26$^{\text{th}}$ TJIMO

Alexandria, Virginia

Round: **Guts**

Set 1

1. How many integers x are there such that $|3x - 13| < 14$?
2. Let H be a regular hexagon of area $\sqrt{3}$ and S be a square of area 1. Find the ratio of the perimeter of H to the perimeter of S.
3. How many prime numbers are there between 40 and 80?
4. Timely, Charming Timberman/Chopper Timothy Chachacha is currently at the origin (0, 0) and trying to get to a forest at (4, 4). If he can only move either up one unit or right one unit at a time, in how many ways can Timothy get to the forest?

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ALEXANDRIA, VIRGINIA

Round: **Guts**

Set 2

5. An unfair coin can land on heads, tails, or its edge. If the probability that the coin doesnt land on heads is $\frac{2}{3}$, and the probability that the coin doesnt land on tails is $\frac{1}{2}$, what is the probability of the coin landing on its edge?

6. How many different ways are there to order the numbers $\{1, 2, 3, 4, 5\}$? For example, $\{3, 2, 5, 4, 1\}$ is one valid ordering.

7. Alice, Bob, Charlie, and Dave agree on two integers x and y, and state the following equations.

   Alice $x + y = 10$

   Bob $x^y = 256$

   Charlie $xy = 21$

   Dave $x^2 + y^2 = 58$

   If exactly one of them is lying, who is the liar?

8. Equilateral triangles are inscribed and circumscribed around a circle. Find the ratio between the area of the smaller triangle to the area of the larger triangle.

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Set 3

9. Two numbers are said to be *relatively prime* if their only common factor is 1. How many positive integers less than 100 are relatively prime to 96?

10. Jason is thirsty, so he is randomly choosing drinks from a cooler containing 100 lemonades, 100 love potions, and 100 water bottles. If he wants to make sure he has either at least 5 lemonades, at least 25 love potions, or at least 12 water bottles, what is the minimum number of drinks that Jason must randomly pull out from the cooler?

11. There are two real numbers such that the square of their sum is 612 and the square of their difference is 272. Find their product.

12. What is the ratio of the area of a regular hexagon of side length 4 to the area of an equilateral triangle with side length 1?

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Round: **Guts**

Set 4

13. Find the positive difference between the sum of the first 1000 odd positive integers and the sum of the first 1000 even positive integers.
14. Kevin starts with the number 1000. Every second, he can either divide the number by 2 (rounding down), or subtract 10. What is the minimum number of seconds it takes to get the number below 0?
15. Find the number of triangles created from three vertices of a cube that are not on a face of the cube.
16. If $(22)(34) = 718$ in base b, find b.

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Round: **Guts**

Set 5

17. If $\log_{16}(x^2) + (\log_{16} x)^2 = -1$, find the value of x.
18. If a and b are positive integers, $ab = 2500$, and $\gcd(a, b) = 10$, find the largest possible value of a.
19. Three circles A, B, and C are mutually externally tangent (each pair of them only shares one point in common). If the radii of the three circles are 6, 7, and 8, find the area of triangle ABC.
20. PokéFarmer Kevy is raising a Pokémon population of two-legged Mienshao and four-legged Bulbasaur. During breeding season, however, Kevy lost track of how many Pokémon he had. However, counting their footprints, he determined that his Pokémon had 148 legs among them. Help Kevy determine the difference between the minimum and maximum number of Pokémon he could have.

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Round: **Guts**

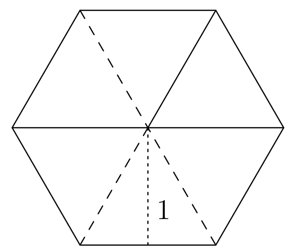
Set 6

21. A 100-sided die (yes, they exist) is weighted so that the probability of rolling at most n is proportional to n. Find the probability of rolling a 42.

22. If I receive $1 + 2 + \cdots + n$ gifts on the nth day of Christmas, how many total gifts do I have by the end of the 12th day of Christmas? (For example, I have $1 + (1 + 2) = 4$ gifts at the end of the second day.)

23. Find the volume of a 10-cube (a cube in 10 dimensions) with side length 2.
Hint: Look at the lower dimension cubes first (1-cube = line, 2-cube = square, 3-cube = cube).

24. A certain strip of paper can be twisted and folded into a Möbius strip (connected at the dotted line near the 1 in the diagram) that resembles a regular hexagon when flattened (see diagram). If the width of the strip is 1, what is the length of the strip?



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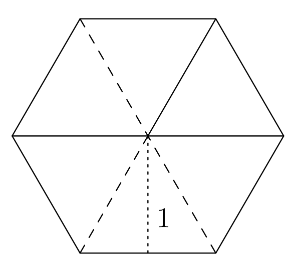

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Round: **Guts**

Set 7

25. A pizza of radius 6 inches is made. If $\frac{2}{3}$ of it is covered with chicken and $\frac{1}{2}$ is covered with mushrooms, what is the maximum possible area on the pizza that is only covered by chicken? Note that any area on the pizza may be covered by one topping, both, or neither. Express your answer in terms of π.

26. Find the sum of all whole numbers less than 100 that have a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, and a remainder of 4 when divided by 5.

27. Whenever Low calls his friend High, they always begin their conversation with Hi, High and Yo, Low. Let N be the number of ways to arrange the letters in HIHIGHYOLOW. Find the last 3 digits of N.

28. Let S be the set $\{1, 2, 3, ..., 30\}$ and N be the number of subsets s of S such that the sum of the elements in s is greater than 232. Find the number of factors of N.

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Round: **Guts**

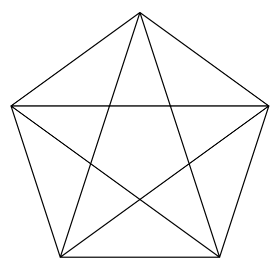
Set 9

33. Mildew Dim and his archrival, Grizzly Absky, are fighting for the heart of their mutual love in a game of Prussian Poulet. Each turn, players alternate eating a piece of chicken, which has a $\frac{1}{5}$ chance being poisoned, thus killing the player. If Mildew goes first, what is the probability that he will not be poisoned before Grizzly?

34. Find the sum of all x such that $2014x^3 - 4513x^2 + 991x - 42 = 0$.

35. Compute the number of positive integers n such that $10n + 1$ divides $2014n + 47,530$.

36. A regular pentagon has area 1. When all its diagonals are drawn, a smaller regular pentagon is outlined within the original pentagon. Find the area of the smaller pentagon.



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