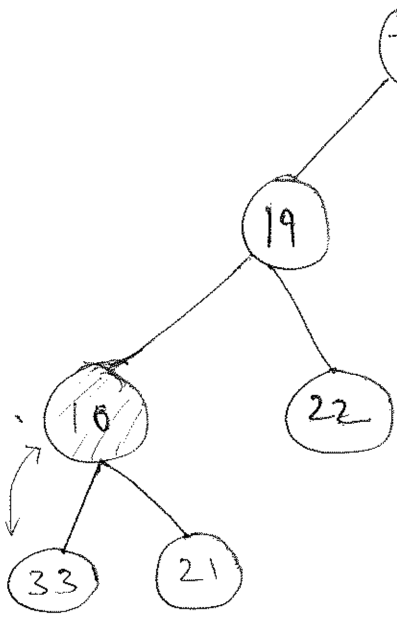
# HW #4 Solutions

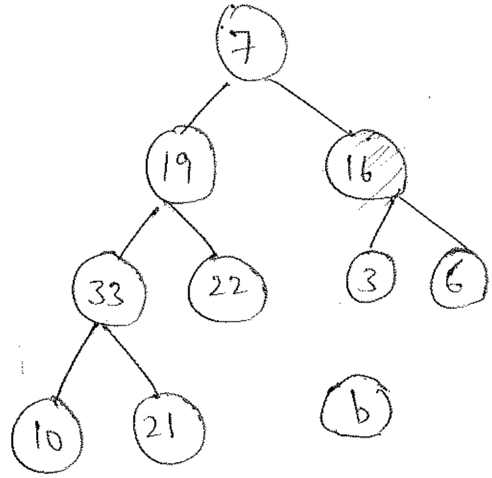


1. A =

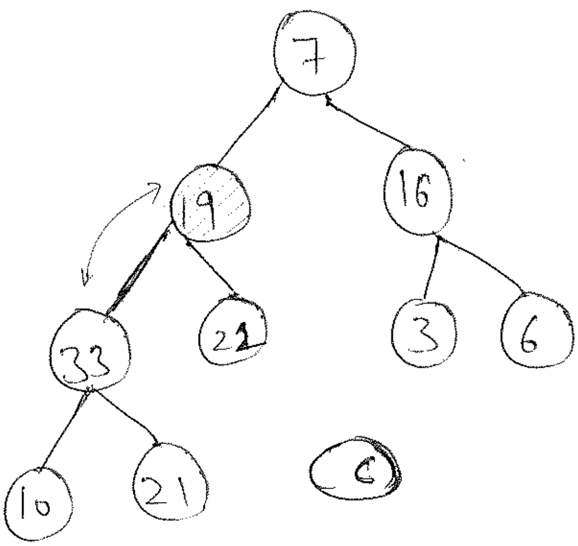
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 19 | 16 | 10 | 22 | 3 | 6 | 33 | 21 |



