L6: Framework of Reinforcement Learning (I)

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(slides prepared by Tongzhou Mu)

Contents are based on Reinforcement Learning: An Introduction *from Prof. Richard S. Sutton and Prof. Andrew G. Barto, and* COMPM050/COMPGI13 *taught at UCL by Prof. David Silver.*

Agenda

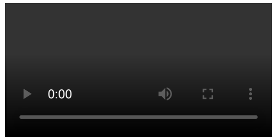
- Examples
- Environment Description and Learning Objective
- Inside an RL Agent

click to jump to the section.

Examples

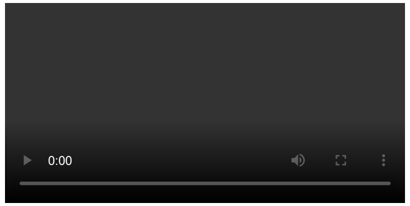
RL Applications

Control a humannoid in Mujoco.



https://gym.openai.com/envs/Humanoid-v2/

Play Atari games.

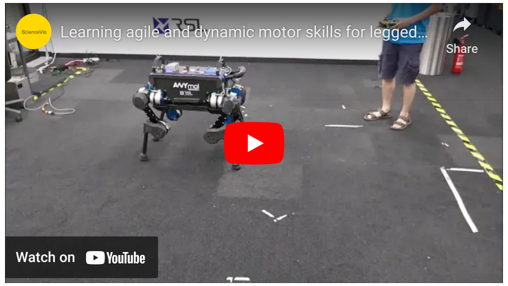


https://gym.openai.com/envs/Enduro-v0/

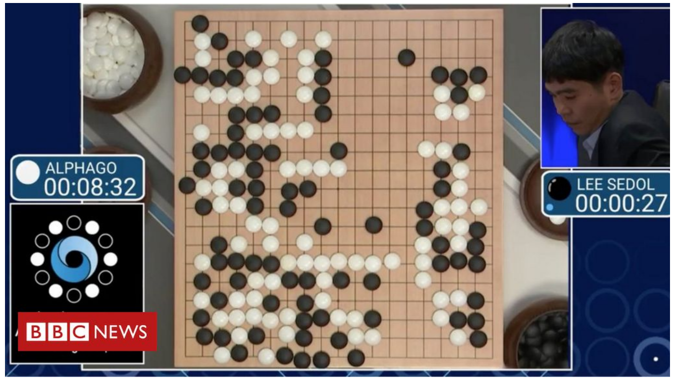
RL Applications

Play Go.

Learn motor skills for legged robots



https://www.youtube.com/watch?v=ITfBKjBH46E



Agent-Environment Interface

- **Agent**: learner and decision maker.
- **Environment**: the thing agent interacts with, comprising everything outside the agent.
- **Action**: how agent interacts with the environment.
- In engineers' terms, they are called controller, controlled system (or plant), and control signal.

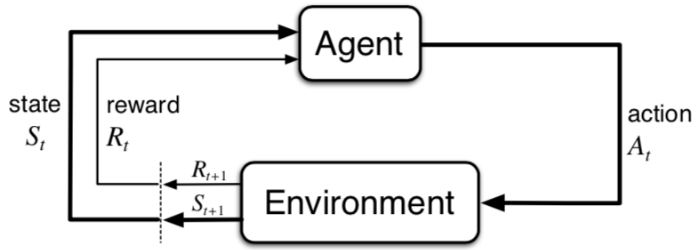
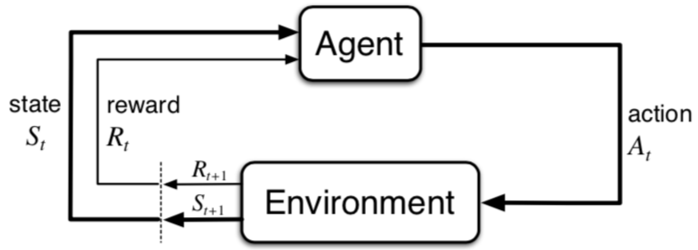


Figure 3.1: The agent–environment interaction in a Markov decision process.

