Design Theory of Dielectric-Coated Circular Metallic Waveguides for Infrared Transmission

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Abstract—A circular metallic hollow waveguide with inner dielectric multilayers is designed with the emphasis on low-loss transmission of the HE_{11} mode for the infrared. The effects of absorptions and variations of thickness of coated dielectric materials are discussed and are shown to be small on the minimum attainable transmission losses. Mode structure and transmission properties are also clarified for the general class of hollow waveguides by using a normalized surface impedance and admittance.

I. Introduction

DEVELOPMENT of low-loss flexible waveguides is of great concern for transmitting infrared CO_2 laser light. Although significant progress has been made for infrared dielectric fibers as cited in the previous paper [1], hollow-core waveguides seem to be potential media from the viewpoint of high-power operation [2]. There have been proposed and fabricated several kinds of hollow waveguides for practical use. Garmire *et al.* extensively analyzed and fabricated a planar ribbon-like metallic waveguide called the FIT guide [3] which transmits the TE_0 mode in the slab geometry. This guide can be bent but only in one direction. Therefore, a tube of circular cross section will be suitable to bend waveguides in arbitrary directions. In conventional circular metallic waveguides, however, the mode with the lowest attenuation is the TE_{01} mode, not the HE_{11} mode whose field distribution resembles that of commercially available CO_2 lasers. Marhic *et al.* [4] obtained TE_{0q} operation of a CO_2 laser and achieved low-loss transmission through a metallic tube. However, the TE_{0q} mode suffers substantial loss increase due to bends as far as metallic waveguides are concerned [5], [6]. As an open-type waveguide, a helical-circular waveguide has also been fabricated [7].

As studies of HE_{11} propagation in hollow dielectric guides [8], numerous studies have been performed on the subject for waveguide lasers [9]. However, the structures are essentially straight and short in length, and they are not practical as flexible waveguides. In order to realize flexible waveguides, two types of circular waveguides have so far been proposed and fabricated to transmit the HE_{11} mode with low loss. One is a waveguide made with oxide glass functioning as a cladding whose refractive index is lower than unity [10], [11]. Another is a dielectric-coated metallic hollow waveguide [12]. As far as a metallic waveguide with a single inner dielectric layer is concerned, transmission characteristics have been analyzed in the late 1950's, and more recently [13]-[17], for the transmission of the TE_{01} mode. However, basic analyses must be done for the infrared because metals are properly characterized by a complex refractive index in the infrared rather than by a large conductivity. Furthermore, we are interested in the transmission of HE_{1m} modes, not the TE_{01} mode [5], [18], in a circular waveguide. In our previous papers [1], [19], basic transmission properties of the dielectric-coated metallic waveguides were analyzed for a slab geometry and also some analyses were done for circular structures. Recent fabrication techniques show that total power losses of 0.35 dB are attained for nickel pipes of 1.5 mm diam and 1 m long with an inner germanium layer at the CO_2-laser wavelength [20].

In this paper, we systematically clarify the mode structure and transmission properties of general hollow-core waveguides characterized by a normalized surface impedance and admittance [21], [22]. Mode transition, which has never been mentioned before, is theoretically made clear when the surface impedance and admittance change. Next, we shall present a design theory of a circular metallic waveguide with inner dielectric multilayers consisting of two kinds of dielectrics in order to reduce the attenuation constant of the HE_{11} mode. It is shown that attenuation constants of the TE_{0q} and TM_{0q} modes are also reduced drastically compared with those in ordinary metallic waveguides when the attenuation constants of the hybrid modes are minimized. Effects of absorptions of coating materials are discussed and shown to be small on the minimum attainable transmission losses. Finally, an optimum design theory is presented to reduce loss increase due to random changes of thickness of dielectric layers.

Readers who are only interested in dielectric-coated circular metallic waveguides may skip the Sections II and III.

II. Mode Structure and Attenuation Constants in General Hollow-Core Waveguides

Consider a waveguide consisting of a circular cylinder of radius T and refractive index n_0 ($\simeq 1$) embedded in other media consisting of several kinds of dielectrics or metal as shown in Fig. 1. When the condition

$$n_0 k_0 T >> 1 \tag{1}$$

is satisfied, the axial phase constant β is approximated by $n_0 k_0$ for lower order modes and electromagnetic fields in $r \leqslant T$ are

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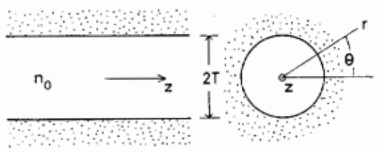


Fig. 1. Hollow-core waveguides.

described as follows [23]:

$$E_r = \left[\frac{1-P}{2} J_{n-1}\left(u\frac{r}{T}\right) - \frac{1+P}{2} J_{n+1}\left(u\frac{r}{T}\right)\right] \cdot \cos(n\theta + \theta_0) \tag{2}$$

$$E_\theta = -\left[\frac{1-P}{2} J_{n-1}\left(u\frac{r}{T}\right) + \frac{1+P}{2} J_{n+1}\left(u\frac{r}{T}\right)\right] \cdot \sin(n\theta + \theta_0) \tag{3}$$

$$E_z = j\frac{u}{n_0 k_0 T} J_n\left(u\frac{r}{T}\right) \cos(n\theta + \theta_0) \tag{4}$$

$$H_r = \frac{n_0 k_0}{\omega\mu_0} E_\theta \tag{5}$$

$$H_\theta = \frac{n_0 k_0}{\omega\mu_0} E_r \tag{6}$$

$$H_z = -j\left(\frac{n_0 k_0}{\omega\mu_0}\right)\frac{u}{n_0 k_0 T} P J_n\left(u\frac{r}{T}\right)\sin(n\theta + \theta_0) \tag{7}$$

where n is the integer describing the azimuthal dependence and P is an arbitrary constant to be determined from the boundary conditions. Time and z dependences of the form $\exp(j(\omega t - \beta z))$ are suppressed. u/T is the complex transverse phase constant related to β via

$$\beta^2 + \left(\frac{u}{T}\right)^2 = (n_0 k_0)^2. \tag{8}$$

Introducing a normalized surface impedance z_{TE} and normalized surface admittance y_{TM} defined at $r = T$ [1], one can express the boundary conditions as follows:

$$\left.\frac{E_\theta}{H_z}\right)_{r=T} = \frac{\omega\mu_0}{n_0 k_0} z_{\mathrm{TE}}, \quad \left.\frac{H_\theta}{E_z}\right)_{r=T} = -\frac{n_0 k_0}{\omega\mu_0} y_{\mathrm{TM}} \tag{9}$$

which lead to

$$\frac{n}{u} J_n(u) - \left[J_n'(u) + j\frac{u}{n_0 k_0 T} z_{\mathrm{TE}} J_n(u)\right] P = 0 \tag{10}$$

$$\frac{n}{u} J_n(u) P - \left[J_n'(u) + j\frac{u}{n_0 k_0 T} y_{\mathrm{TM}} J_n(u)\right] = 0. \tag{11}$$

By eliminating the parameter P which is used to designate mode nomenclature [23], one obtains the characteristic equation as follows [14]:

$$\left[\frac{J_n'(u)}{u J_n(u)} + j\frac{z_{TE}}{n_0 k_0 T}\right]\left[\frac{J_n'(u)}{u J_n(u)} + j\frac{y_{\mathrm{TM}}}{n_0 k_0 T}\right] = \frac{n^2}{u^4} \tag{12}$$

or

$$\frac{J_n'(u)}{u J_n(u)} = -j\frac{1}{2}\left(\frac{z_{\mathrm{TE}}}{n_0 k_0 T} + \frac{y_{\mathrm{TM}}}{n_0 k_0 T}\right) \pm j\left[\frac{1}{4}\left(\frac{z_{\mathrm{TE}}}{n_0 k_0 T} - \frac{y_{\mathrm{TM}}}{n_0 k_0 T}\right)^2 - \frac{n^2}{u^4}\right]^{1/2}. \tag{13}$$

One should note that the impedance method where the boundary conditions are given by (9) is an approximate one for cylindrical structures. It requires that $(a_{\min}^2 - 1)^{1/2}\, n_0 k_0 \gg u/T$ and the Hankel functions associated with wave equations can be replaced by their asymptotic expressions, where $a_{\min} n_0$ is the minimum refractive index in $r > T$. It can also be applied to waveguides no matter how the number of layers is increased as in Section IV.

