to\infty} e^{-\rho t}\lambda(t) k(t) = 0. \tag{8}$$

If $k(t) \to k^{ss}$ and $\lambda(t) \to \lambda^{ss}$, then the latter condition holds if $\rho > 0$.

9. Competitive equilibrium (again!)

Given a tax policy $(\tau_c, \tau_k, \tau_n,)$ and initial capital stock (per capita) $k_0 > 0$,

a CE consists of $\{c(t), n(t), R(t), w(t), S(t), k(t), \lambda(t),\ t \geq 0\}$ such that:

> $\{w, R\}$ satisfy (1), $\{S\}$ satisfies (4), $\{c, n\}$ satisfy (6), and $\{k, \lambda\}$ satisfy the ODE's (3) and (7), where λ_0 is chosen so that the TCs in (8) hold (the system converges to a steady state).

10. Solving the system

Since F has CRS, we can define

$$f(k) \equiv F(k,1),$$

and write output and factor returns as

$$\begin{aligned} F(k,n) &= nf(k/n), \\ R &= f'(k/n), \\ w &= f(k/n) - (k/n)f'(k/n). \end{aligned}$$

Use these facts, and use (4) to substitute for S, to write (3) and (7) as

$$\begin{aligned} \dot{k}(t) &= n(t)f\left[k(t)/n(t)\right] - \delta k(t) - c(t), \\ \frac{\dot{\lambda}}{\lambda} &= \rho + \delta - (1-\tau_k)\, f'\left[k(t)/n(t)\right], \quad \text{all } t. \end{aligned} \tag{9}$$

Use (6) to define functions

$$c = \Gamma(\lambda), \qquad n = \Omega(\lambda),$$

satisfying

$$\begin{aligned} u_1(\Gamma(\lambda), 1-\Omega(\lambda)) &= (1+\tau_c)\,\lambda, \\ u_2(\Gamma(\lambda), 1-\Omega(\lambda)) &= \lambda\,(1-\tau_n)\,w. \end{aligned} \tag{10}$$

the assumption that both goods are normal implies that $\Gamma(\lambda)$ and $1-\Omega(\lambda)$ are decreasing in λ. A higher MV for capital implies consumption & leisure fall.

11. Characterize the CE

1. show the existence and uniqueness of a steady state;
2. use a phase diagram to describe the qualitative behavior of capital and its costate (and hence quantities and factor returns) way from SS (stability);
3. calculate the speed of adjustment in the neighborhood of steady state;
4. "comparative steady states," i.e., qualitative and quantitative changes in steady state as parameters of the model are changed.