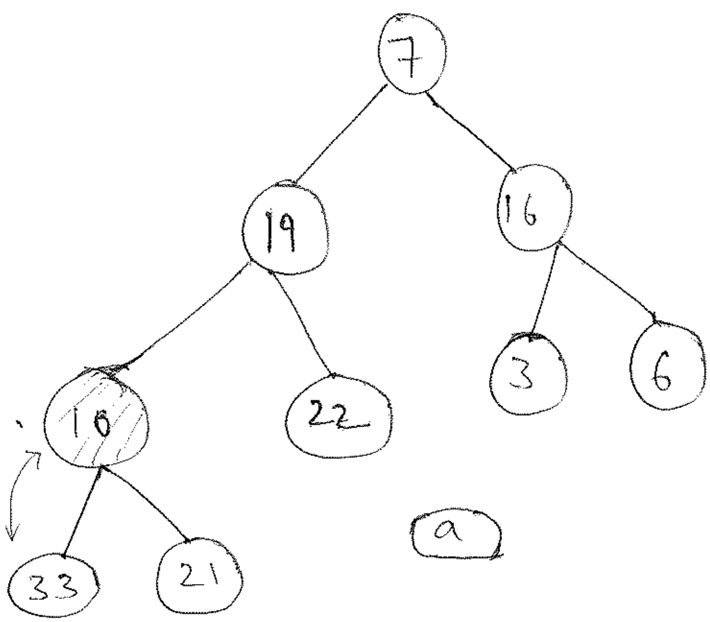
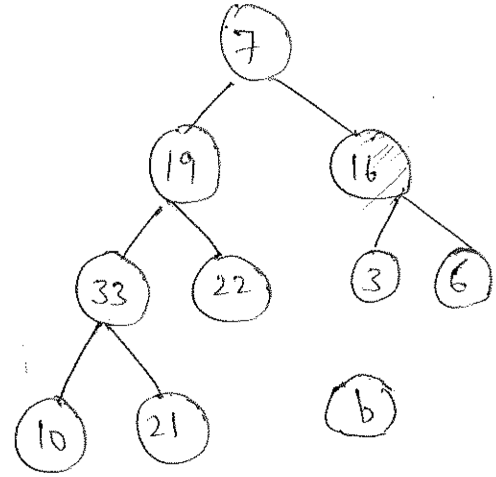
HW #4 Solutions