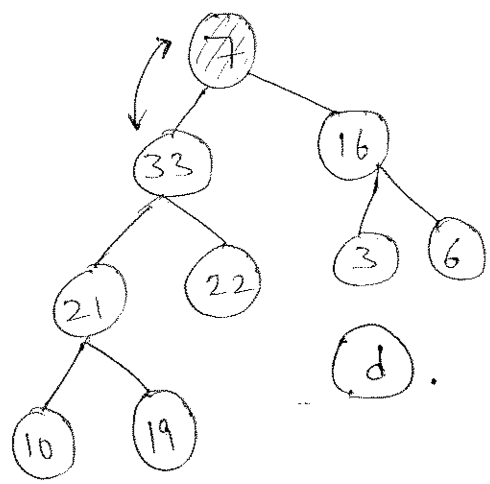
a



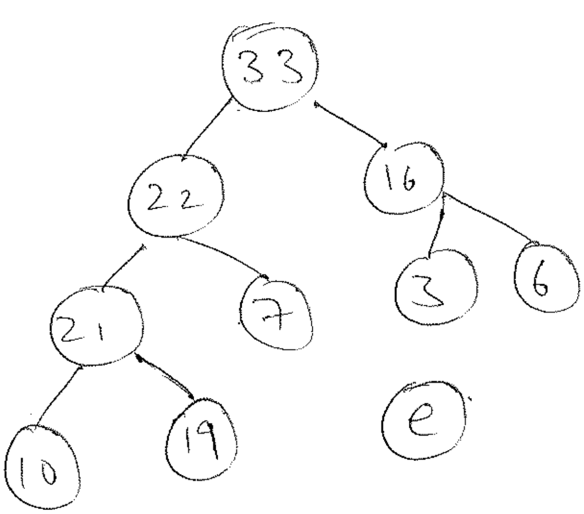
b



c

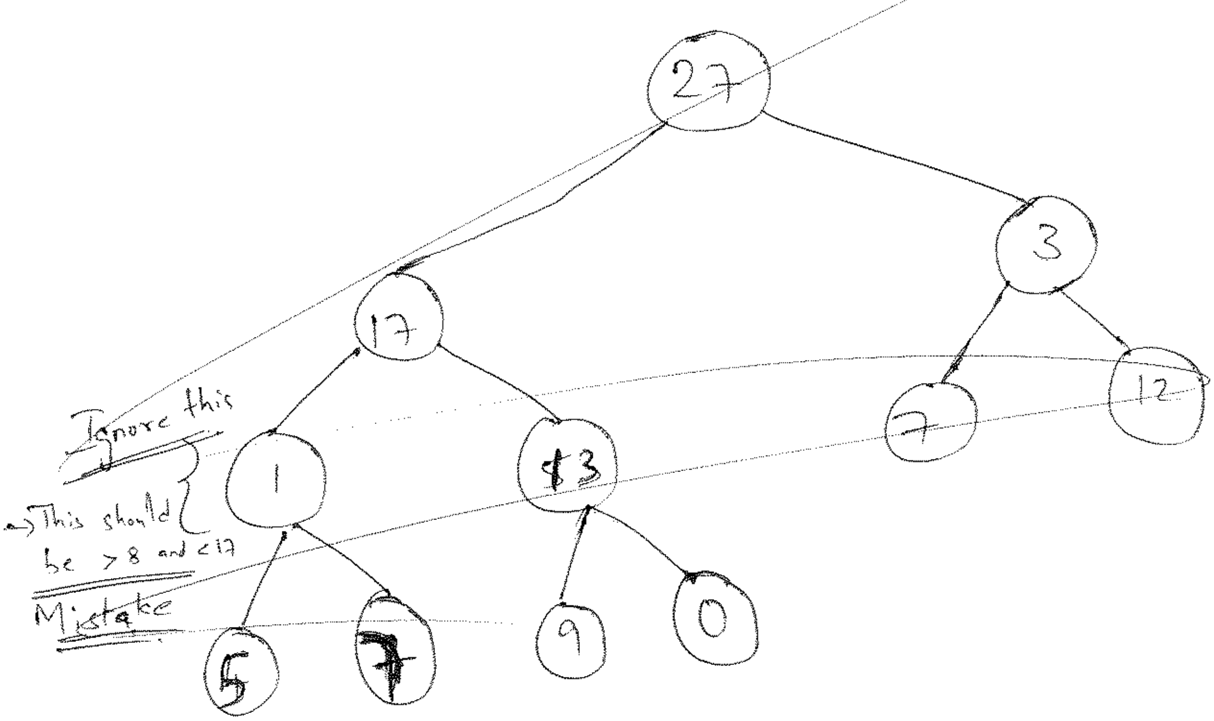


d



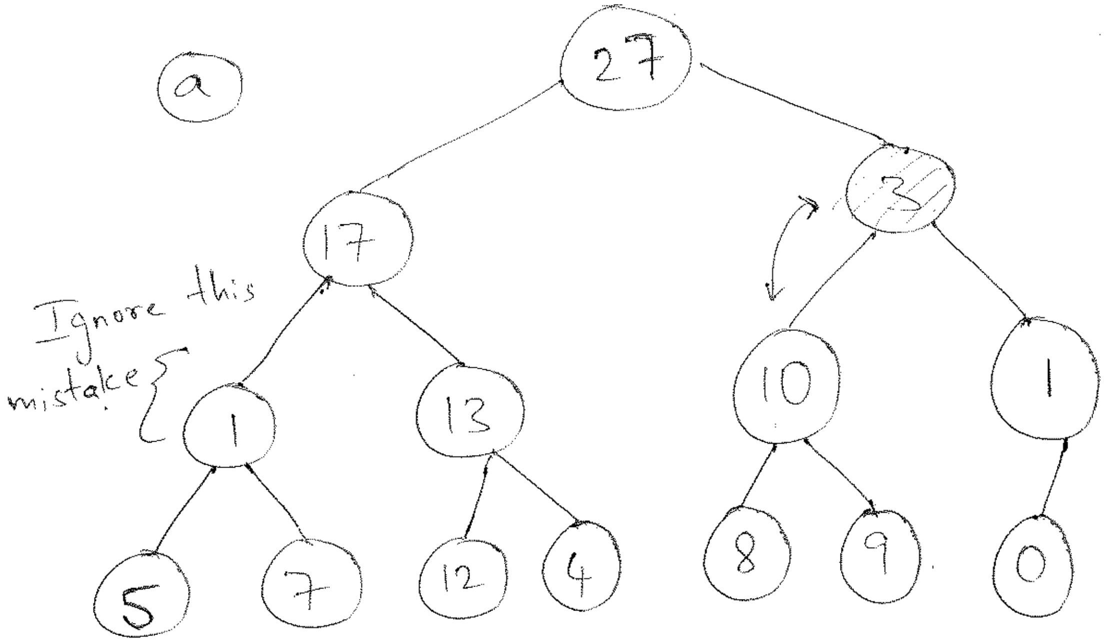
e

