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HW#8

(1) $A = \begin{bmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{bmatrix}$ $B = \begin{bmatrix} b_{11}, & b_{12} \\ b_{21}, & b_{22} \end{bmatrix}$

Traditional method.

$C_{11} = a_{11} b_{11} + a_{12} b_{21}$

$C_{12} = a_{11} b_{12} + a_{12} b_{22}$

$C_{21} = a_{21} b_{11} + a_{22} b_{21}$

$C_{22} = a_{21} b_{12} + a_{22} b_{22}$

Strassen method

$X_1 = (a_{11}+a_{22}) * (b_{11}+b_{22})$

$X_2 = (a_{21}+a_{22}) * b_{11}$

$X_3 = a_{11} * (b_{12}-b_{22})$

$X_4 = a_{22} * (b_{21}-b_{11})$

$X_5 = (a_{11}+a_{12}) * b_{22}$

$X_6 = (a_{21}-a_{11}) * (b_{11}+b_{12})$

$X_7 = (a_{12}-a_{22}) * (b_{21}+b_{22})$

$C_{11} = X_1 + X_4 - X_5 + X_7 = (a_{11}+a_{22}) * (b_{11}+b_{22}) + a_{22} * (b_{21}-b_{11}) - (a_{11}+a_{12}) * b_{22} + (a_{12}-a_{22}) * (b_{21}+b_{22})$

$= a_{11} b_{11} + \cancel{a_{11} b_{22}} + \cancel{a_{22} b_{11}} + \cancel{a_{22} b_{22}}$
$+ \cancel{a_{22} b_{21}} - \cancel{a_{22} b_{11}} - \cancel{a_{11} b_{22}} - \cancel{a_{12} b_{22}}$
$+ a_{12} b_{21} + \cancel{a_{12} b_{22}} - \cancel{a_{22} b_{21}} - \cancel{a_{22} b_{22}}$

$= a_{11} b_{11} + a_{12} b_{21}$

$C_{12} = X_3 + X_5$

$$C_{12} = a_{11}(b_{12} - b_{22}) + (a_{11} + a_{12})b_{22}$$
$$= a_{11}b_{12} \cancel{- a_{11}b_{22}} \cancel{+ a_{11}b_{22}} + a_{12}b_{22}$$
$$= a_{11}b_{12} + a_{12}b_{22}$$

$$C_{21} = (a_{21} + a_{22})b_{11} + a_{22}(b_{21} - b_{11})$$
$$= a_{21}b_{11} + \cancel{a_{22}b_{11}} + a_{22}b_{21} \cancel{- a_{22}b_{11}}$$
$$= a_{21}b_{11} + a_{22}b_{21}$$

$$C_{22} = (a_{11} + a_{22})(b_{11} + b_{22}) + a_{11}(b_{12} - b_{22}) +$$
$$- (a_{21} + a_{22})b_{11} + (a_{21} - a_{11})(b_{11} + b_{12})$$
$$= \cancel{a_{11}b_{11}} + \cancel{a_{11}b_{22}} + \cancel{a_{22}b_{11}} + a_{22}b_{22} + \cancel{a_{11}b_{12}} \cancel{- a_{11}b_{22}}$$
$$\cancel{- a_{21}b_{11}} \cancel{- a_{22}b_{11}} + \cancel{a_{21}b_{11}} + a_{21}b_{12} \cancel{- a_{11}b_{11}} \cancel{- a_{11}b_{12}}$$
$$= a_{21}b_{12} + a_{22}b_{22}$$

$\therefore$ Traditional method & normal method get the same result

② $T(n) = 7T(n/2) + n^2$ $\quad n = 2^k$

$T(n/2) = 7T(\frac{n}{4}) + (\frac{n}{2})^2 \Rightarrow T(n) = 7\cdot\left(7T(\frac{n}{4}) + (\frac{n}{2})^2\right) + n^2$

$= 7^2 T(\frac{n}{4}) + 7(\frac{n}{2})^2 + n^2$

$T(\frac{n}{4}) = 7T(\frac{n}{8}) + (\frac{n}{4})^2$

$T(n) = 7^2\left(7T(\frac{n}{8}) + (\frac{n}{4})^2\right) + 7(\frac{n}{2})^2 + n^2$

$= 7^3 T(\frac{n}{8}) + 7^2(\frac{n}{4})^2 + 7(\frac{n}{2})^2 + n^2$

$= 7^k T(\frac{n}{2^k}) + \sum_{i=0}^{k-1} 7^i (\frac{n}{2^i})^2$

$= 7^k T(\frac{n}{2^k}) + n^2 \sum_{i=0}^{k-1} (\frac{7}{4})^i$

$= 7^k T(\frac{n}{2^k}) + n^2 \sum_{i=1}^{k} (\frac{7}{4})^{i-1}$

$\Downarrow$

$= 7^k T(\frac{n}{2^k}) + n^2 \frac{(\frac{7}{4})^k - 1}{7/4 - 1}$

$n = 2^k$

$\frac{n}{2^k} = 1 \quad = n = 2^k \quad k = \log n$

$= 7^{\log n} + n^2 \frac{(\frac{7}{4})^{\log n} - 1}{(3/4)}$

$= n^{\log 7} + n^2 \frac{n^{\log(7/4)} - 1}{3/4} = \boxed{n^{0.85} + \frac{n^2(n^{0.24} - 1)}{3/4}}$

③ $M(n) = 7M(n/2)$ $M(1) = 1$

$M(n/2) = 7M\left(\frac{n}{4}\right) \Rightarrow M(n) = 7^2 M\left(\frac{n}{4}\right)$

$M\left(\frac{n}{4}\right) = 7M\left(\frac{n}{8}\right) \Rightarrow M(n) = 7^3 M\left(\frac{n}{8}\right)$

$M(n) = 7^k M\left(\frac{n}{2^k}\right)$

$n/2^k = 1 \Rightarrow n = 2^k = k = \log n$

$M(n) = 7^{\log n} \; 1$

$= n^{\log 7}$

$\boxed{M(n) = n^{0.85}}$