By dividing u into real and imaginary parts as

$$u = u_0 + j u_i \tag{14}$$

and assuming that $u_0 \gg u_i$, one obtains the attenuation constants α of modes by help of (8) as

$$\alpha = \frac{u_0 u_i}{n_0 k_0 T^2}. \tag{15}$$

For $n = 0$, (12) separates into the TE_{0q} ($P = \infty$) and TM_{0q} ($P = 0$) modes, whose characteristic equations are

$$\frac{J_1(u)}{u J_0(u)} = j\frac{1}{n_0 k_0 T} \times \begin{cases} z_{\mathrm{TE}}, & \mathrm{TE}_{0q} \text{ modes} \\ y_{\mathrm{TM}}, & \mathrm{TM}_{0q} \text{ modes.}\end{cases} \tag{16}$$

In the special case of $|z_{\mathrm{TE}}| \ll z_0$ or $|z_{\mathrm{TE}}| \gg z_0$, one obtains the attenuation constant of the TE_{0q} mode by using a perturbation method as

$$\alpha = \begin{cases} n_0 k_0 \dfrac{{u_0}^2}{(n_0 k_0 T)^3} \mathrm{Re}\,(z_{\mathrm{TE}}), & |z_{\mathrm{TE}}| \ll z_0 \\[2ex] \dfrac{1}{T} \mathrm{Re}\left(\dfrac{1}{z_{\mathrm{TE}}}\right), & |z_{\mathrm{TE}}| \gg z_0 \end{cases} \tag{17}$$

where z_0 is defined by

$$z_0 = n_0 k_0 T / u_0 \tag{18}$$

and u_0 is determined by $J_1(u_0) = 0$ for $|z_{\mathrm{TE}}| \ll z_0$ and $J_0(u_0) = 0$ for $|z_{\mathrm{TE}}| \gg z_0$ as seen from (16).

Similarly, one obtains the attenuation constant of the TM_{0q} mode as

$$\alpha = \begin{cases} n_0 k_0 \dfrac{{u_0}^2}{(n_0 k_0 T)^3} \mathrm{Re}(y_{\mathrm{TM}}), & |y_{\mathrm{TM}}| \ll y_0 \\[2ex] \dfrac{1}{T} \mathrm{Re}\left(\dfrac{1}{y_{\mathrm{TM}}}\right), & |y_{\mathrm{TM}}| \gg y_0 \end{cases} \tag{19}$$

where

$$y_0 = z_0 = \frac{n_0 k_0 T}{u_0}. \tag{20}$$

We next derive the attenuation constants of the hybrid modes, i.e., modes for $n \neq 0$. Three extreme cases for z_{TE} and

TABLE I
MODE DESIGNATION AND THE PARAMETER F DESCRIBING THE ATTENUATION CONSTANTS OF MODES.

$\lvert z_{TE}\rvert \ll z_0,\ \lvert y_{TM}\rvert \gg y_0$			$\lvert z_{TE}\rvert \ll z_0,\ \lvert y_{TM}\rvert \ll y_0$			$\lvert z_{TE}\rvert \gg z_0,\ \lvert y_{TM}\rvert \ll y_0$		
MODE	F	u_0	MODE	F	u_0	MODE	F	u_0
TE_{0q}	$\mathrm{Re}(z_{TE})$	j_{1q}	TE_{0q}	$\mathrm{Re}(z_{TE})$	j_{1q}	TE_{0q}	$\left(\frac{n_0k_0T}{u_0}\right)^2 \mathrm{Re}\left(\frac{1}{z_{TE}}\right)$	j_{0q}
TE_{nq}	$\frac{1}{1-\left(\frac{n}{u_0}\right)^2} \mathrm{Re}\left[z_{TE} + \frac{n^2}{u_0^4} \frac{(n_0k_0T)^2}{y_{TM}}\right]$	j'_{nq}	HE_{nq}	$\frac{1}{2} \mathrm{Re}(z_{TE} + y_{TM})$	$j_{n-1\,q}$	TE_{nq}		j_{nq}
TM_{nq}	$\left(\frac{n_0k_0T}{u_0}\right)^2 \mathrm{Re}\left(\frac{1}{y_{TM}}\right)$	j_{nq}	EH_{nq}		$j_{n+1\,q}$	TM_{nq}	$\frac{1}{1-\left(\frac{n}{u_0}\right)^2} \mathrm{Re}\left[y_{TM} + \frac{n^2}{u_0^4} \frac{(n_0k_0T)^2}{z_{TE}}\right]$	j'_{nq}
TM_{0q}		j_{0q}	TM_{0q}	$\mathrm{Re}(y_{TM})$	j_{1q}	TM_{0q}	$\mathrm{Re}(y_{TM})$	j_{1q}

Note: j_{nq} and j'_{nq} represents the qth zero of $J_n(u_0)$ and $J'_n(u_0)$ except for zero, respectively. Mode transition is symbolically shown in bold solid, solid, and dashed lines. Bold solid lines indicate that mode designation (TE_{0q} and TM_{0q} modes) can never change no matter how the magnitude of z_{TE} or y_{TM} changes. Solid lines cocrespond to the mode transition when z_{TE} or y_{TM} approaches ∞ (or $j\infty$), and dashed lines correspond to that when z_{TE} or y_{TM} approaches $-\infty$ (or $-j\infty$).

y_{TM} are discussed which lead to low-loss waveguides

i) $|z_{TE}| << z_0$ and $|y_{TM}| << y_0$.

By neglecting higher order terms of $|z_{TE}|^2/(n_0k_0T)^2$ and $|y_{TM}|^2/(n_0k_0T)^2$ in (13), one has

$$\frac{J_{n\mp 1}(u)}{u J_n(u)} = \mp j \frac{1}{2n_0k_0T} (z_{TE} + y_{TM}) \tag{21}$$

which leads to

$$\alpha = \frac{1}{2} \frac{n_0k_0u_0^2}{(n_0k_0T)^3} \mathrm{Re}(z_{TE} + y_{TM}) \tag{22}$$

where u_0 is determined from

$$J_{n\mp 1}(u_0) = 0. \tag{23}$$

By use of (10) and (21), one gets $P \simeq \mp 1$, which shows that upper and lower signs in (13), (21), and (23) correspond to the HE_{nq} and EH_{nq} modes, respectively [23].

ii) $|z_{TE}| << z_0$ and $|y_{TM}| >> y_0$.