2. (A, 3)  A = ⟨27, 17, 3, 1, 5, 7, 7, 12, 4, 8, 9, 0⟩.

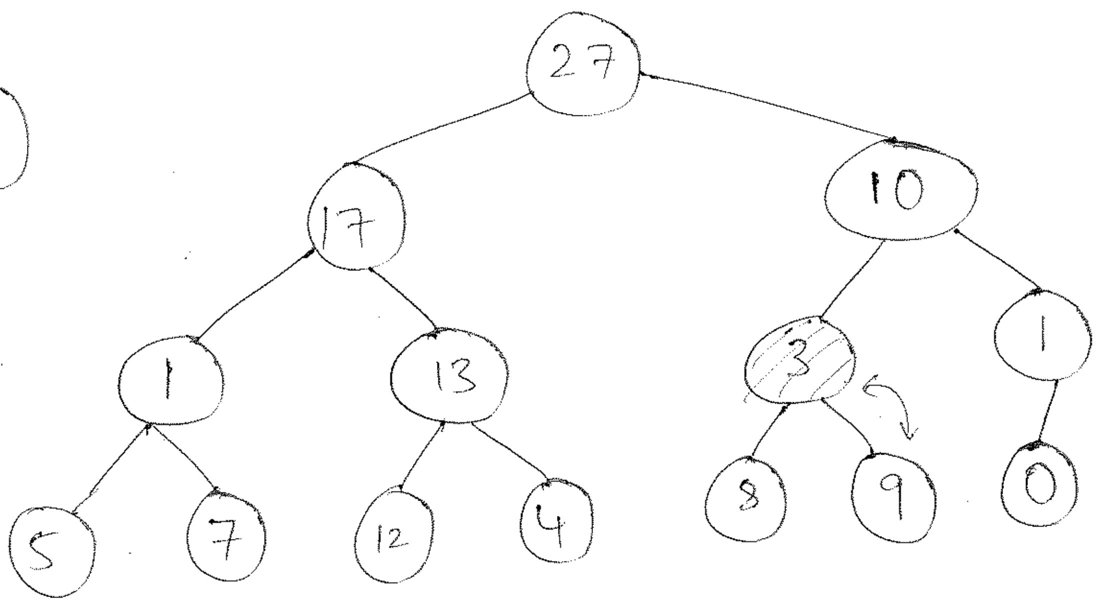


2. $A = \langle 27, 17, 3, 1, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 \rangle$ 2