Agent-Environment Interface

- At each step t the agent
  - Executes action A_t
  - Receives state S_t
  - Receives scalar reward R_t
- The environment
  - Receives action A_t
  - Emits state S_{t+1}
  - Emits scalar reward R_{t+1}



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RL: A Sequential Decision Making Problem

- **Goal: select actions to maximize total future reward**
- Actions may have long-term consequences
- Reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward
- Examples:
  - A financial investment (may take months to mature)
  - Refuelling a helicopter (might prevent a crash in several hours)
  - Blocking opponent moves (might help winning chances many moves from now)

Environment Description and Learning Objective

State

- State: A representation of the entire environment, it may contain
  - Description about the external environment
  - Description about the agent
  - Description about the desired task / goal
  - ...
- As in control problems, we can use a vector $\boldsymbol{s} \in \mathbb{R}^n$ to represent the state.
- We can also use advanced data structures, such as images, small video clips, sets, and graphs.

Transition

- State transition functions can be deterministic or stochastic. More generally, we use a stochastic transition function.
- A state transition function is defined as
  - $\mathcal{P}_{s,s'}^{a} = P(s'|s,a) = \Pr(S_{t+1} = s'|S_t = s, A_t = a)$
- $\mathcal{P}$ defines the dynamics of the environment.

Markov Property

- "The future is independent of the past given the present"
- Markov state
    - A state S_t is Markov if and only if
        - $\Pr(S_{t+1}|S_t, A_t) = \Pr(S_{t+1}|S_1, A_1, \ldots, S_t, A_t)$
    - The state captures all relevant information from the history
    - Once the state is known, the history may be thrown away
    - i.e. The state is a sufficient statistic of the future

Observation

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  - A robot with camera vision isn't told its absolute location
  - A trading agent only observes current prices
  - A poker playing agent only observes public cards

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- In RL community, observation and state are sometimes used interchangeably, but "state" is more like to be Markov state, "observation" is more like to be non-Markov state.

Reward

- A reward R_{t+1} is a **scalar random variable** about the feedback signal
  - Indicates how well agent is doing at step t
  - Like a negative concept of "cost" in optimal control
- The agent's job is to maximize cumulative reward
- Examples:
  - Make a humanoid robot walk
    - + reward for forward motion
    - - reward for falling over
  - Playing Go
    - +/− reward for winning/losing a game
  - Manage an investment portfolio
    - + reward for each $ in bank

Probabilistic Description of Environment: Markov Decision Processes

- A Markov decision process (MDP) is a Markov process with rewards and decisions.
- Definition:
- A **Markov decision process** is a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R})$
  - $\mathcal{S}$ is a set of states (discrete or continuous)
  - $\mathcal{A}$ is a set of actions (discrete or continuous)
  - $\mathcal{P}$ is a state transition probability function
    - $\mathcal{P}_{s,s'}^{a} = P(s'|s,a) = \Pr(S_{t+1} = s'|S_t = s, A_t = a)$
  - $\mathcal{R}$ is a reward function
    - $\mathcal{R}_s^a = R(s,a) = \mathbb{E}[R_{t+1}|S_t = s, A_t = a]$
  - Sometimes, an MDP also includes an initial state distribution μ

Probabilistic Description of Environment: Markov Decision Processes

- Markov decision processes formally describe an environment for reinforcement learning
- Almost all RL problems can be formalized as MDPs, e.g.
  - Optimal control primarily deals with continuous MDPs
  - Partially observable problems can be converted into MDPs
  - Bandits are MDPs with one state (we won't discuss this in our class)
- In this course, our RL algorithms are based on the MDP assumption (i.e., fully observable states).

Return

- Infinite-horizon return (total discounted reward) from time-step t (for a given policy):
  - $G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$
  - $\gamma \in [0, 1]$: discount factor
  - Note: G_t is a **random variable**, because reward is a random variable
  - *Does it remind you the concept of "cost-to-go" function?*

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- Most Markov reward and decision processes are discounted. Why?
  - Mathematically, total reward gets bounded (if step rewards are bounded).
  - Uncertainty about the future may not be fully represented
  - Animal/human behaviour shows preference for immediate reward

Episode

- Agent-environment interaction usually breaks naturally into subsequences, which we call episodes, e.g.,
  - plays of a game
  - trips through a maze
- Termination of an episode
  - Each episode ends in a special state called the *terminal state*, followed by a **reset** to a standard starting state or to a sample from a standard distribution of starting states.
  - Even if you think of episodes as ending in different ways, such as winning and losing a game, the next episode begins independently of how the previous one ended.
  - **The time of termination**, T, is a random variable that normally varies from episode to episode.
- Tasks with episodes of this kind are called *episodic tasks*.
- In episodic tasks, returns will be truncated to finite-horizon.