As the following approximation is made

$$\left[\frac{1}{4}\left(\frac{z_{TE}}{n_0k_0T} - \frac{y_{TM}}{n_0k_0T}\right)^2 - \frac{n^2}{u^4}\right]^{1/2} \simeq \frac{1}{2}\left(\frac{y_{TM}}{n_0k_0T} - \frac{z_{TE}}{n_0k_0T}\right) - \frac{n^2}{u^4} \frac{n_0k_0T}{y_{TM}} \tag{24}$$

(13) is simplified as

$$\frac{J_n'(u)}{u J_n(u)} = \begin{cases} -j\left(\frac{z_{TE}}{n_0k_0T} + \frac{n^2}{u^4} \frac{n_0k_0T}{y_{TM}}\right) & \text{(25a)} \\ -j \frac{y_{TM}}{n_0k_0T} & \text{(25b)} \end{cases}$$

and the parameter P is approximately expressed by

$$P = \begin{cases} j \frac{u^2}{n} \frac{y_{TM}}{n_0k_0T} & \text{(26a)} \\ j \frac{n}{u^2} \frac{n_0k_0T}{y_{TM}}. & \text{(26b)} \end{cases}$$

Equation (26) indicates that $|P|$ is very large in (26a) and small in (26b), which means that modes characterized by upper and lower lines in (25) resemble the TE_{nq} and TM_{nq} modes, respectively. As the parameter P is not exactly ∞ or 0, one cannot call the hybrid modes in ii) the TE_{nq} or TM_{nq} modes in the exact sense of words. However, we hereafter call them TE_{nq} and TM_{nq} modes for convenience' sake. Generally speaking, it is rather complicated to clarify the mode transition from the HE_{nq} or EH_{nq} modes in i) to the TE_{nq} or TM_{nq} modes in ii). However, for some special cases, we can analytically show this mode transition. When y_{TM} is purely real or imaginary and increases to ∞ or $j\infty$, the HE_{nq} (EH_{nq}) modes gradually become the TE_{nq} (TM_{nq}) modes. On the other hand, if y_{TM} approaches $-\infty$ or $-j\infty$, the HE_{nq} (EH_{nq}) modes gradually change to the TM_{nq} (TE_{nq}) modes.

By using a perturbation method, one has

$$J_n'(u_0) = 0, \quad TE_{nq} \text{ modes} \tag{27}$$

$$J_n(u_0) = 0, \quad TM_{nq} \text{ modes} \tag{28}$$

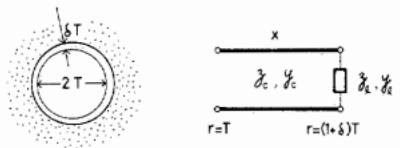
and the attenuation constants of the modes as

$$\alpha = \begin{cases} n_0k_0 \frac{u_0^2}{(n_0k_0T)^3} \frac{1}{1-\left(\frac{n}{u_0}\right)^2} \cdot \mathrm{Re}\left[z_{TE} + \frac{n^2}{u_0^4} \frac{(n_0k_0T)^2}{y_{TM}}\right], & TE_{nq} \text{ modes} \\ \frac{1}{T} \mathrm{Re}\left(\frac{1}{y_{TM}}\right), & TM_{nq} \text{ modes.} \end{cases} \tag{29}$$

iii) $|z_{TE}| >> z_0$ and $|y_{TM}| << y_0$.

Following the similar procedures in ii), one can deduce that the HE_{nq} (EH_{nq}) modes gradually become the TM_{nq} (TE_{nq}) modes when z_{TE} approaches ∞ or $j\infty$, whereas the HE_{nq} (EH_{nq}) modes become the TE_{nq} (TM_{nq}) modes when z_{TE} approaches $-\infty$ or $-j\infty$.

Table I summarizes the attenuation constants of the modes



(a) (b)

Fig. 2. Waveguide satisfying the conditions $|z_{TE}| << z_0$ and $|y_{TM}| << y_0$. (b) represents a transverse transmission line model corresponding to (a) the structure [1].

by representing α as

$$\alpha = n_0 k_0 \frac{u_0^2}{(n_0 k_0 T)^3} \quad F \tag{30}$$

in the three extreme cases, as well as mode designation, mode transition, and values of u_0.

III. Principle to Reduce Attenuation Constants of Hybrid Modes

Generally speaking, it can be possible to achieve low-loss waveguides for the hybrid modes including the HE_{11}, TE_{11}-like, or TM_{11}-like modes in cases i), ii), or iii), respectively. Actually, the condition ii) is satisfied in conventional metallic waveguides in microwave regions. However, in infrared waveguides where $n_0 k_0 T$ is generally large and the normalized admittance of metal is not so large as that in microwave regions, it is much easier and efficient to achieve low-loss waveguides for the He_{11} mode by the case i) [1]. As it is easily possible to have a waveguide with a large normalized impedance z_l and small admittance y_l or vice versa [1], we design the waveguide so as to satisfy the condition i) by adding a dielectric layer with thickness of δT as shown in Fig. 2, where a corresponding transverse transmission line model is also shown.

Letting the normalized impedance and admittance of the added layer be z_c and y_c, and a transverse electric length of the medium x, one obtains z_{TE} as

$$z_{TE} = z_c \frac{z_l + j z_c \tan(x)}{z_c + j z_l \tan(x)}. \tag{31}$$

When the condition $|z_l| << z_c$ or $|z_l| >> z_c$ is satisfied, Re (z_{TE}) is approximated by

$$\mathrm{Re}\,(z_{TE}) = \begin{cases} \dfrac{2\,\mathrm{Re}\,(z_l)}{1 + \cos(2x)}, & |z_l| << z_c \\[2ex] 2\,\dfrac{z_c^2}{|z_l|^2}\,\dfrac{\mathrm{Re}\,(z_l)}{1 - \cos(2x)}, & |z_l| >> z_c \end{cases} \tag{32}$$

and similarly

$$\mathrm{Re}\,(y_{TM}) = \begin{cases} \dfrac{2\,\mathrm{Re}\,(y_l)}{1 + \cos(2x)}, & |y_l| << y_c \\[2ex] 2\,\dfrac{y_c^2}{|y_l|^2}\,\dfrac{\mathrm{Re}\,(y_l)}{1 - \cos(2x)}, & |y_l| >> y_c \end{cases} \tag{33}$$

where z_c and y_c are assumed to be real, i.e., the added layer is free from absorption.

TABLE II
Loss Factor F_{min}, the Normalized Impedance, and Admittance in the Case when the Attenuation Constants of the Hybrid Modes are Minimized.

		$\|z_l\| \ll z_c,\ \|y_l\| \gg y_c$	$\|z_l\| \gg z_c,\ \|y_l\| \ll y_c$
F_{min}	HE_{nq} EH_{nq}	$\frac{1}{2}(1+c)^2 \mathrm{Re}(z_l)$	$\frac{1}{2}(1+c)^2 \mathrm{Re}(y_l)$
	TE_{0q}	$(1+c)\,\mathrm{Re}(z_l)$	$c(1+c)\,\mathrm{Re}(y_l)$
	TM_{0q}	$c(1+c)\,\mathrm{Re}(z_l)$	$(1+c)\,\mathrm{Re}(y_l)$
$\|z_{TE}\|$		$z_c\, c^{1/2}$	$z_c\, c^{-1/2}$
$\|y_{TM}\|$		$y_c\, c^{-1/2}$	$y_c\, c^{1/2}$

By introducing the parameter c defined by

$$C = \begin{cases} \dfrac{y_c}{|y_l|}\left[\dfrac{\mathrm{Re}\,(y_l)}{\mathrm{Re}\,(z_l)}\right]^{1/2}, & |z_l| << z_c \text{ and } |y_l| >> y_c \\[2ex] \dfrac{z_c}{|z_l|}\left[\dfrac{\mathrm{Re}\,(z_l)}{\mathrm{Re}\,(y_l)}\right]^{1/2}, & |z_l| >> z_c \text{ and } |y_l| << y_c \end{cases} \tag{34}$$

F of (30) describing the attenuation constants of the hybrid modes becomes

$$F = \left[\frac{1}{1 + \cos(2x)} + \frac{c^2}{1 - \cos(2x)}\right] \times \begin{cases} \mathrm{Re}\,(z_l); & |z_l| \ll z_c \text{ and } |y_l| \gg y_c \\ \mathrm{Re}\,(y_l); & |z_l| \gg z_c \text{ and } |y_l| \ll y_c \end{cases} \tag{35}$$

and x_{min} which minimizes F is obtained as

$$\cos(2x_{min}) = (1 - c)/(1 + c) \tag{36}$$

or

$$x_{min} = \pm \tan^{-1}(c^{1/2}) + s\pi \qquad (s = 0, 1, \cdots). \tag{37}$$

F's of the hybrid modes, TE_{0q}, and TM_{0q} modes which are expressed by F_{min} are calculated and summarized in Table II as well as with $|z_{TE}|$ and $|y_{TM}|$. It is clear that the attenuation constants of the hybrid modes as well as those of the TE_{0q} and TM_{0q} modes can be reduced when waveguides with smaller $|z_l|$ ($|y_l|$) and larger $|y_l|$ ($|z_l|$) are attained. One can easily see how F's are changed by adding dielectric layers in the next section.

