

## HW #2 Solutions

1. Machine speed =  $10^{-9}$  sec/op =  $10^9$  op/sec.

$$T(n) = 15n^2, 8n^3, 2^n, 3^n, n!, n \log n.$$

Total operations in 1 day =  $86400 \times 10^9$  operations.

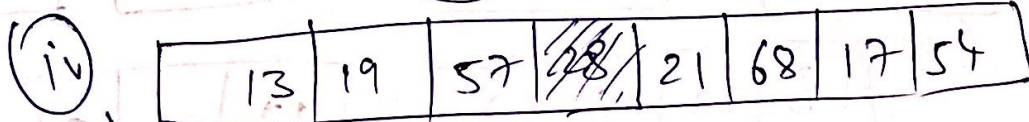
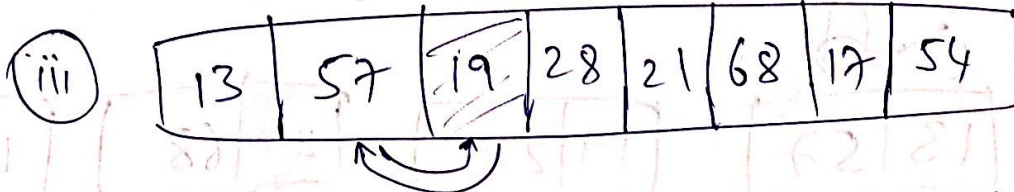
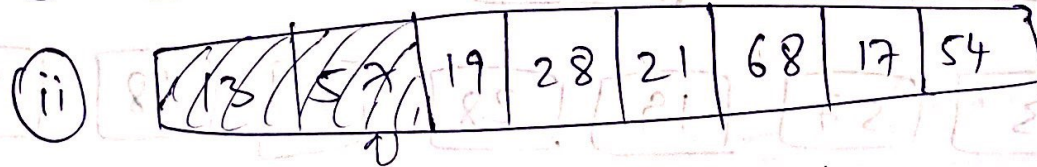
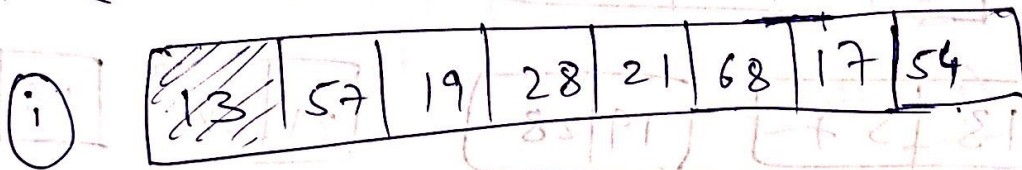
$$15n^2 = 86400 \times 10^9$$

$$n = \sqrt{\frac{86400 \times 10^9}{15}}$$

You have to simplify using calculator.

Similarly for  $8n^3, 2^n, 3^n$ .  
For  $n!, n \log n$  try using any programming language. (We will not give such functions in examination).

2.  $A = \langle 13, 57, 19, 28, 21, 68, 17, 54 \rangle$



V

13	19	28	57	<del>21</del>	68	17	54
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VI

13	19	21	28	57	<del>68</del>	17	54
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VII

13	19	21	28	57	68	<del>17</del>	54
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VIII

13	17	19	21	28	57	68	<del>54</del>
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IX

13	17	19	21	28	<del>57</del>	57	68
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Merge

13	57	19	28	21	68	17	54
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13	57	19	28
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21	68	17	54
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13	57
----	----

19	28
----	----

21	68
----	----

17	54
----	----

13	57	19	28
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21	68	17	54
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13	57
----	----

19	28
----	----

21	68
----	----

17	54
----	----

13	19	28	57
----	----	----	----

17	21	54	68
----	----	----	----



13	17	19	21	28	54	57	68
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Similarly for array B.

(3) Direct from slides. (Average case complexity for sequential search).

Prove.  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

Base case:  $n=1$   
 $1 = \frac{1(1+1)}{2}$

Assuming it is true for 'n':  
 $1 = 1$

Inductive step.

$$\begin{aligned} \sum_{i=1}^n i + (n+1) &= \frac{n(n+1)}{2} + (n+1) \\ &= \frac{n(n+1) + 2(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$



$\therefore$  It is true for  $(n+1)$ .

Hence  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

4. Plot graphs using any programming language.  
 From  $n \geq 44$ ,  $T_2(n) \leq T_1(n)$ .

$\therefore [44, \infty) \rightarrow$  algorithm 2 is better performing.

To obtain the above values,  $T$  have assumed  $\log \text{ base } \rightarrow 2$ .

$$(1 + \frac{1}{5})^1 = 1.2$$

Assuming it is true for  $n$ .  
 Inductive step.

$$(1+n) + \frac{(1+n)n}{5} = (1+n) + i \sum_{i=1}^n$$

$$\frac{(1+n)5 + (1+n)n}{5} =$$

$$\frac{(5+n)(1+n)}{5} =$$

$$(1+n) \text{ ref. value at } T_1(n) \text{ is } \frac{(1+n)n}{5} + i \sum_{i=1}^n$$