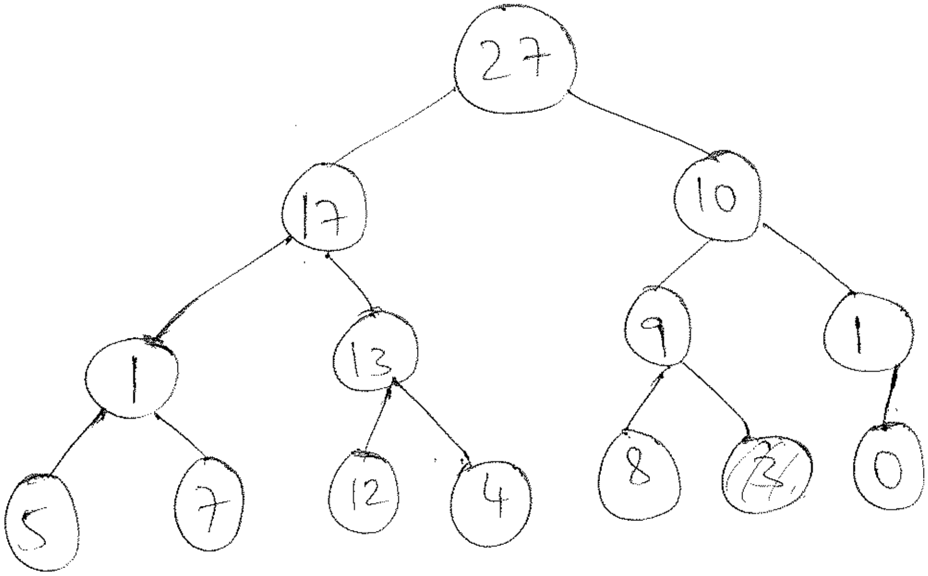
a




b




c



3. Given in Appendix - A. (Module - 1).

$$A(n) = \frac{2}{n}\sum_{i=2}^{n-1} A(i) + (n-1)$$

$$\vdots$$

$$A(n) = O(n\log n).$$

4. a) $T(n) = 2T(n/4) + 1, \quad T(1) = 1$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{4^2}\right) + 1.$$

$$T(n) = 2\left(2T\left(\frac{n}{4^2}\right) + 1\right) + 1$$

$$= 2^2\, T\left(\frac{n}{4^2}\right) + 2 + 2^0.$$

$$T(n) = 2^r \cdot T\left(\frac{n}{4^r}\right) + \sum_{i=0}^{r-1} 2^i$$

Assume $n = 2^k$, we know $\frac{n}{4^r} = 1$

$\boxed{k = \log_2 n}$

$\boxed{r = \log_4 n}$

$$\Rightarrow 2^k = 4^r.$$

$$\boxed{k = 2r}$$

$$= 2^{\log_4 n}(1) + (2^r - 1).$$

$$= 2^{\log_4 n} + 2^{\log_4 n} - 1$$

$$= 2n^{\log_4 2} - 1 = 2n^{\frac{1}{2}} - 1$$

$$= O(\sqrt{n})$$

$a = 1, \; r = 2$

$$S_r = \frac{a(r^r - 1)}{r - 1}$$

$$= \frac{1(2^r - 1)}{2 - 1}$$

$$= (2^r - 1).$$

$\sum_{i=0}^{r-1} 2^i \rightarrow$ contains total $r$ terms with $a = 1, r = 2$.

$$n = 4^r$$

$$n = 2^{2r}$$

$$2^{\log_4 n} = n^{\log_4 2}$$

$$n^{\log_4 2} = n^{\frac{1}{\log_2 4}} = n^{\frac{1}{2}}$$

(b) $T(n) = 2T\left(\frac{n}{4}\right) + n^2$, $T(1) = 1$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{4^2}\right) + \left(\frac{n}{4}\right)^2$$

$$T(n) = 2\left(2T\left(\frac{n}{4^2}\right) + \frac{n^2}{4^2}\right) + n^2 = 2^2T\left(\frac{n}{4^2}\right) + 2\cdot\frac{n^2}{4^2} + n^2$$

$$T\left(\frac{n}{4^2}\right) = 2T\left(\frac{n}{4^3}\right) + \left(\frac{n}{4^2}\right)^2$$

$$T\left(\frac{n}{4^2}\right) = 2T\left(\frac{n}{4^3}\right) + \frac{n^2}{4^4}$$

$$T(n) = 2^2\left(2T\left(\frac{n}{4^3}\right) + \frac{n^2}{4^4}\right) + 2\cdot\frac{n^2}{4^2} + n^2$$

$$= 2^3T\left(\frac{n}{4^3}\right) + 2^2\cdot\frac{n^2}{4^4} + 2\cdot\frac{n^2}{4^2} + n^2$$

$$T\left(\frac{n}{4^3}\right) = 2T\left(\frac{n}{4^4}\right) + \frac{n^2}{4^6}$$

$$T(n) = 2^3\left(2T\left(\frac{n}{4^4}\right) + \frac{n^2}{4^6}\right) + 2^2\cdot\frac{n^2}{4^4} + 2\cdot\frac{n^2}{4^2} + n^2$$

$$= 2^4T\left(\frac{n}{4^4}\right) + 2^3\cdot\frac{n^2}{4^6} + 2^2\cdot\frac{n^2}{4^4} + 2\cdot\frac{n^2}{4^2} + n^2$$

$$T(n) = 2^r\cdot T\left(\frac{n}{4^r}\right) + n^2\sum_{i=1}^{r}\frac{2^{i-1}}{4^{2(i-1)}}$$

$$= 2^{r} T\left(\frac{n}{4^{r}}\right) + n^{2} \cdot \sum_{i=1}^{r} \frac{2^{i-1}}{4^{2(i-1)}}$$

$$= n^{\frac{1}{2}}(\text{from } 4(a)) + n^{2} \cdot \sum_{i=1}^{r} \frac{2^{i-1}}{4^{2(i-1)}}.$$