Learning Objective of RL

- Formally, the objective of an RL agent is to maximize its *expected return*
- Given an MDP, find a policy π to maximize the expected return induced by π
  - We use τ to denote a trajectory $s_0, a_0, r_1, s_1, a_1, r_2, ...$ generated by π
    - The conditional probability of τ given π is

$$\begin{aligned}
\Pr(\tau|\pi) &= \Pr(s_0, a_0, r_1, s_1, a_1, r_2, ...|\pi) \\
&= \Pr(S_0 = s_0)\Pr(a_0, r_1, s_1, a_1, r_2, ...|\pi, s_0) \\
&= \Pr(S_0 = s_0)\Pr(a_0, r_1, s_1|\pi)\Pr(a_1, r_2, ...|\pi, s_1) \qquad \text{// by Markovian property} \\
&= \Pr(S_0 = s_0)\pi(a_0|s_0)P(s_1|s_0, a_0)\Pr(r_1|s_0, a_0)\Pr(a_1, r_2, ...|\pi, s_1) \\
&= \ ... \\
&= \Pr(S_0 = s_0)\prod_t \pi(a_t|s_t)P(s_{t+1}|s_t, a_t)\Pr(r_{t+1}|s_t, a_t)
\end{aligned}$$

Learning Objective of RL

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- Given an MDP, find a policy π to maximize the expected return induced by π
  - We use τ to denote a trajectory $s_0, a_0, r_1, s_1, a_1, r_2, ...$ generated by π
  - Optimization problem: $\max_\pi J(\pi)$

$$
\begin{aligned}
J(\pi) &= \mathbb{E}_{\tau \sim \pi}[R_1 + \gamma R_2 + ...] \\
&= \sum_\tau \Pr(\tau|\pi)(r_1 + \gamma r_2 + ...) \\
&= \sum_\tau \left(\Pr(S_0 = s_0) \prod_t \left(\pi(a_t|s_t) P(s_{t+1}|s_t, a_t) \Pr(r_{t+1}|s_t, a_t) \right) (r_1 + \gamma r_2 + ...) \right)
\end{aligned}
$$

Data Collection in Supervised Learning and Reinforcement Learning

- In supervised learning,
  - A dataset $\mathcal{D} = \{(x_i, y_i)\}$ is usually given and fixed
  - where x_i is input of a data sample, y_i is the corresponding label
- In reinforcement learning,
  - The "dataset" $\mathcal{D} = \{(s_t, a_t, r_{t+1}, s_{t+1})\}$ is sampled by the agent itself by its policy π
  - And the data distribution will shift according to the change of π
- This difference introduces a core problem in RL: exploration, which we will elaborate later in this course.

Relationship between Optimal Control and Reinforcement Learning

- Optimal Control
  - Controller
  - Controlled System
  - Control Signal
  - State
  - Cost
  - Cost-to-go function
- Reinforcement Learning
  - Agent
  - Environment
  - Action
  - State / Observation
  - Reward
  - Return
- Differences
  - Environment dynamics is usually known in optimal control, but likely to be unknown in RL.
  - RL extends the ideas from optimal control to non-traditional control problems.
  - RL is more data-driven while optimal control is model-driven.

Inside an RL Agent

Major Components of an RL Agent

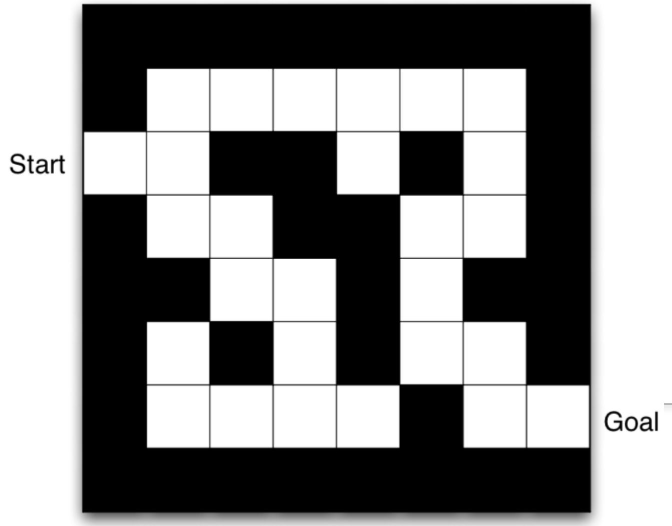
- An RL agent may include one or more of these components:
  - **Model**: agent's representation of the environment
  - **Policy**: agent's behaviour function
  - **Value function**: how good is each state and/or action

Model

- In RL community, the term "model" has a specific meaning
- A **model** predicts what the environment will do next
- $\mathcal{P}$ predicts the next state
  - $\mathcal{P}^a_{s,s'} = \Pr(S_{t+1} = s' | S_t = s, A_t = a)$
  - Sometimes this is also called *dynamics model*
- $\mathcal{R}$ predicts the next (immediate) reward
  - $\mathcal{R}^a_s = \mathbb{E}[R_{t+1} | S_t = s, A_t = a]$
  - Sometimes this is also called *reward model*
- If the agent maintains a model of the environment to learn policies and value, we call its learning method is *model-based*.
- It is also possible for the agents to learn about policies and environments without an environment model. Then, it is called *model-free*.