IV. Minimum Losses in Metallic Hollow Waveguides with Inner Dielectric Multilayers

As shown in Fig. 3, if we coat two kinds of dielectrics alternately with refractive indices of $a_i n_0$ ($i = 1, 2$) and a thickness of $\delta_i T (i = 1, 2)$ satisfying

$$\delta_i (a_i^2 - 1)^{1/2}\, n_0 k_0 T \equiv \delta_i u_i = \frac{\pi}{2} \qquad (i = 1, 2) \tag{38}$$

inside a metallic wall with a complex refractive index $n_0 (n - j\kappa)$, these laminated dielectrics play the role of the impedance or admittance transformer [1], [14], [18] in the transmission line model [1]. They make a normalized impedance z_l or ad-

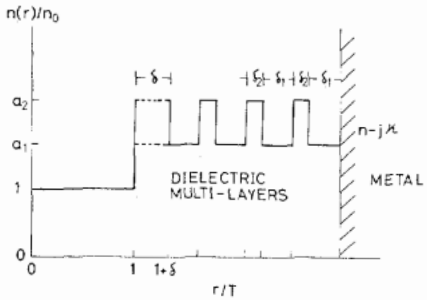


Fig. 3. Refractive-index profile for achieving a low-loss waveguide for the hybrid mode, where $a_2 > a_1$ and the refractive index of the adjacent dielectric layer to the metal is $a_1 n_0$.

mittance y_l much smaller or larger than that of the original metal. Therefore, by coating a further layer with a thickness of δT, we can reduce the attenuation constants of the hybrid modes by the general method in Section III.

We first assume that the dielectrics are free from absorptions. Let the refractive index of the adjacent dielectric be $a_1 n_0$ and $a_2 > a_1$. We define the normalized impedance z_i and admittance y_i corresponding to the dielectric media by

$$z_i = (a_i^2 - 1)^{-1/2} \qquad (i = 1, 2) \tag{39}$$

$$y_i = a_i^2 (a_i^2 - 1)^{-1/2} \qquad (i = 1, 2). \tag{40}$$

When a total number m of dielectric layers including the added one is $2m_p + 1$, the normalized impedance $z_{\mathrm{TE}}{}^{(0)}$ and $y_{\mathrm{TM}}{}^{(0)}$ of the metal are transformed to z_l and y_l defined at $r = (1 + \delta)T$, respectively, as

$$z_l = \left(\frac{z_2}{z_1}\right)^{2m_p} z_{\mathrm{TE}}{}^{(0)} \simeq \frac{n + j\kappa}{n^2 + \kappa^2} \, C^{m_p} \tag{41}$$

$$y_l = \left(\frac{y_2}{y_1}\right)^{2m_p} y_{\mathrm{TM}}{}^{(0)} \simeq (n - j\kappa) \left(\frac{a_2}{a_1}\right)^{4m_p} C^{m_p} \tag{42}$$

where $z_{\mathrm{TE}}{}^{(0)}$, $y_{\mathrm{TM}}{}^{(0)}$, and C are

$$z_{\mathrm{TE}}{}^{(0)} = [(n - j\kappa)^2 - 1]^{-1/2} \simeq (n - j\kappa)^{-1} \tag{43}$$

$$y_{\mathrm{TM}}{}^{(0)} = (n - j\kappa)^2 \, z_{\mathrm{TE}}{}^{(0)} \simeq n - j\kappa \tag{44}$$

$$C = (a_1^2 - 1)/(a_2^2 - 1) < 1 \tag{45}$$

and it is assumed that $n, \kappa \gg 1$. It is clear that the larger m_p, the smaller $|z_l|$ and the larger $|y_l|$. Therefore, by assigning

$$z_c = z_1, \qquad y_c = y_1 \tag{46}$$

$$x = \delta (a_1^2 - 1)^{1/2} \, n_0 k_0 T \tag{47}$$

and using (37) and the first column of Table II, one can see that the attenuation constants of the hybrid modes are minimized at

$$\delta (a_1^2 - 1)^{1/2} \, n_0 k_0 T = \pm \tan^{-1} \left[\frac{a_1}{(a_1^2 - 1)^{1/4}} \left(\frac{a_1}{a_2}\right)^{m_p} C^{-m_p/2} \right] + s\pi \tag{48}$$

and $F_{\min}$ is expressed by

$$F_{\min} = F_{\mathrm{metal}} \times \begin{cases} C^{m_p} \left[1 + \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}} \left(\dfrac{a_1}{a_2}\right)^{2m_p} C^{-m_p} \right], & \mathrm{TE}_{0q} \text{ modes} \\[2ex] \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}} \left(\dfrac{a_1}{a_2}\right)^{2m_p} \cdot \left[1 + \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}} \left(\dfrac{a_1}{a_2}\right)^{2m_p} C^{-m_p} \right], & \mathrm{TM}_{0q} \text{ modes} \\[2ex] \dfrac{1}{2} C^{m_p} \cdot \left[1 + \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}} \left(\dfrac{a_1}{a_2}\right)^{2m_p} C^{-m_p} \right]^2, & \mathrm{HE}_{nq} \text{ and } \mathrm{EH}_{nq} \text{ modes} \end{cases} \tag{49}$$

where

$$F_{\mathrm{metal}} = \frac{n}{n^2 + \kappa^2} \tag{50}$$

and $|z_{\mathrm{TE}}|$ is

$$|z_{\mathrm{TE}}| = \frac{a_1}{(a_1^2 - 1)^{3/4}} \left(\frac{a_1}{a_2}\right)^{m_p} C^{-m_p}. \tag{51}$$

Similarly, when a total number m of dielectric layers is $2m_p + 2$, one has

$$z_l = \frac{n - j\kappa}{a_1^2 - 1} \, C^{-m_p} \tag{52}$$

$$y_l = \frac{a_1^4}{a_1^2 - 1} \, \frac{n + j\kappa}{n^2 + \kappa^2} \left(\frac{a_2}{a_1}\right)^{4m_p} C^{-m_p} \tag{53}$$

and the condition which minimizes the attenuation constants of the hybrid modes as

$$\delta (a_2^2 - 1)^{1/2} \, n_0 k_0 T = \pm \tan^{-1} \left[\frac{(a_1^2 - 1)^{1/2}}{a_1 (a_2^2 - 1)^{1/4}} \left(\frac{a_1}{a_2}\right)^{m_p} C^{m_p/2} \right] + s\pi. \tag{54}$$