1. A =

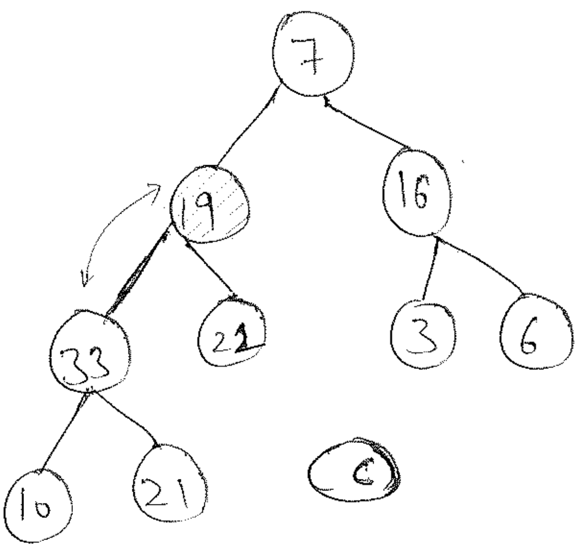
7	19	16	10	22	3	6	33	21



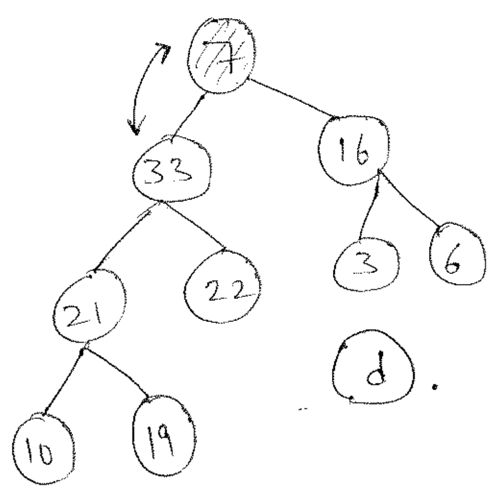
a



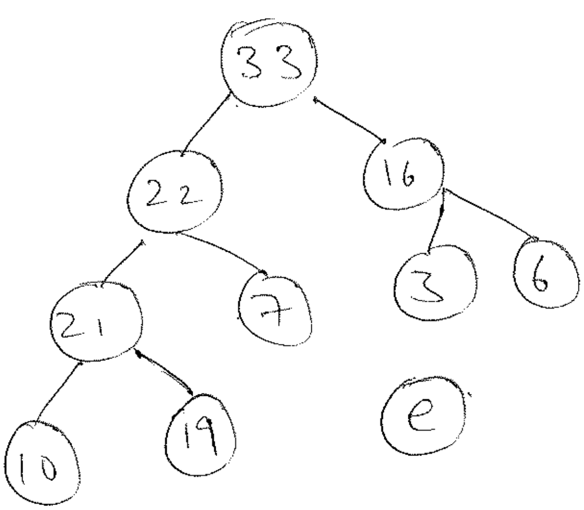
b



c



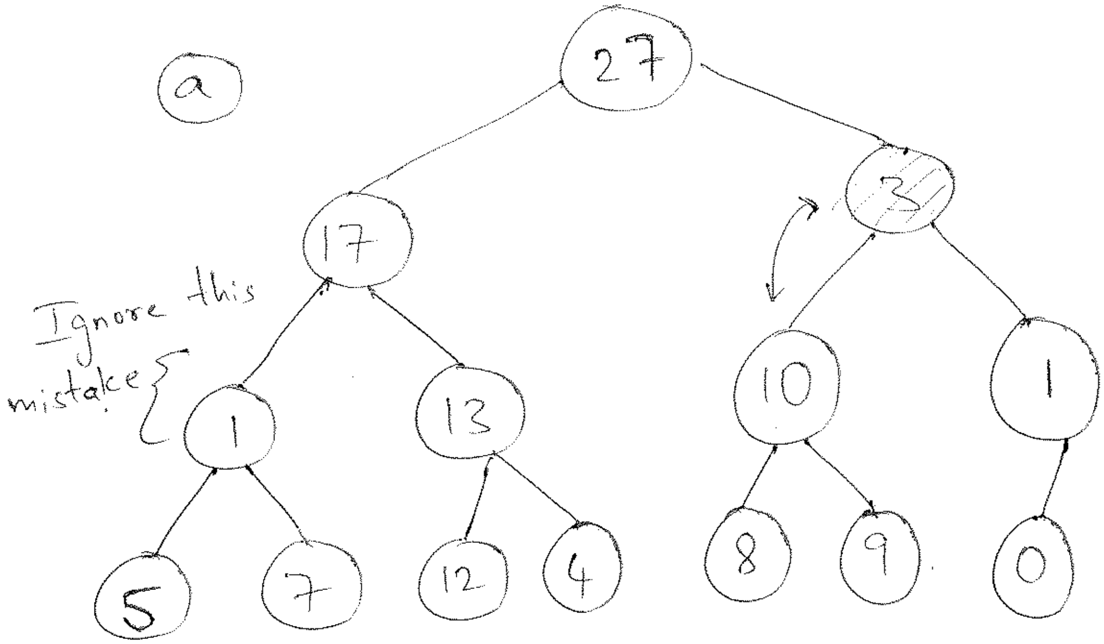
d



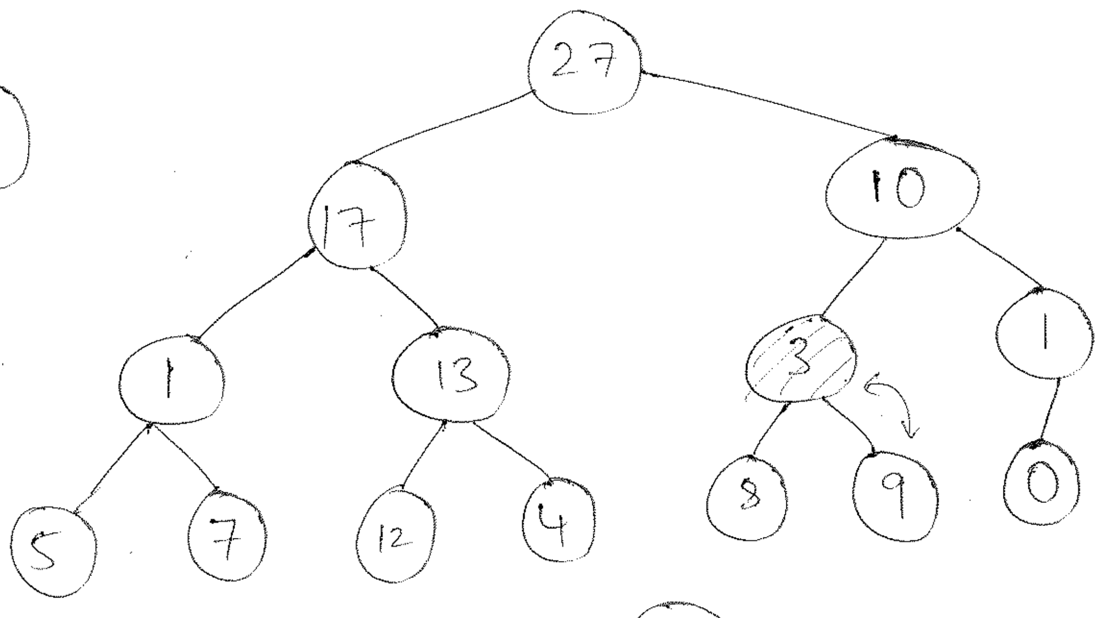
e

2. $A = \langle 27, 17, 3, 1, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 \rangle$ 2