Maze Example

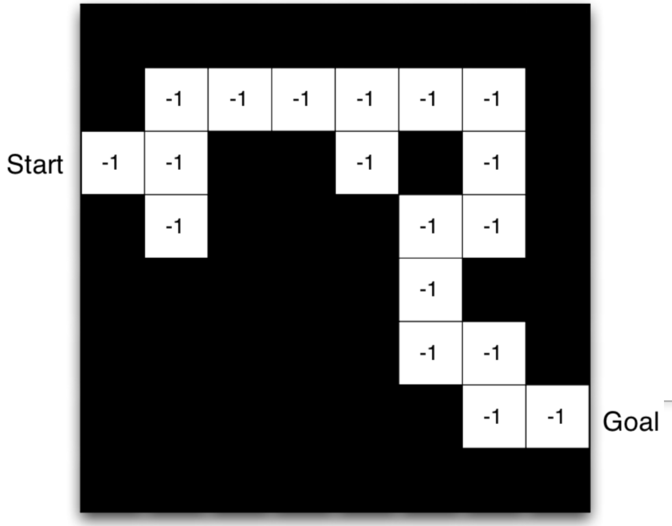
- States: Agent's location
- Actions: N, E, S, W, stay
- Reward: -1 per time-step
- Termination: Reach goal



Maze Example: Model

- Agent may have an internal model of the environment
- Dynamics: how actions change the state
- Rewards: how much reward from each state

- In the right figure:
  - Grid layout represents transition model $\mathcal{P}^a_{s,s'}$
  - Numbers represent immediate reward $\mathcal{R}^a_s$ from each state s (same for all a)

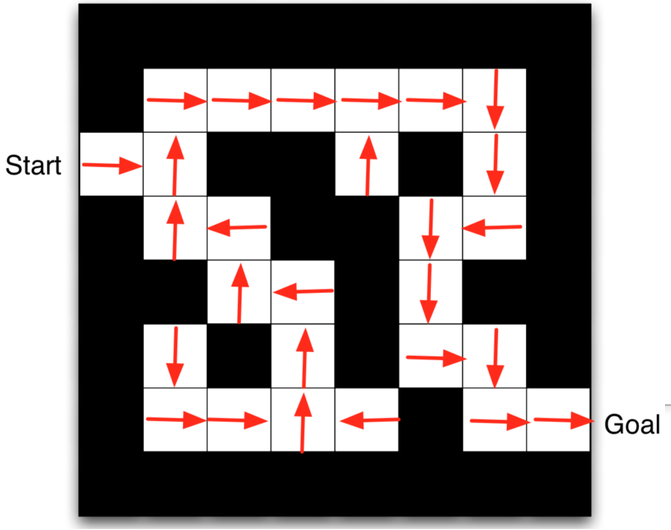


Policy

- A policy is the agent's behaviour
- It is a map from state to action, e.g.,
  - Deterministic policy: $a = \pi(s)$
  - Stochastic policy: $\pi(a|s) = \Pr(A_t = a|S_t = s)$

Maze Example: Policy

- Arrows represent policy $\pi(s)$ for each state s
- This is the optimal policy for this Maze MDP



Value Function

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- **State-value function**
  - The state-value function $V_\pi(s)$ of an MDP is the expected return starting from state s, following the policy π
  - $V_\pi(s) = \mathbb{E}_\pi[G_t | S_t = s]$ (assuming infinite horizon here)

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  - $Q_\pi(s, a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a]$ (assuming infinite horizon here)
- Notation explanation:
  - In this lecture, when we write $\mathbb{E}_\pi$, it means we take expectation over all samples/trajectories generated by running the policy π in the environment
  - So it counts for all the randomness from policy, initial state, state transition, and reward.