Correspondingly, $F_{\min}$ and z_{TE} are obtained as follows:

$$F_{\min} = F_{\mathrm{metal}} \times \begin{cases} \dfrac{a_1^2}{(a_2^2-1)^{1/2}} \left(\dfrac{a_1}{a_2}\right)^{2m_p} \cdot \left[1 + \dfrac{(a_1^2-1)}{a_1^2(a_2^2-1)^{1/2}} \left(\dfrac{a_2}{a_1}\right)^{2m_p} C^{m_p}\right], & \mathrm{TE}_{0q} \text{ modes} \\ \dfrac{a_1^4}{a_1^2-1} \left(\dfrac{a_1}{a_2}\right)^{4m_p} C^{-m_p} \cdot \left[1 + \dfrac{(a_1^2-1)}{a_1^2(a_2^2-1)^{1/2}} \left(\dfrac{a_2}{a_1}\right)^{2m_p} C^{m_p}\right], & \mathrm{TM}_{0q} \text{ modes} \\ \dfrac{1}{2}\dfrac{a_1^4}{a_1^2-1} \left(\dfrac{a_1}{a_2}\right)^{4m_p} C^{-m_p} \cdot \left[1 + \dfrac{(a_1^2-1)}{a_1^2(a_2^2-1)^{1/2}} \left(\dfrac{a_2}{a_1}\right)^{2m_p} C^{m_p}\right]^2, & \mathrm{HE}_{nq} \text{ and } \mathrm{EH}_{nq} \text{ modes} \end{cases} \tag{55}$$

$$|z_{\mathrm{TE}}| = \frac{a_1}{(a_1^2-1)^{1/2}} \frac{1}{(a_2^2-1)^{1/4}} \left(\frac{a_1}{a_2}\right)^{m_p} C^{-m_p/2}. \tag{56}$$

For conventional metallic waveguides without dielectric layers, the attenuation constants α_H of the modes are expressed by

$$\alpha_H = n_0 k_0 \frac{u_0^2}{(n_0 k_0 T)^3} F^{(0)} \tag{57}$$

where

$$F^{(0)} = \begin{cases} \dfrac{n}{n^2+\kappa^2}, & \mathrm{TE}_{0q} \text{ modes} \\ n, & \mathrm{TM}_{0q} \text{ modes} \\ \dfrac{n}{2}, & \mathrm{HE}_{nq} \text{ and } \mathrm{EH}_{nq} \text{ modes.} \end{cases} \tag{58}$$

In order to see the reduction of the attenuation constants by coating dielectric layers whose thickness is determined by the method mentioned above, typical examples of α/α_H of the hybrid modes as well as the TE_{0q} and TM_{0q} modes are shown in Fig. 4 as a function of the total number m of dielectric layers for

$$a_1 = 2.4 \text{ (ZnSe)}, \quad a_2 = 4.0 \text{ (Ge)},$$
$$n = 20.5, \quad \kappa = 58.6 \text{ (Al)}. \tag{59}$$

It is seen that a drastic reduction in the attenuation constants can be made in the hybrid and TM_{0q} modes by coating only a few layers of dielectrics. When the number of layers exceeds three in this particular example, the attenuation constant of the TE_{0q} mode is also reduced compared with that in an aluminum hollow waveguide. Computer calculations using a minimization technique for F are also conducted and it turns out that α/α_H cannot be distinguished in the figures presented,

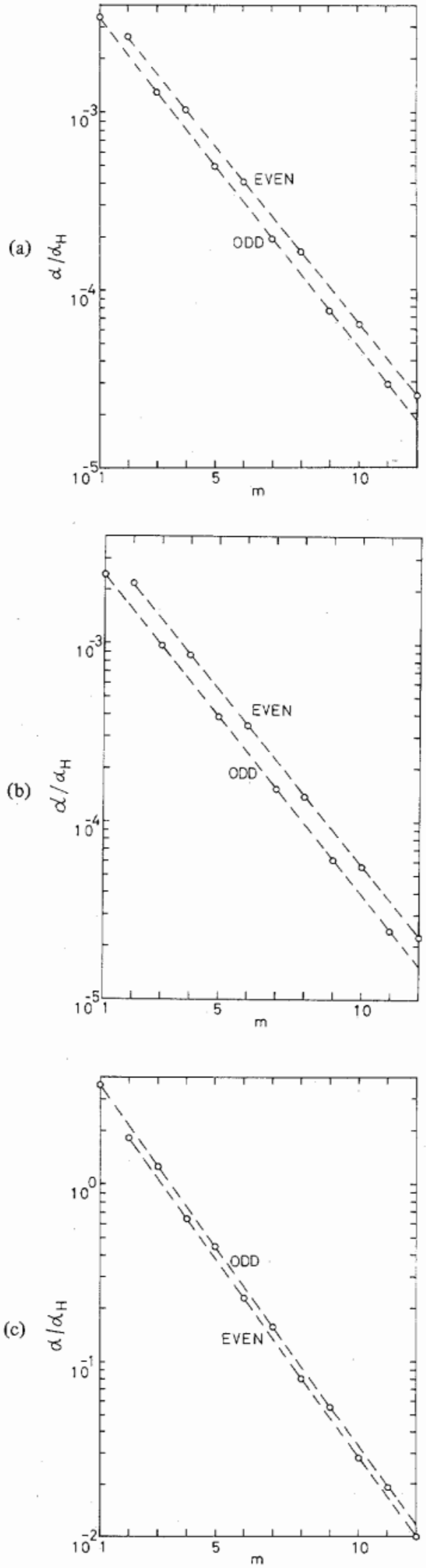


Fig. 4. Loss reduction ratio α/α_H for the (a) hybrid, (b) TM_{0q}, and (c) TE_{0q} modes as a function of m when the thickness of each dielectric layer is chosen such that (38) and (48) or (54) are satisfied, where it is assumed that $a_1 = 2.4$ (ZnSe), $a_2 = 4.0$ (Ge), $n = 20.5$, and $\kappa = 58.6$ (Al).

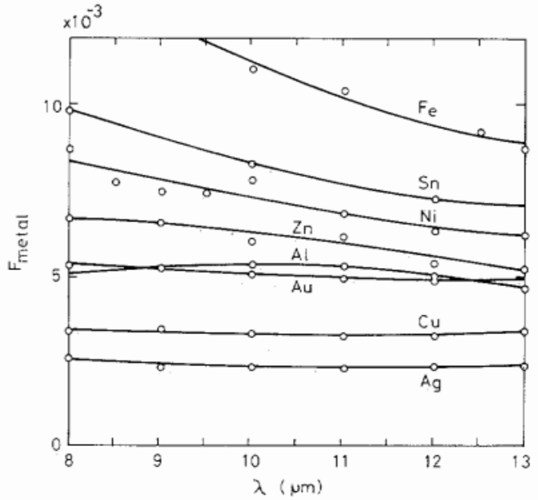


Fig. 5. Wavelength dependence of F_{metal}. Solid lines represent minimum least square curves for the reported values denoted by small circles [24], [25]. Coefficients for the curves are summarized in Table III as well as those for n and κ.

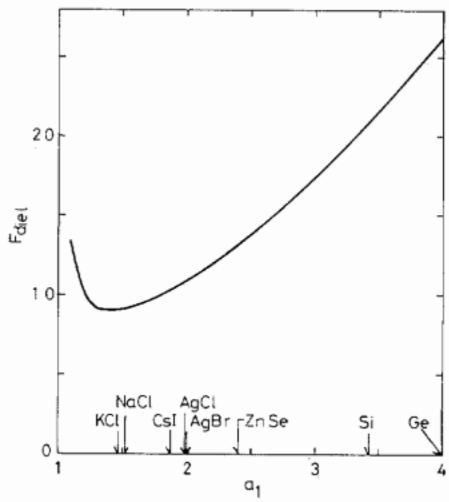


Fig. 6. F_{diel} as a function of the refractive index a_1 adjacent to a metal, where refractive indexes of several dielectrics at 10.6 μm are depicted by arrows.

although an electric length of each medium obtained numerically is a little bit different from analytical one. This fact indicates that the present method of loss reduction is not merely one of possible loss-reduction techniques but the approximate method for the exact minimization and also magnitudes of minimum attainable losses are rather insensitive to variations of thickness of each dielectric layer.

