

# Bayes' theorem in AI | Example 1

1. **Goal:** Calculate  $P(B|A)$  (likelihood it will rain given there are clouds). **(likelihood)**

2. What we know:

- $P(A)$ : How likely it is to rain in general **(Prior Knowledge)**.
- $P(A|B)$ : How likely it is to rain if there are clouds **(Evidence)**.

3. **Question:**

- What is the likelihood of clouds given it rains ( $P(B|A)$ )? **(posterior)**

## What We Know

1.  $P(A)$ : Probability it will rain >>> **30%** chance it rains today.

2.  $P(A|B)$ : Probability it will rain if there are clouds. >>> **80%** chance of rain.

3.  $P(B)$ : Probability of clouds **50%** chance of clouds on any given day.

What is  $P(B|A) = ?$

# Bayes' theorem in AI | Example 2

1. Given clouds in the morning, what's the probability of rain in the afternoon?
2. 80% of rainy afternoons start with cloudy mornings.
3. 40% of days have cloudy mornings.
4. 10% of days have rainy afternoons.

$$P(\text{rain}|\text{clouds}) = \frac{P(\text{clouds}|\text{rain}) P(\text{rain})}{P(\text{clouds})}$$
$$\frac{0.8 \cdot 0.1}{0.4} = 0.2$$

# Bayes' theorem in AI

**Knowing**

$P(\text{cloudy morning} \mid \text{rainy afternoon})$

**we can calculate**

$P(\text{rainy afternoon} \mid \text{cloudy morning})$

# Bayes' theorem in AI

## Knowing

$P(\text{visible effect} \mid \text{unknown cause})$

we can calculate

$P(\text{unknown cause} \mid \text{visible effect})$

# Bayes' theorem in AI | Example 3

## What We Know

1. **P(A)**: Probability it will rain >>> **20%** chance it rains today.
2. **P(A|B)**: Probability it will rain if there are clouds. >>> **70%** chance of rain.
3. **P(B)**: Probability of clouds **50%** chance of clouds on any given day.

What is **P(B|A) = ?**

## Example Calculation

1. What we know:

- P(A) (rain): 0.20
- P(A|B) (rain if clouds): 0.70
- P(B) (clouds): 0.50

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$P(B|A) = \frac{0.70 \cdot 0.50}{0.20} = 87.5\%$$

## Reducing Uncertainty:

- Initially, the probability of clouds was 50%. After observing rain, this updates to 87.5%, making the prediction much more confident.

# Bayes' theorem in AI

$P(\text{Temperature-High}) = \mathbf{30\%}$  (likelihood of high temperature on any day).

- $P(\text{Rain}|\text{Clouds}) = \mathbf{70\%}$  (probability of rain given clouds).
- $P(\text{Temperature-High} | \text{Clouds}) = \mathbf{20\%}$
- $P(\text{Temperature-High} | \text{Rain}) = \mathbf{50\%}$

What is  $P(\text{Clouds} | \text{Rain}, \text{Temperature-High})$ ?

$$P(B|A, C) = \frac{P(A|B) \cdot P(C|B) \cdot P(B)}{P(A, C)}$$

If it rained and the temperature is high, the probability of clouds **decreases to 70%**

# Bayes' theorem | Practice Example

Company A supplies 40% of the computers sold and is late 5% of the time.  
Company B supplies 30% of the computers sold and is late 3% of the time.  
Company C supplies another 30% and is late 2.5% of the time. A computer arrives late - what is the probability that it came from Company A?

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

# Bayes' theorem | Practice Example

Company A supplies 40% of the computers sold and is late 5% of the time.

Company B supplies 30% of the computers sold and is late 3% of the time.

Company C supplies another 30% and is late 2.5% of the time.

A computer arrives late - what is the probability that it came from Company A?

$$P(A) = 0.4$$

$$P(B) = 0.3$$

$$P(C) = 0.3$$

$$P(\text{Late} | A) = 0.05$$

$$P(\text{Late} | B) = 0.03$$

$$P(\text{Late} | C) = 0.025$$

$$P(A | \text{Late}) = \frac{P(\text{Late}|A) P(A)}{P(\text{late})}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Joint Probability

$$P(\text{Late}|A)*P(A) + P(\text{Late} |B)P(B) + P(\text{Late} | C) P(C) = 0.0365$$

$$p(A | \text{Late}) = \frac{(0.05)(0.4)}{0.0365} = 0.54 = 54\%$$

$$P(\text{late}) =$$

Joint Probability

$$P(\text{Late}|A)*P(A) + P(\text{Late}$$



# Bayes' theorem | Practice Example

A mechanic knows these facts:

If a car has engine trouble, it makes a loud noise 80% of the time.

The chance of a car having engine trouble is 0.000033.

The chance of a car making a loud noise is 2%.

What is the chance that a car making a loud noise actually has engine trouble?

# Bayes' theorem | Example Simplified

A mechanic knows these facts:

If a car has engine trouble, it is likely to make a loud noise 80% of the time.

(This means  $P(A \mid B)=0.80$ , where **A = loud noise and B = engine trouble**).

The chance of a car having engine trouble is very small, only 0.000033.

(This means  $P(B)=0.000033$ ).

The chance of any car making a loud noise is 2%.

(This means  $P(A)=0.02$ ).

The question is:

If a car is making a loud noise, what is the chance it actually has engine trouble?

(You need to find  $P(B \mid A)$ )

# Bayes' theorem | Example Solution

$$P(B|A) = \frac{0.80 \cdot 0.000033}{0.02} = 0.0013\%$$