Bellman Expectation Equation

- Value functions satisfy recursive relationships:

$$V_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma V_\pi(S_{t+1}) | S_t = s] \quad \text{(Bellman expectation equation)}$$

Proof:

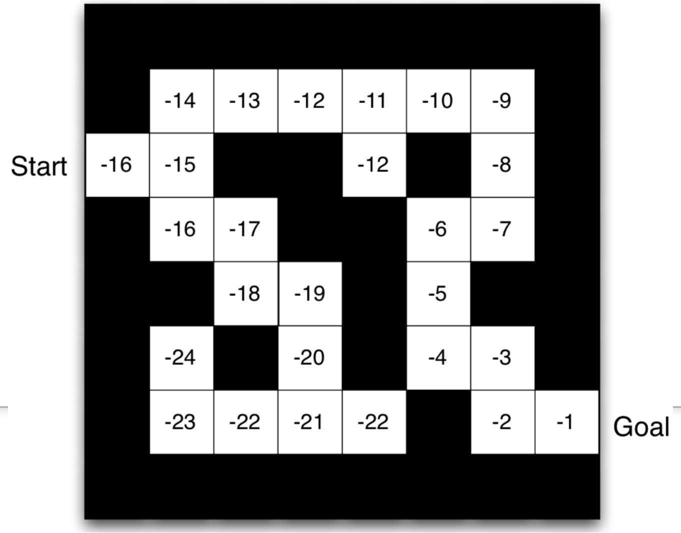
$$\begin{aligned} V_\pi(s) &= \mathbb{E}_\pi[G_t | S_t = s] = \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots | S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma(R_{t+2} + \gamma R_{t+3} + \ldots) | S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} | S_t = s] \end{aligned}$$

- The value function can be decomposed into two parts:
  - immediate reward R_{t+1}
  - discounted value of successor state $\gamma V_\pi(S_{t+1})$
- The action-value function can similarly be decomposed:

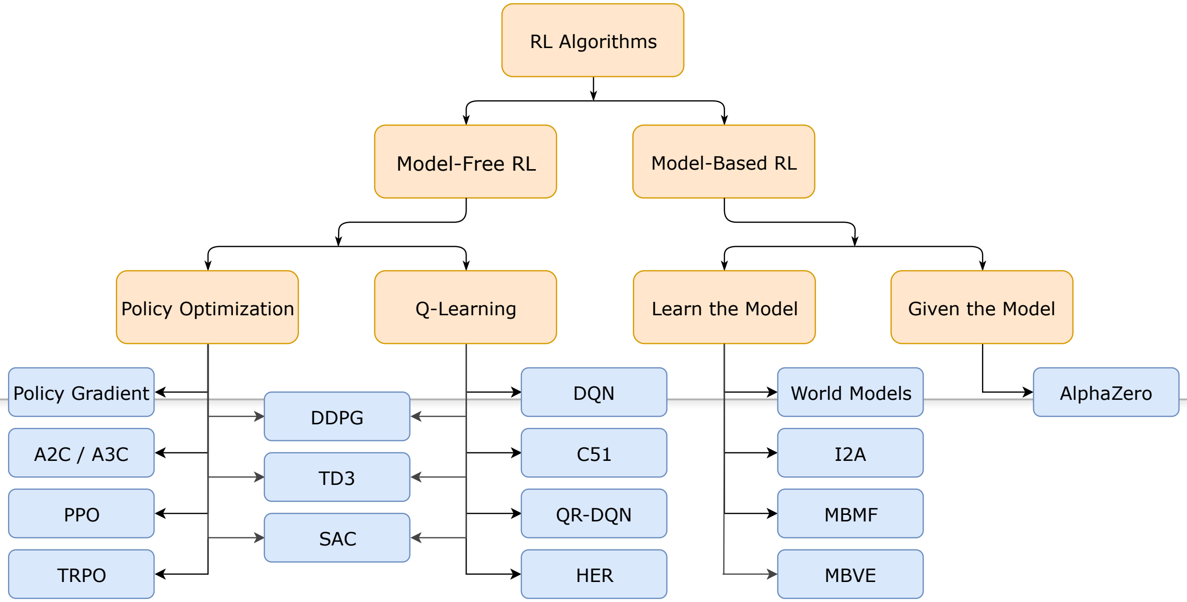
$$Q_\pi(s, a) = \mathbb{E}_\pi[R_{t+1} + \gamma Q_\pi(S_{t+1}, A_{t+1}) | S_t = s, A_t = a]$$

Maze Example: Value Function

- Numbers represent value $V_\pi(s)$ of each state s
- This is the value function corresponds to the optimal policy we showed previously



A Taxonomy of RL Algorithms and Examples



OpenAI Spinning Up

End