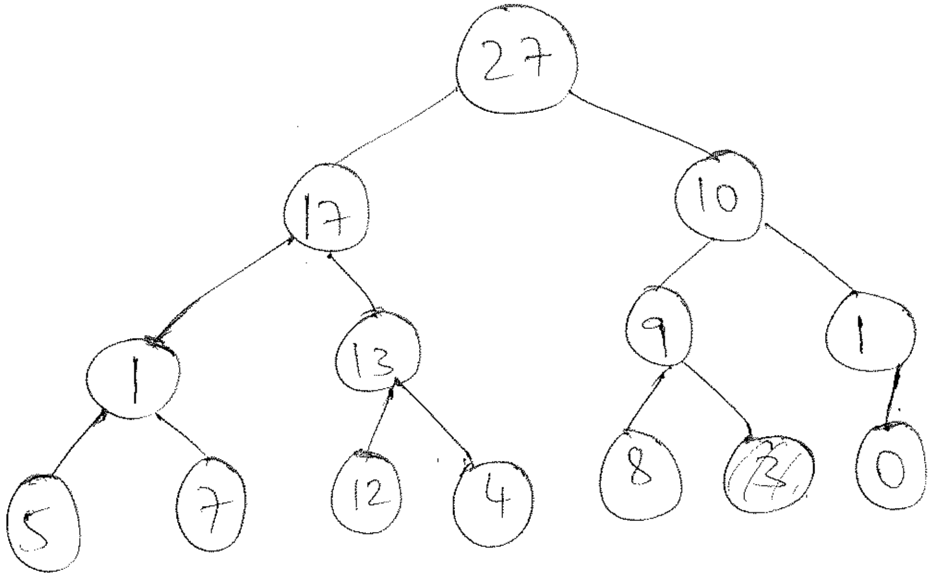
a



b



c



(b) $T(n) = 2T\left(\frac{n}{4}\right) + n^2$, $T(1) = 1$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{4^2}\right) + \left(\frac{n}{4}\right)^2$$

$$T(n) = 2\left(2T\left(\frac{n}{4^2}\right) + \frac{n^2}{4^2}\right) + n^2 = 2^2T\left(\frac{n}{4^2}\right) + 2\cdot\frac{n^2}{4^2} + n^2.$$