We next consider minimization of losses of the waveguide for actual dielectrics and metals. As α/α_H approximately decreases exponentially with m_p, to achieve low-loss waveguides it is necessary to make C small, i.e., to make a_2 much larger than a_1 and to reduce $F_{\min}^{(1)}$ of hybrid modes, where

$$F_{\min}^{(1)} = \frac{1}{2}\, F_{\text{metal}} \left[1 + \frac{a_1^2}{(a_1^2 - 1)^{1/2}}\right]^2$$

$$= \frac{1}{2}\, F_{\text{metal}} \cdot F_{\text{diel}}. \tag{60}$$

Fig. 5 shows F_{metal} of various metals as a function of the wavelength near 10.6 μm. (See also Table III). It can be seen that it is desirable to choose Ag or Cu as a metal.

Fig. 6 shows F_{diel} as a function of a_1. When a refractive index of the most adjacent dielectric to the metal is $2^{1/2} = 1.414$, the minimum loss is attained.

Fig. 7 shows $F_{\min}/F_{\text{metal}}$ for various combinations of dielectrics as a function of odd m. The right-hand scale corresponds to the loss of the HE_{11} mode in the case of

$$T = 500\ \mu\text{m},\quad \lambda = 10.6\ \mu\text{m},\quad n = 20.5,\quad \kappa = 58.6. \tag{61}$$

By considering the fact that the transmission loss of the corresponding aluminum waveguide is around 11.7 dB/m, one can see how a large amount of loss reduction can be made in the dielectric-coated metallic waveguide.

We finally mention that the maximum number of dielectric layers beyond which the basic assumption of the analysis, i.e., $|z_{TE}| << z_0$ and $|y_{TM}| << y_0$ is no longer satisfied. As seen from (49), (56), and Table II, $|z_{TE}|$ which is larger than $|z_{TE}{}^{(0)}|$ for a few dielectric layers increases with m or m_p, whereas $|y_{TM}|$ which is smaller than $|y_{TM}{}^{(0)}|$ decreases. In the metallic

TABLE III

COEFFICIENTS c_0, c_1, AND c_2 WHEN n, κ, AND F_{metal} ARE APPROXIMATED BY THE LEAST SQUARE CURVE, i.e., BY $c_0 + c_1\,(\lambda\text{-}10.6) + c_2\,(\lambda\text{-}10.6)^2$, WHERE λ IS THE WAVELENGTH IN MICROMETERS.

		c_0	c_1	c_2	Ref.
Ag	n	13.5	1.73	0.061	(24)
	κ	75.3	5.35	-0.29	
	F_{metal} $(\times 10^{-3})$	2.31	-0.052	0.025	
Al	n	20.5	0.94	0.10	(24)
	κ	58.6	1.84	0.70	
	F_{metal} $(\times 10^{-3})$	5.32	-0.11	-0.074	
Au	n	17.1	2.41	0.0021	(24)
	κ	55.9	4.29	-0.18	
	F_{metal} $(\times 10^{-3})$	5.02	-0.012	0.010	
Cr	n	11.8	1.01	0.011	(25)
	κ	25.5	1.68	-0.091	
	F_{metal} $(\times 10^{-3})$	15.0	-0.87	0.14	
Cu	n	14.1	2.12	0.093	(24)
	κ	64.3	4.80	-0.18	
	F_{metal} $(\times 10^{-3})$	3.27	-0.030	0.016	
Fe	n	9.63	1.16	0.0083	(25)
	κ	28.5	2.97	-0.021	
	F_{metal} $(\times 10^{-3})$	10.6	-1.04	0.14	
Ni	n	9.08	1.06	-0.0091	(25)
	κ	34.8	3.03	-0.047	
	F_{metal} $(\times 10^{-3})$	7.03	-0.44	0.034	
Sn	n	17.4	1.88	0.025	(24)
	κ	43.5	3.70	-0.063	
	F_{metal} $(\times 10^{-3})$	7.90	-0.56	0.078	
W	n	10.7	0.817	-0.029	(25)
	κ	31.0	2.40	0.075	
	F_{metal} $(\times 10^{-3})$	9.94	-0.74	-0.011	
Zn	n	15.8	1.88	0.068	(24)
	κ	48.7	4.11	0.23	
	F_{metal} $(\times 10^{-3})$	6.04	-0.32	-0.023	

waveguide it is necessary that

$$|y_{TM}{}^{(0)}| = (n^2 + \kappa^2)^{1/2} << y_0. \tag{62}$$

Therefore, for the dielectric-coated metallic waveguide, a maximum number m or m_p of dielectric layers could be decided by

$$(n^2 + \kappa^2)^{1/2} \geqslant |z_{TE}|. \tag{63}$$

Examples shown in Figs. 4 and 5 clearly satisfy (63).

V. Loss Increase in Dielectric-Coated Metallic Waveguides due to Absorptions of Dielectrics

Let the complex refractive indices of coated dielectric materials be $(a_i - ja_i')\, n_0$ $(i = 1, 2)$ and $a_i >> a_i'$. For simpli-

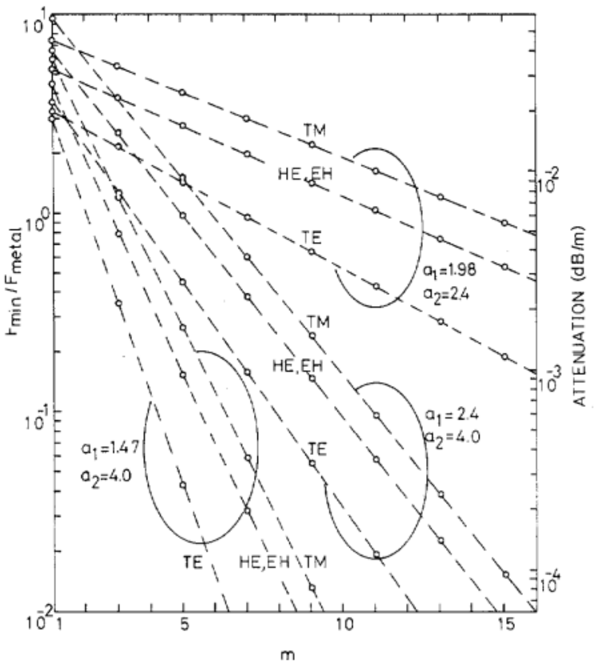


Fig. 7. F_{min}/F_{metal} denoted by small circles for various combinations of dielectrics as a function of odd m. Right-hand scale corresponds to the power loss of the HE_{11} mode for (61).

city, we treat the case $m = 2m_p + 1$. By replacing the transverse phase constant u_i/T, the normalized surface impedance z_i and admittance y_i as

$$\frac{u_i}{T} \to \frac{u_i}{T}\left(1 - j\,\frac{a_i\,a_i'}{a_i^2 - 1}\right) \tag{64}$$

$$z_i \to z_i\left(1 + j\,\frac{a_i\,a_i'}{a_i^2 - 1}\right) \tag{65}$$

$$y_i \to y_i\left(1 - j\,\frac{a_i^2 - 2}{a_i^2 - 1}\,\frac{a_i'}{a_i}\right) \tag{66}$$

and x as

$$x \to \delta\,(a_1^2 - 1)^{1/2}\,n_0 k_0 T\left(1 - j\,\frac{a_1\,a_1'}{a_1^2 - 1}\right) \tag{67}$$

in the transverse transmission line model [1] and after a lot of calculations which consider (38) and (48), we finally express Re (z_{TE}) and Re (y_{TM}) as follows:

$$\mathrm{Re}\,(z_{TE}) = F_{metal}\cdot C^{m_p}\left[1 + \frac{a_1^2}{(a_1^2 - 1)^{1/2}}\left(\frac{a_1}{a_2}\right)^{2m_p} C^{-m_p}\right]\cdot\left(1 + \frac{n^2 + \kappa^2}{n}\,\epsilon_{TE}\right) \tag{68}$$