$a = 1$ (first term in G.P)

common ratio $= \frac{2}{4^2} = \frac{1}{8}$.

$$S_r = \frac{1\left(1-\left(\frac{1}{8}\right)^{r}\right)}{1-\frac{1}{8}} = \frac{8}{7}\left(1-\left(\frac{1}{8}\right)^{r}\right).$$

$$= \frac{8}{7}\left(1-\left(\frac{1}{8}\right)^{\log_4 n}\right)$$

$$= \frac{8}{7}\left(1 - n^{\log_4\left(\frac{1}{8}\right)}\right)$$

$$= \frac{8}{7}\left(1 - n^{-\log_4 8}\right) = \frac{8}{7}\left(1-n^{-1.5}\right)$$

$$= n^{\frac{1}{2}} + n^{2}\left(\frac{8}{7}\left(1-n^{-1.5}\right)\right)$$

$$= n^{0.5} + \frac{8n^2}{7} - \frac{8}{7}n^{0.5}.$$

$$= O(n^2)$$

c) $T(n) = 3T(n/2) + 5 \cdot n \ , \ T(1) = 1$

$$T\left(\frac{n}{2}\right) = 3T\left(\frac{n}{2^2}\right) + 5 \cdot \frac{n}{2}$$

$$T(n) = 3\left(3T\left(\frac{n}{2^2}\right) + 5 \cdot \frac{n}{2}\right) + 5 \cdot n$$

$$= 3^2 T\left(\frac{n}{2^2}\right) + 3 \cdot 5 \cdot \frac{n}{2} + 5 \cdot n$$

$$T\left(\frac{n}{2^2}\right) = 3T\left(\frac{n}{2^3}\right) + 5 \cdot \frac{n}{2^2}$$

$$T(n) = 3^2\left(3T\left(\frac{n}{2^3}\right) + 5 \cdot \frac{n}{2^2}\right) + 3 \cdot 5 \cdot \frac{n}{2} + 5 \cdot n.$$

$$= 3^3 T\left(\frac{n}{2^3}\right) + 3^2 \cdot 5 \cdot \frac{n}{2^2} + 3 \cdot 5 \cdot \frac{n}{2} + 5 \cdot n$$

$$= 3^r T\left(\frac{n}{2^r}\right) + \frac{3^{r-1}}{2^{r-1}} \cdot 5n + \text{- - - -}$$

$$= 3^r T\left(\frac{n}{2^r}\right) + 5n \sum_{i=0}^{r-1} \left(\frac{3}{2}\right)^i$$

$a = 1, \quad r = \frac{3}{2}$

$$S_r = \frac{1(1.5^r - 1)}{0.5} = 2(1.5^r - 1)$$

$$n = 2^k, \quad \frac{n}{2^r} = 1 \Rightarrow \boxed{k = r}$$

$$= 3^{\log n} + 5n\left(2(1.5^r - 1)\right) = 3^{\log n} + 10n\left(1.5^{\log n} - 1\right)$$

$$= n^{\log 3} + 10n\left(n^{\log 1.5} - 1\right).$$

$$= O(n^{1.58}).$$

(d) $T(n) = 2T\left(\frac{n}{2}\right) + (n-1)$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2} - 1\right)$$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2} - 1\right)\right) + (n-1)$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + (n-2) + (n-1)$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2} - 1\right)$$

$$T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2} - 1\right)\right) + (n-2) + (n-1)$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + (n-4) + (n-2) + (n-1)$$

$$= 2^r \cdot T\left(\frac{n}{2^r}\right) + (r.n - 2^{r-1}) + \text{-----}$$

$$= 2^r \cdot T\left(\frac{n}{2^r}\right) + r.n - \left(\sum_{i=0}^{r-1} 2^i\right)$$

$a = 1$, $r = 2$.

$$S_r = \frac{1(2^r - 1)}{2-1} = (2^r - 1)$$

$$= 2^r \cdot T\left(\frac{n}{2^r}\right) + r.n - (2^r - 1)$$

$$\frac{n}{2^r} = 1 \quad , \quad n = 2^k.$$

$$\boxed{k = r}$$

$$= \cancel{2^{\log n}} + n\log n - \cancel{2^{\log n}} + 1$$

$$= \cancel{2n + n\log n + 1} = n\log n + 1$$

$$= O(n\log n).$$

(b) $\cancel{T(n) = 2T\left(\frac{n}{4}\right) + n^2, \quad T(1) = 1}$

$\cancel{T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{4^2}\right) + \frac{n^2}{4^2}}$.

$\cancel{T(n) = 2\left(2T\left(\frac{n}{4}\right.\right.}$