$$T\left(\frac{n}{4^2}\right) = 2T\left(\frac{n}{4^3}\right) + \left(\frac{n}{4^2}\right)^2$$

$$T\left(\frac{n}{4^2}\right) = 2T\left(\frac{n}{4^3}\right) + \frac{n^2}{4^4}.$$

$$T(n) = 2^2\left(2T\left(\frac{n}{4^3}\right) + \frac{n^2}{4^4}\right) + 2\cdot\frac{n^2}{4^2} + n^2.$$

$$= 2^3T\left(\frac{n}{4^3}\right) + 2^2\cdot\frac{n^2}{4^4} + 2\cdot\frac{n^2}{4^2} + n^2$$

$$T\left(\frac{n}{4^3}\right) = 2T\left(\frac{n}{4^4}\right) + \frac{n^2}{4^6}.$$

$$T(n) = 2^3\left(2T\left(\frac{n}{4^4}\right) + \frac{n^2}{4^6}\right) + 2^2\cdot\frac{n^2}{4^4} + 2\cdot\frac{n^2}{4^2} + n^2$$

$$= 2^4T\left(\frac{n}{4^4}\right) + 2^3\cdot\frac{n^2}{4^6} + 2^2\cdot\frac{n^2}{4^4} + 2\cdot\frac{n^2}{4^2} + n^2.$$

$$T(n) = 2^r\cdot T\left(\frac{n}{4^r}\right) + n^2\sum_{i=1}^{r}\frac{2^{i-1}}{4^{2(i-1)}}$$

(c) $T(n) = 3T(n/2) + 5 \cdot n \,, \; T(1) = 1$

$$T\left(\frac{n}{2}\right) = 3T\left(\frac{n}{2^2}\right) + 5 \cdot \frac{n}{2}$$

$$T(n) = 3\left(3T\left(\frac{n}{2^2}\right) + 5 \cdot \frac{n}{2}\right) + 5 \cdot n$$

$$= 3^2 T\left(\frac{n}{2^2}\right) + 3 \cdot 5 \cdot \frac{n}{2} + 5 \cdot n$$

$$T\left(\frac{n}{2^2}\right) = 3T\left(\frac{n}{2^3}\right) + 5 \cdot \frac{n}{2^2}$$

$$T(n) = 3^2\left(3T\left(\frac{n}{2^3}\right) + 5 \cdot \frac{n}{2^2}\right) + 3 \cdot 5 \cdot \frac{n}{2} + 5 \cdot n.$$

$$= 3^3 T\left(\frac{n}{2^3}\right) + 3^2 \cdot 5 \cdot \frac{n}{2^2} + 3 \cdot 5 \cdot \frac{n}{2} + 5 \cdot n$$

$$= 3^r T\left(\frac{n}{2^r}\right) + \frac{3^{r-1}}{2^{r-1}} \cdot 5n + \ldots$$

$$= 3^r T\left(\frac{n}{2^r}\right) + 5n \sum_{i=0}^{r-1} \left(\frac{3}{2}\right)^i.$$

$a = 1, \; r = \frac{3}{2}$

$$S_r = \frac{1(1.5^r - 1)}{0.5}$$

$$= 2(1.5^r - 1)$$

$$n = 2^k, \quad \frac{n}{2^r} = 1 \Rightarrow \boxed{k = r}$$

$$= 3^{\log n} + 5n\left(2(1.5^r - 1)\right) = 3^{\log n} + 10n\left(1.5^{\log n} - 1\right)$$

$$= n^{\log 3} + 10n\left(n^{\log 1.5} - 1\right).$$

$$= O(n^{1.58}).$$

$$\frac{n}{2^{\gamma}} = 1 \quad , \quad n = 2^{k}$$

$$\boxed{k = \gamma}$$

$$= \cancel{2^{\log n}} + n\log n - \cancel{2^{\log n}} + 1$$

$$= \cancel{2n + n\log n + 1} = n\log n + 1$$

$$= O(n\log n).$$