$$\mathrm{Re}\,(y_{TM}) = F_{metal}\cdot\frac{a_1^2}{(a_1^2 - 1)^{1/2}}\left(\frac{a_1}{a_2}\right)^{2m_p}\cdot\left[1 + \frac{a_1^2}{(a_1^2 - 1)^{1/2}}\left(\frac{a_1}{a_2}\right)^{2m_p} C^{-m_p}\right]\cdot\left(1 + \frac{n^2 + \kappa^2}{n}\,\epsilon_{TM}\right) \tag{69}$$

where the excess loss factors ϵ_{TE} and ϵ_{TM} are defined by

$$\epsilon_{TE} = C^{-m_p}\,\frac{a_1\,a_1'}{a_1^2 - 1}\,n_0 k_0 \delta T + \frac{C^{-m_p} - 1}{C^{-1} - 1}\cdot\left[\frac{a_1\,a_1'}{(a_1^2 - 1)^{1/2}} + \frac{a_2\,a_2'}{(a_2^2 - 1)^{1/2}}\right]\frac{1}{a_1^2 - 1}\cdot\frac{\pi}{2} \tag{70}$$

$$\epsilon_{TM} = D^{-m_p}\,\frac{a_1'}{a_1}\,n_0 k_0 \delta T + \frac{D^{-m_p} - 1}{D^{-1} - 1}\cdot\left[\frac{1}{(a_1^2 - 1)^{1/2}}\,\frac{a_1'}{a_1} + \frac{1}{(a_2^2 - 1)^{1/2}}\,\frac{a_2'}{a_2}\,D^{-1}\right]\frac{\pi}{2} \tag{71}$$

$$D = \left(\frac{a_1}{a_2}\right)^4 \frac{a_2^2 - 1}{a_1^2 - 1}. \tag{72}$$

In deriving (69), it is assumed that $\kappa >> n$ and therefore we have approximated $\kappa^2 - n^2$ by $\kappa^2 + n^2$ in the multiplier of ϵ_{TM} to simplify further calculations for the hybrid modes. When ϵ_{TE} or ϵ_{TM} is equal to $n/(n^2 + \kappa^2)$, losses of the TE_{0q} or TM_{0q} modes in the case of no dielectric absorptions are doubled.

For the hybrid modes, the factor F is expressed by

$$F = F_{min}\left(1 + \frac{n^2 + \kappa^2}{n}\,\epsilon\right) \tag{73}$$

where

$$\epsilon = \frac{\left[\epsilon_{TE} + \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}}\left(\dfrac{a_1}{a_2}\right)^{2m_p} C^{-m_p}\,\epsilon_{TM}\right]}{\left[1 + \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}}\left(\dfrac{a_1}{a_2}\right)^{2m_p} C^{-m_p}\right]} \tag{74}$$

which shows that ϵ is between ϵ_{TE} and ϵ_{TM}. Therefore, the condition which doubles the minimum attainable losses is given by

$$\epsilon = \frac{n}{n^2 + \kappa^2}. \tag{75}$$

For the special case of $m_p = 0$, the absorption coefficient a_1' to satisfy (75) is

$$a_1' = \frac{n}{n^2 + \kappa^2}\,\frac{a_1^2 + (a_1^2 - 1)^{1/2}}{1 + (a_1^2 - 1)^{1/2}}\cdot\frac{(a_1^2 - 1)^{1/2}}{a_1}\,\frac{1}{n_0 k_0 \delta T}. \tag{76}$$

By using (48) with $s = 0$, one can express

$$a_1' = \frac{n}{n^2 + \kappa^2}\,\frac{a_1^2 + (a_1^2 - 1)^{1/2}}{1 + (a_1^2 - 1)^{1/2}}\cdot\frac{(a_1^2 - 1)}{a_1}\,\frac{1}{\tan^{-1}\,[a_1/(a_1^2 - 1)^{1/4}]} \tag{77}$$

to obtain

$$a_1' = \begin{cases} 6.8 \times 10^{-3}, & a_1 = 1.47\ (\mathrm{KCl}) \\ 2.6 \times 10^{-2}, & a_1 = 2.4\ (\mathrm{ZnSe}) \\ 7.3 \times 10^{-2}, & a_1 = 4.0\ (\mathrm{Ge}) \end{cases} \tag{78}$$

for $n = 20.5$ and $\kappa = 58.6$. Equation (78) shows that very lossy dielectrics can be permitted as a coating material.

TABLE IV
ABSORPTION COEFFICIENT a' WHICH DOUBLES THE MINIMUM ATTAINABLE POWER LOSSES OF THE HYBRID MODES FOR $n = 20.5$ AND $\kappa = 58.6$ (Al)

NUMBER OF DIELECTRIC LAYERS	WAVEGUIDE STRUCTURE		
	$a_1=1.47$ $a_2=4.0$	$a_1=2.4$ $a_2=4.0$	$a_1=1.98$ $a_2=2.4$
5	7.4×10^{-5}	1.1×10^{-3}	9.1×10^{-4}
7	1.1×10^{-5}	4.3×10^{-4}	6.5×10^{-4}
11	2.2×10^{-7}	6.2×10^{-5}	3.3×10^{-4}

For the waveguide with multilayered dielectrics, to see a rough estimation of the effect of material absorptions, one approximates $\delta n_0 k_0 T$ of (48) by

$$\delta n_0 k_0 T = \frac{1}{(a_1^2 - 1)^{1/2}} \frac{\pi}{2} \tag{79}$$

then

$$\epsilon_{\mathrm{TE}} = C^{-m_p} \left\{ \frac{a_1 a_1'}{(a_1^2 - 1)^{3/2}} + \frac{1}{a_2^2 - a_1^2} \cdot \left[\frac{a_1 a_1'}{(a_1^2 - 1)^{1/2}} + \frac{a_2 a_2'}{(a_2^2 - 1)^{1/2}} \right] \right\} \frac{\pi}{2} \tag{80}$$

$$\epsilon_{\mathrm{TM}} = D^{-m_p} (1 - D)^{-1} \cdot \left[\frac{a_1'}{a_1 (a_1^2 - 1)^{1/2}} + \frac{a_2'}{a_2 (a_2^2 - 1)^{1/2}} \right] \frac{\pi}{2} \tag{81}$$

where we have used $C^{m_p} \ll 1$ in (80) to overestimate the loss increase.

Letting $a_1' = a_2' = a'$ for simplicity, the absorption coefficient a' which doubles the minimum attainable power losses of the hybrid modes is listed in Table IV for typical combinations of dielectrics. From the viewpoint of art of purification of dielectric materials [1] and a fabrication technique [20], it could be possible to fabricate waveguides with small power losses without being affected by absorptions of dielectric materials too much.

VI. Loss Increase in Dielectric-Coated Metallic Waveguides due to Thickness Changes of Dielectric Layers

From the viewpoint of actual fabrication techniques, it is desirable to design a waveguide relatively insensitive to variations of thickness of each dielectric layer. In the discussion of Section III, when the parameter c of (34) for the dielectric-coated waveguide

$$c = \begin{cases} \dfrac{a_1^2}{(a_1^2 - 1)^{1/2}} \left(\dfrac{a_1}{a_2}\right)^{2m_p} C^{-m_p}, & m = 2m_p + 1 \\[2ex] \dfrac{a_1^2 - 1}{a_1^2 (a_2^2 - 1)^{1/2}} \left(\dfrac{a_2}{a_1}\right)^{2m_p} C^{m_p}, & m = 2m_p + 2 \end{cases} \tag{82}$$

becomes small, i.e., especially for $m = 2m_p + 2$, F exhibits a large asymmetry as a function of x, the electric length of dielectric layer adjacent to air. Therefore, it is preferable to weaken the asymmetry as far as possible.

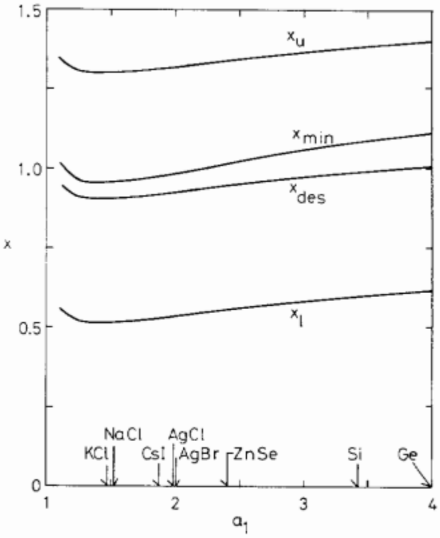


Fig. 8. Various electric lengths of the dielectric layer as a function of a_1 in the metallic waveguide with a single dielectric layer.

Let electric lengths of the medium adjacent to air be x_l and x_u which make, say, twice the minimum attainable power losses and the electric length x_{des} be

$$x_{\mathrm{des}} = (x_l + x_u)/2. \tag{83}$$

We temporarily call the waveguide an optimized one with the electric length x_{des} for the dielectric adjacent to air and (38) for the other dielectrics, which are expressed symbolically by

$$x_{\mathrm{des}}^{(i)}, \quad i = 1, 2, \cdots, m. \tag{84}$$

Fig. 8 shows x_{min}, x_{des}, x_l, and x_u for the waveguide with a single dielectric layer. A significant difference is not found between x_{min} and x_{des} in this case. One should note that ±40-percent errors of the electric length or thickness of the layer can be permissible within which the power losses of the waveguide cannot be doubled.

In order to see the loss increase in the waveguide with multilayered dielectrics, computer simulations are carried out by changing the electric length x_i of the ith layer by

$$x_i = x_{\mathrm{des}}^{(i)} (1 + A r_i), \qquad i = 1, 2, \cdots, m \tag{85}$$

where A is an amplitude to indicate the measure of variations and three types of variables are assigned for r_i as follows:

1) random variables uniform in $[-1, 1]$,
2) random variables with -1 or 1, and
3) definite values with -1 and 1, respectively.

The factor F's of dielectric-coated metallic waveguides are numerically computed for cases 1) and 2). It turns out that increments of power loss from the minimum attainable one are very small, although they tend to increase with the number of layers in the presence of random variations of thickness of dielectric layer. Let us show an example for the aluminum waveguide coated by ten layers of dielectrics with $a_1 = 2.4$ and $a_2 = 4.0$. If the randomness of thickness change A is within 10 percent, waveguides can be realized with the loss increment factor F/F_{min} less than 1.3 for the case 1) and 1.7 for the case 2) with probability of 90 percent.

The most strict case is case 3) to evaluate loss increase due to

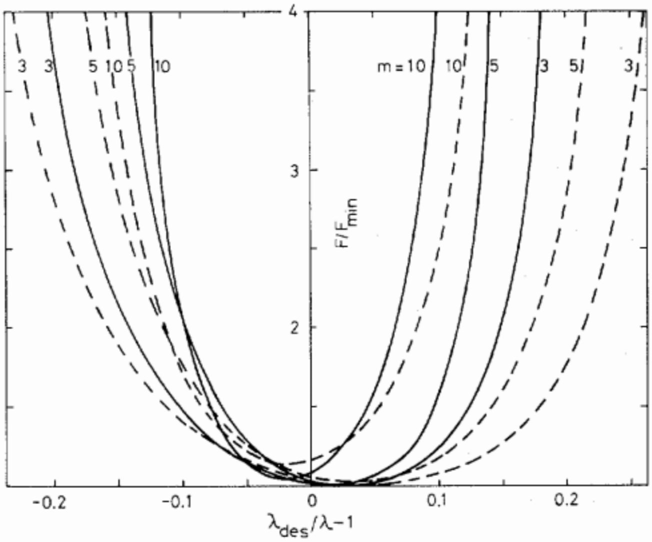


Fig. 9. Loss increase factor $F/F_{\min}$ as a function of λ_{des}/λ-1 or A in case 3), where solid and dashed lines correspond to a waveguide with a_1 = 2.4 (ZnSe) and a_2 = 4.0 (Ge) and one with a_1 = 1.47 (KCl) and a_2 = 4.0 (Ge), respectively, for n = 20.5 and κ = 58.6 (Al).

thickness changes. The uniform change of each dielectric layer is also equivalent to the change of the wavelength. When the thickness of each layer is optimized, parameter A is given by

$$A = \lambda_{des}/\lambda - 1 \tag{86}$$

where λ_{des} is the wavelength, say 10.6 μm, corresponding to (83) and at which power loss is approximately minimized and λ is the operating wavelength. Fig. 9 shows $F/F_{\min}$ in this case 3) by changing A or $\lambda_{des}/\lambda - 1$. It is shown that loss increase in the waveguide with $a_1 = 1.47$ and $a_2 = 4.0$, which has smaller power loss, is rather insensitive compared with that in the waveguide with $a_1 = 2.4$ and $a_2 = 4$. Fig. 9 also shows that if the thickness change of each dielectric layer or the operating wavelength is controlled within ±10 percent from the designed one, the loss increase in the waveguide is not too much even if ten layers of dielectrics are coated.

In closing this section, we add some comments on bending losses of the dielectric-coated metallic waveguides. To evaluate the bending losses of metal or dielectric hollow-core waveguides, the famous formula given by Marcatili and Schmeltzer [8] has been used in optical through submillimeter wavelengths [2], [8], [26]. After the first question on the validity of the bending loss formula was raised in [27], recent studies [4]-[6] show that the formula is wrong and cannot be properly applied to evaluation of the bending losses in circular hollow waveguides. Therefore, we shall discuss the curvature losses of the dielectric-coated metallic waveguides in more detail elsewhere based on the formula presented in [6], which is the only formula available at present to the hollow waveguides in infrared wavelengths where the media are characterized by the complex refractive index $n_0(n - j\kappa)$ without restriction on n and κ. One should also note that fabricated germanium-coated nickel waveguides can be bent more sharply than the ordinary nickel waveguides [20].

Finally, we would like to give a comment on Bragg fibers [28], [29]. To realize low-loss waveguides with a finite number of dielectric layers, dielectric-coated metallic waveguides seem to be preferable because large reflection is expected at the surface of a metal.

VII. Conclusion

Transmission characteristics of circular metallic waveguides with multilayered dielectrics are discussed for the infrared. Minimum attainable transmission losses are analytically deduced for the hybrid modes including the HE_{11} mode in the waveguide. The effects of absorptions and variations of thickness of dielectric layers are discussed and are shown to be small on the minimum attainable power losses.

Curvature losses of the dielectric-coated waveguide will be reported elsewhere [30].

Acknowledgment

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Role of the Fusion Splice in the Concatenation Problem

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Abstract—**Near-field measurements used to derive an empirical model of a fusion splice have indicated that fusion splices generate very little mode coupling between multimode graded-index fibers of identical parameters. Results of dispersion measurements confirm this. In particular it is found that the bandwidth of a concatenated link of identical fibers (respliced fibers from the same preform) is determined by longitudinal profile variations and not splice characteristics. Fusion splices between nonidentical fibers can generate a larger degree of mode coupling. This unequal parameter-induced mode coupling has a measurable effect on the dispersion properties of concatenated fiber links.**

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I. Introduction

THE basic working principle of the fusion splice was well described by Kohanzadeh in his pioneering work on the subject: "The essential idea behind hot splices is simple and basic to glass: heat the fibers and fuse them together" [1].